NUMB3RS Activity: Restaurant Ranking
Episode: "Take Out"

Topic: Probability and directed graphs **Grade Level:** 10 - 12
Objective: Students will analyze a graph network and determine which vertices are the most important.
Time: 20 minutes

Introduction

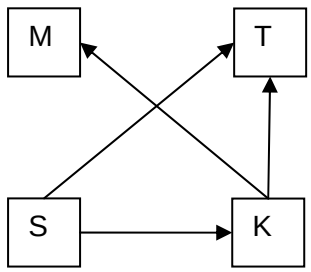
After a series of restaurant invasions turns deadly, the FBI asks Charlie to help find the next target. Charlie develops a restaurant predictive model by applying "a mixture of multi-dimensional graph Laplacians and page-rank matrices." He goes on to say, "It's a way of investigating a group of things—in this case, restaurants—and identifying their shared properties. Think of a refrigerator filled with groceries. Steaks, apples, wine and a pie. Each item is unique, and you can buy them separately at any number of places. But more likely, the family made a single shopping trip. Basically, I figured out where the robbers went shopping for their targets. They're using the *L.A. Restaurant Guide* for their list of targets. They're looking for diners with the fattest credit cards." In this activity, students will use directed graphs and probabilities to find which restaurant is most likely to be struck next by the criminals.

To do this, Charlie must determine which restaurants are most important to the killers and identify the relationship between restaurants. For this activity, we will focus on restaurant referrals (e.g., "if you like this restaurant, you will like these others as well").

Discuss with Students

For this exercise, a graph does not mean a collection of coordinates plotted on *x*- and *y*-axes. The graphs in this activity are represented by a collection of vertices connected with edges that show relationships. The vertices represent people or restaurants, depending on the example. To illustrate this, use the following example.

Suppose Kevin is younger than Michelle and Tom, and Susan is younger than Kevin and Tom. To illustrate this age network, the following directed graph could be drawn.



Directed graphs contain arrows showing the direction of movement. In this case, they are drawn from the younger person to the older person. To reinforce this concept, students could be put in groups of four and create a similar age-relationship graph with their group partners.

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Student Page Answers:

1. *Restaurant B refers patrons to restaurants D and C, while restaurant A refers no patrons*. **2.** $\frac{1}{4}$

3. *R(B) = 0* **4.** *R(C)* = $\frac{1}{2}$ × *P(B) = 0.125*

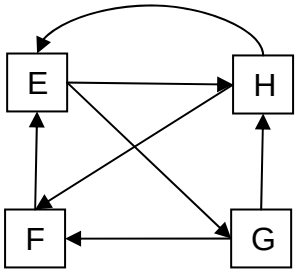
5.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0.1 & 0.067 \\ 0.1 & 0 & 0 & 0.1 & 0.067 \\ 0.1 & 0.1 & 0 & 0 & 0.067 \\ 0 & 0.1 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.167 \\ 0.267 \\ 0.267 \\ 0.3 \\ 0.0 \end{bmatrix}$$

Don should check restaurant D. However, because the probability differences are not very large, restaurants B and C should be checked as well.

6.

$$\begin{array}{c} \\ E \\ F \\ G \\ H \end{array}\begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 0.25 & 0 & 0.125 \\ 0 & 0 & 0.125 & 0.125 \\ 0.125 & 0 & 0 & 0 \\ 0.125 & 0 & 0.125 & 0 \end{bmatrix} \end{array} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.25 \\ 0.125 \\ 0.25 \end{bmatrix}$$



Don should check restaurant A.

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Name: _____ Date: _____

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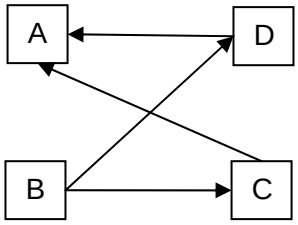
To do this, Charlie must determine which restaurants are most important to the killers and identify the relationship between restaurants. For this activity, we will focus on restaurant referrals (e.g., "if you like this restaurant, you will like these others as well").

