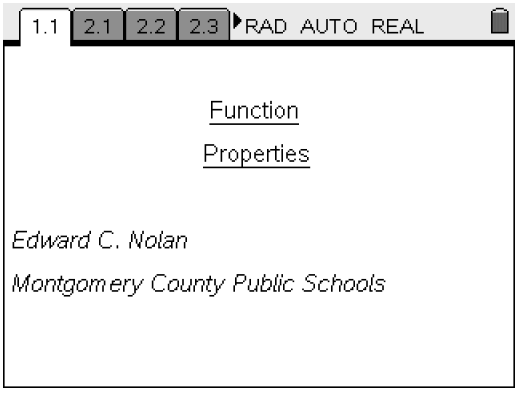
TI-Nspire Activity: Function Properties Activity
By: Edward C. Nolan



Activity Overview

In this activity, students explore the meaning of relations and functions. In addition, they investigate different representations for relations and functions. They examine how to determine whether a relation is a function and investigate many different properties of functions.

Concepts

- *Relations and mappings*
- *Representations of relations and mappings*
- *Functions*
- *Domain and range*
- *Continuity and discrete*
- *Maximum and minimum*
- *Intercepts*
- *Increasing and decreasing*

Teacher Preparation

- *This activity is designed for students to explore the definition and properties of functions. It can be used to introduce students to the topics or as a review prior to summative assessment.*

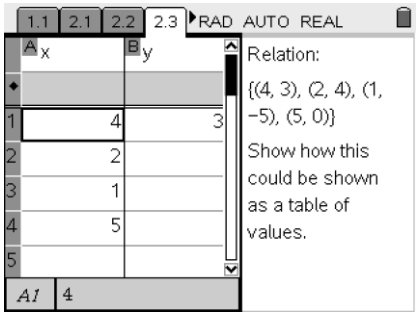
The Classroom.

- *This activity is designed to be done alternating between small group and whole class discussion. Students work on their own and supported by the teacher as they work. There may be occasions where the teacher regains the attention of the class to share/gain observations from the class.*

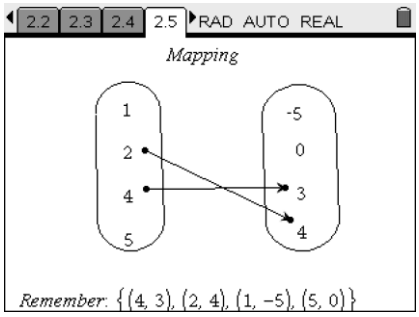
The Lesson

Stage 1: Learning about Relations and Functions

Students start the activity by investigating relations and mappings. The student is given the definition of a relation and asked to change the representation of a relation from ordered pairs to a table.

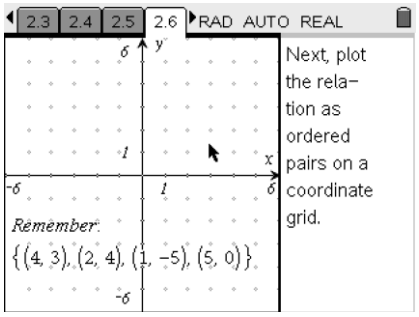


Next, students are asked to change the representation of the relation to a mapping and to a graph.



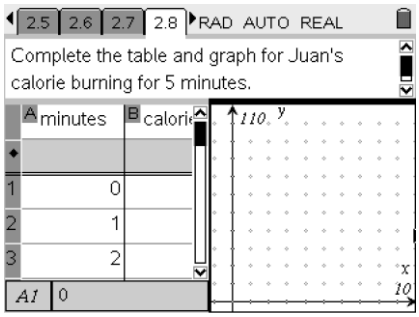
Emphasis should be made by the teacher that when elements are repeated in the domain or the range, they are not repeated in the mapping.

Also, be sure that students are plotting the ordered pairs correctly on the graph, with the first coordinate plotted on the x-axis, the second on the y-axis.