The graph below shows four restaurants where the direction of the referrals is indicated by an arrow.

1. In the graph to the right, what restaurants are referred by restaurant B, and what restaurants are referred by restaurant A?



2. What is the probability of choosing one of these restaurants randomly, noted $P(x)$, if each choice is equally likely?

However, these restaurants refer customers to other restaurants. Therefore, the criminals would not really choose a random restaurant, but rather a restaurant that is more popular than the others (i.e., has the highest referral rating or ranking). To determine the restaurant rank for restaurant A, noted $R(A)$, add all the probabilities of the restaurants that are referred to it. Since restaurant A receives 0.25 from restaurant D and 0.25 from restaurant C, its $R(A) = 0.5$. (This is not to be confused with $P(A)$, which is the probability of randomly choosing restaurant A.)

3. What is the restaurant rank of restaurant B?

Notice that restaurant B splits its referrals between restaurants D and C. Consequently, it only passes $\frac{1}{2}$ of its probability of referring patrons on to each restaurant for their rankings. Therefore, $R(D) = \frac{1}{2} \times P(B) = 0.125$.

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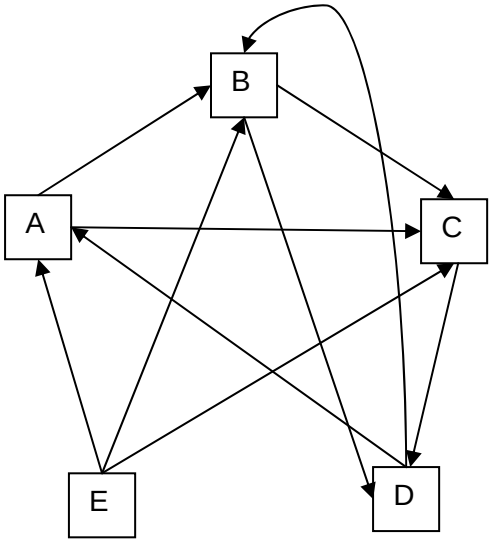
4. What is the restaurant rank for restaurant C?

To find the restaurant rankings for all the restaurants, the information can be translated into a matrix and multiplied, as shown below.

$$\begin{array}{l} \\ \text{Restaurant Referred} \end{array} \begin{array}{c} \text{Referring Restaurant} \\ \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \end{array}$$

$$\begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{bmatrix} 0 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 \\ 0 & 0.125 & 0 & 0 \\ 0 & 0.125 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0.125 \\ 0.125 \end{bmatrix}$$

5. Find the restaurant ranking for each restaurant in the following graph. Use matrix representation and multiplication to find the rankings. Then determine where Charlie should tell Don to look for the criminals.



6. In another scenario, suppose Charlie determines that:

Restaurant E refers customers to restaurants G and H
Restaurant F refers customers to restaurant E
Restaurant G refers customers to restaurants F and H
Restaurant H refers customers to restaurants E and F

Draw a graph for this situation and use matrix multiplication to calculate the restaurant rankings. Then determine the restaurant where Charlie should tell Don to look for the criminals.

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

- When a number is squared, it either becomes very large or close to zero. What happens with matrices? Explore what happens to the restaurant referral matrix when it is squared.

- Sometimes a restaurant would like to know if patrons it refers to other restaurants are ever referred back. This can be seen by looking at the main diagonal of the restaurant referral matrix after it is squared. The numbers along the main diagonal of the matrix indicate how many referrals come back to the restaurant. Square the restaurant referral matrix in question 5 and see if the diagonal of the new matrix matches what the graph tells you.

Additional Resources

- For a humorous look at Google's page ranking system, view the Web page **http://www.google.com/technology/pigeonrank.html**.

- To view Google's true page ranking system, view the Web page **http://www.google.com/technology/index.html**.

- To view how internet search engines work, view the Web page **http://infolab.stanford.edu/~backrub/google.html**.

- To view a mathematical paper on page rankings and their graphs, view the Web page **http://infolab.stanford.edu/~ullman/mining/websearch.pdf**.

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