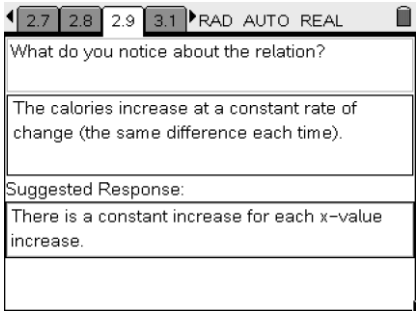


Students then apply the concept of relation to a real world situation.

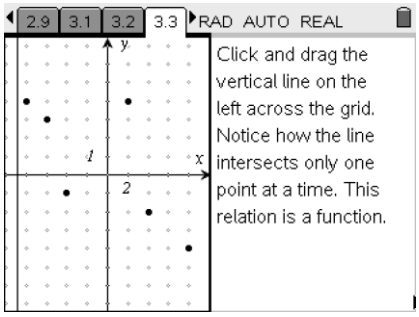
Students then look for a pattern in the real world situation, which they can answer either on the calculator or on the worksheet, depending upon how the teacher wishes to collect student understanding.



Connections can be made to the representation of the constant rate of change and how that appears on the graph (connecting to linear functions).



The students next connect to the definition of functions as a subset of relations.



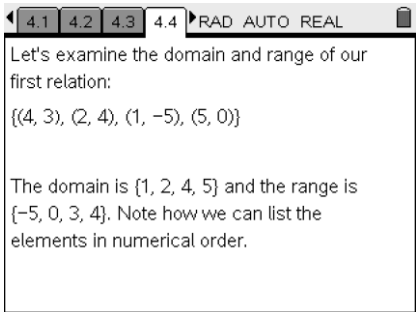
Students may need support in clicking, grabbing, and dragging the vertical line across the graph to test whether the relation is a function.

There is a distinction made between the definition of function and the characteristic of functions using the vertical line test.

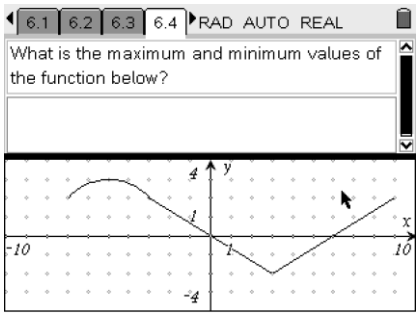
Students then apply the definition of function to the earlier real world situation.

Stage 2: Properties of Functions

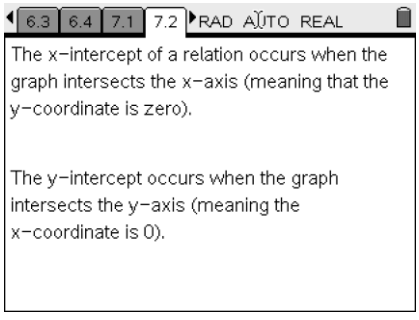
The students begin their study of properties with domain and range. Remember to focus on having the students use correct set notation (using brackets around the sets), placing the elements in numerical order, and not repeating elements.



Next, students learn about continuity and maximum and minimum. Students are asked to identify maxima and minima on relations in both ordered pair and graph form.

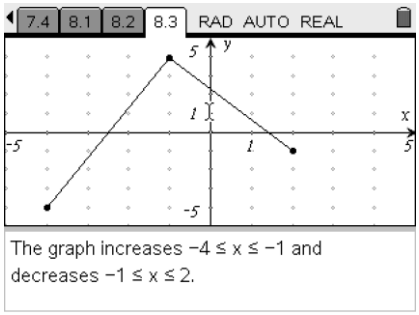


The students then are presented the definitions for intercepts and interpret them from a graph. The teacher can help reinforce with students that x = 0 is the location of the y-intercept and that y = 0 is the x-intercept.



Students conclude their study of the properties of functions with determining the intervals over which the function is increasing or decreasing.

The teacher will need to work with the class to have the students understand interval notation. Having the students shade back to the x-axis can be an effective re-teaching strategy for determining increasing and decreasing.



Analysis/Extension

The teacher can choose to have additional examples of graphs for students to identify function properties. Examples like the second page of the exit card could be created by the teacher to allow for differentiation in the classroom.

Assessment

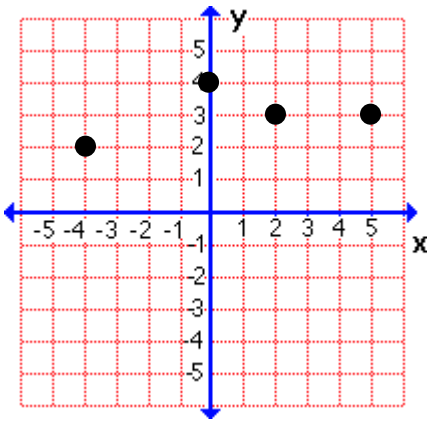
Give each student the Exit Card to complete.

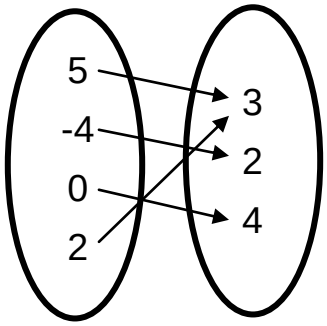
Exit Card Answer Key

Express as a table, a graph, and a mapping.

{ (5, 3), (-4, 2), (0, 4), (2, 3) }

x	y
5	3
-4	2
0	4
2	3



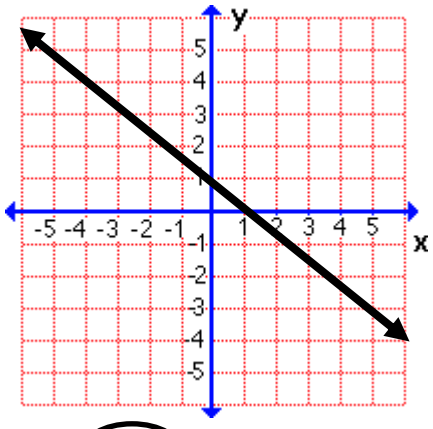


What is the domain and range of this relation?

Domain: { -4, 0, 2, 5 }

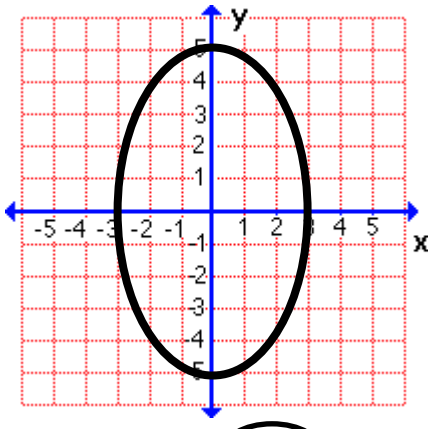
Range: { 2, 3, 4 }

Determine whether each graph represents a function. Explain how you know.



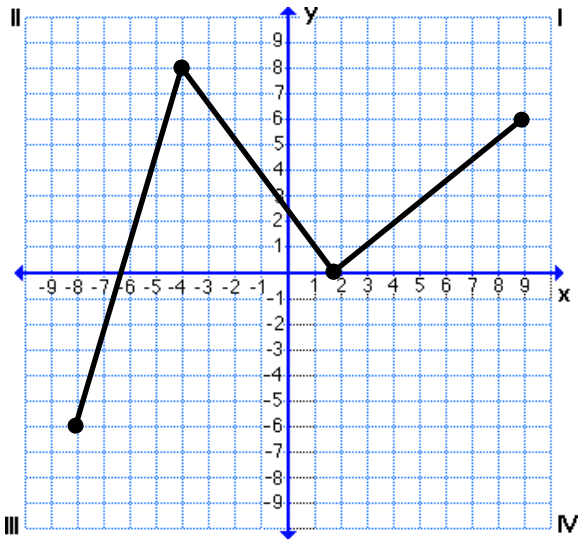
(YES) NO

It is a function because for each x-value there is one and only one y-value.



YES (NO)

It is not a function because for some values of x (like 2), there are two values of y (fails vertical line test).



Domain: -8 ≤ x ≤ 9

Range: -6 ≤ y ≤ 8

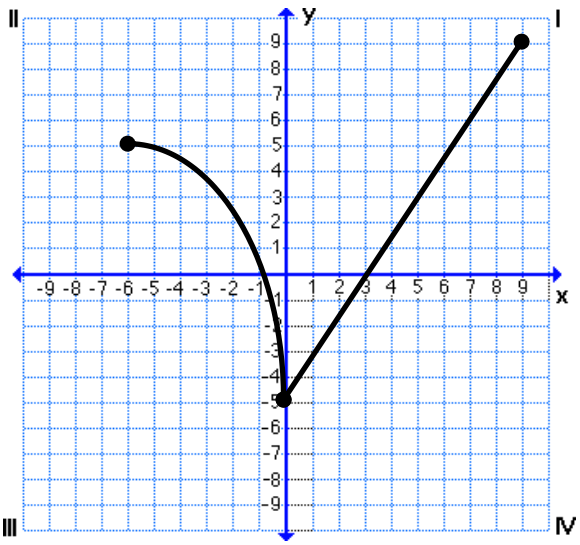
Continuous? (Yes) No

Maximum: 8

Minimum: -6

Increasing: -8 < x < -4 and 2 < x < 9

Decreasing: -4 < x < 2



Domain: -6 ≤ x ≤ 9

Range: -5 ≤ y ≤ 9

Continuous? (Yes) No

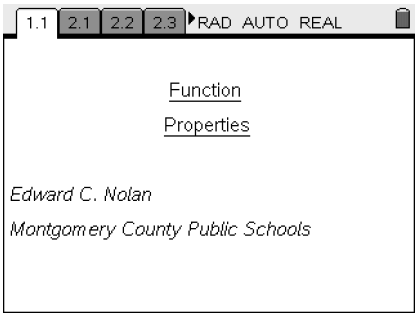
Maximum: 9

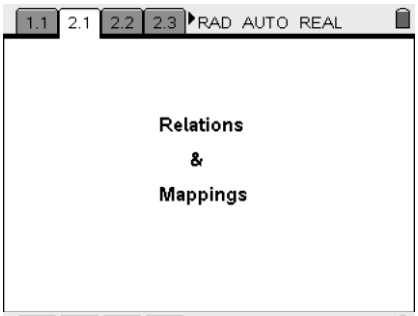
Minimum: -5

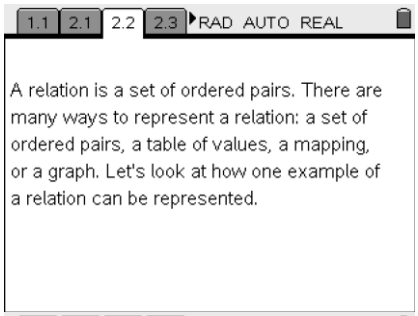
Increasing: 0 < x < 9

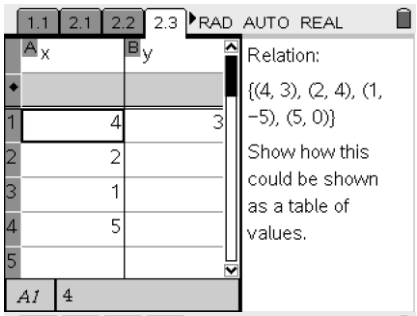
Decreasing: -8 < x < 0

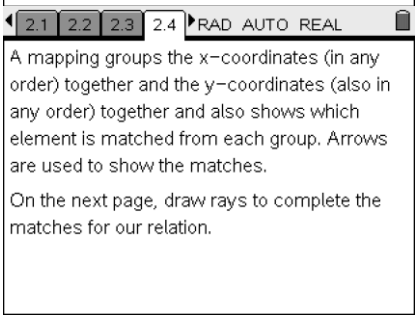
Pages of the Activity

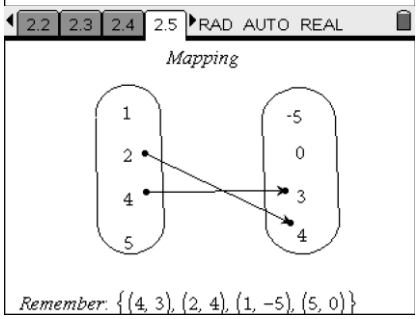


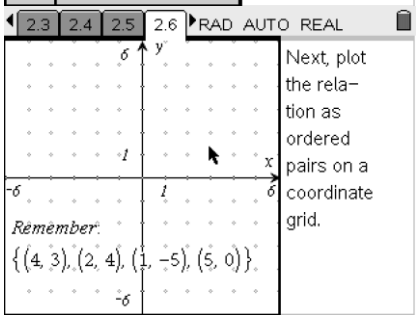












2.4 2.5 2.6 2.7 RAD AUTO REAL

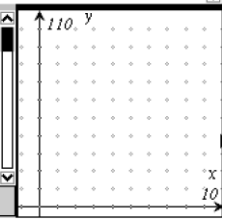
Relations also can be used to represent real world situations. Juan burns 20 calories for every minute he swims. Let x be the number of minutes he swims and y be the number of calories he burns.

2.5 2.6 2.7 2.8 RAD AUTO REAL

Complete the table and graph for Juan's calorie burning for 5 minutes.

	A minutes	B calori
1	0	
2	1	
3	2	

A1 0



2.6 2.7 2.8 2.9 RAD AUTO REAL

What do you notice about the relation?

2.7 2.8 2.9 3.1 RAD AUTO REAL

What do you notice about the relation?

The calories increase at a constant rate of change (the same difference each time).

Suggested Response:

There is a constant increase for each x−value increase.

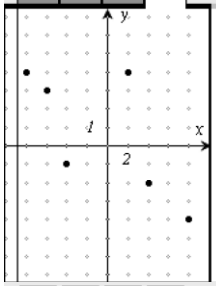
2.7 2.8 2.9 3.1 RAD AUTO REAL

Functions

2.8 2.9 3.1 3.2 RAD AUTO REAL

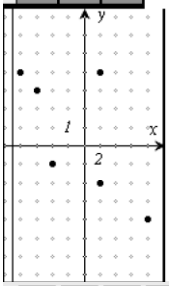
A function is a type of relation where each x−coordinate (sometimes called the 'input') has one and only one y−coordinate (sometimes called the 'output'). One way to check whether a relation is a function is the 'vertical line test.'

2.9 3.1 3.2 3.3 RAD AUTO REAL



Click and drag the vertical line on the left across the grid. Notice how the line intersects only one point at a time. This relation is a function.

3.1 3.2 3.3 3.4 RAD AUTO REAL



Click and drag the vertical line across the grid. Notice what happens just to the right of the y−axis. The line hits two points at the same time. This relation is not a function.

3.2 3.3 3.4 3.5 RAD AUTO REAL

The vertical line test of a function demonstrates when each x−value has no more than one y−value. If the line intersects with more than one point, then the relation is not a function.

3.3 3.4 3.5 3.6 RAD AUTO REAL

Remember, it is important to remember the difference between the definition of a function and a characteristic of a function.

The definition of a function is that each x−value has one and only one y−value.

A characteristic of a function is that it passes the vertical line test.

3.4 3.5 3.6 3.7 RAD AUTO REAL

Is Juan's burning of calories while swimming a function? How do you know?

3.4 3.5 3.6 3.7 RAD AUTO REAL

Is Juan's burning of calories while swimming a function? How do you know?

It is a function because each number of minutes he swims has only one calorie count.

Suggested Response:

Juan's swimming is a function, because each x−value has one and only one y−value.

3.5 3.6 3.7 4.1 RAD AUTO REAL

Domain

&

Range

3.6 3.7 4.1 4.2 RAD AUTO REAL

Let's talk about a couple of terms that deal with some number sets we are already using. These terms are domain and range.

Domain: the set of x−coordinates

Range: the set of y−coordinates

3.7 4.1 4.2 4.3 RAD AUTO REAL

Let's look back at our first relation:
{(4, 3), (2, 4), (1, −5), (5, 0)}
Is it a function?

3.7 4.1 4.2 4.3 RAD AUTO REAL

Let's look back at our first relation:
{(4, 3), (2, 4), (1, −5), (5, 0)}
Is it a function?

Yes, because each x−coordinate has only one corresponding y−coordinate.

Suggested Response:

Yes, it is a function, because each x−coordinate has one and only one y−value.

4.1 4.2 4.3 4.4 RAD AUTO REAL

Let's examine the domain and range of our first relation:

{(4, 3), (2, 4), (1, −5), (5, 0)}

The domain is {1, 2, 4, 5} and the range is {−5, 0, 3, 4}. Note how we can list the elements in numerical order.

4.2 4.3 4.4 4.5 RAD AUTO REAL

What is the domain of {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)}?

4.2 4.3 4.4 4.5 RAD AUTO REAL

What is the domain of {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)}?

{2, 3, 1, −1, 0}

Suggested Response:

The domain is {−1, 0, 1, 2, 3}.

4.3 4.4 4.5 4.6 RAD AUTO REAL

What is the range of {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)}?

4.3 4.4 4.5 4.6 RAD AUTO REAL

What is the range of {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)}?

{4, 9, 1, 0}

Suggested Response:

The range is {0, 1, 4, 9}. Note we do not repeat the same element when it appears in the range.

4.4 4.5 4.6 4.7 RAD AUTO REAL

Is the relation {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)} a function?

4.4 4.5 4.6 4.7 RAD AUTO REAL

Is the relation {(2, 4), (3, 9), (1, 1), (−1, 1), (0, 0)} a function?

Yes, each x−value has one and only one y−value

Suggested Response:

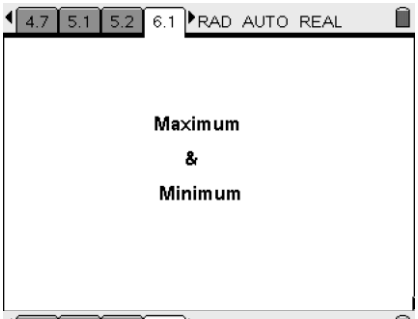
Yes, it is a function. Each x−value has only one y−value (it does not matter how many x−values each y−values has).

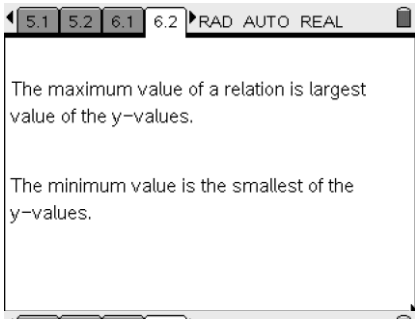
4.5 4.6 4.7 5.1 RAD AUTO REAL

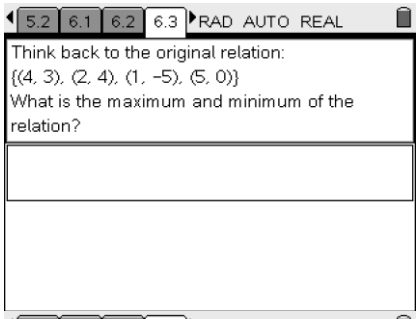
Continuity

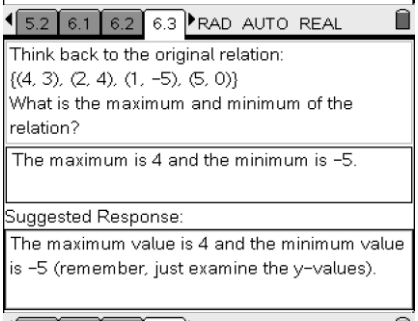
4.6 4.7 5.1 5.2 RAD AUTO REAL

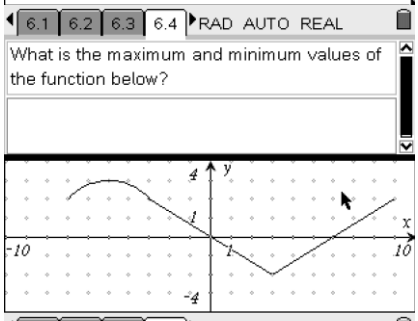
A relation is continuous when it is represented as a line with no gaps, no jumps, and no holes. A characteristic of continuity is being able to draw the relation with a pencil without lifting the pencil. The opposite of continuity is when the relation is discrete, having distinct parts. Relations made of just points is always discrete.

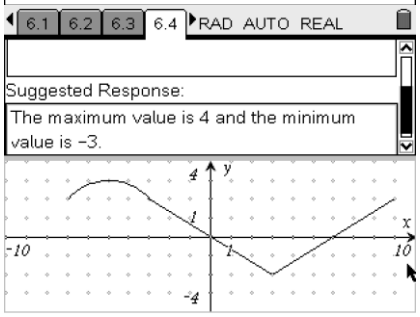

4.7 5.1 5.2 6.1 RAD AUTO REAL
Maximum
&
Minimum

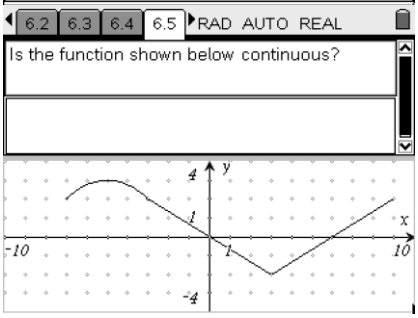

5.1 5.2 6.1 6.2 RAD AUTO REAL
The maximum value of a relation is largest value of the y−values.
The minimum value is the smallest of the y−values.

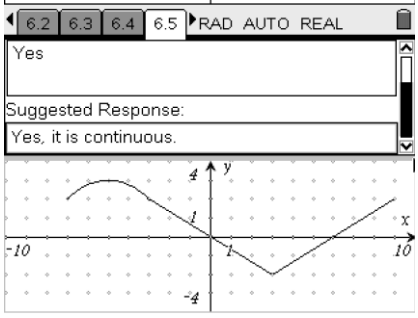

5.2 6.1 6.2 6.3 RAD AUTO REAL
Think back to the original relation:
{(4, 3), (2, 4), (1, −5), (5, 0)}
What is the maximum and minimum of the relation?

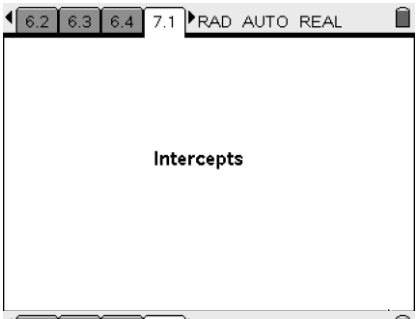

5.2 6.1 6.2 6.3 RAD AUTO REAL
Think back to the original relation:
{(4, 3), (2, 4), (1, −5), (5, 0)}
What is the maximum and minimum of the relation?
The maximum is 4 and the minimum is −5.
Suggested Response:
The maximum value is 4 and the minimum value is −5 (remember, just examine the y−values).

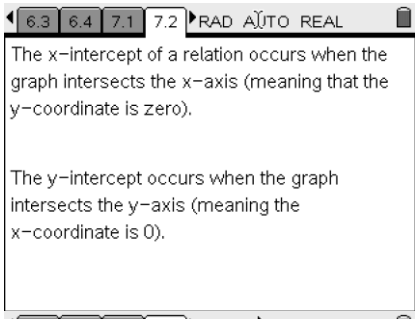

6.1 6.2 6.3 6.4 RAD AUTO REAL
What is the maximum and minimum values of the function below?

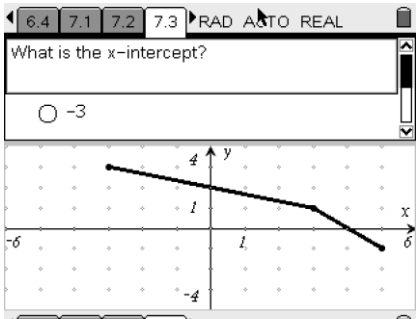

6.1 6.2 6.3 6.4 RAD AUTO REAL
Suggested Response:
The maximum value is 4 and the minimum value is −3.

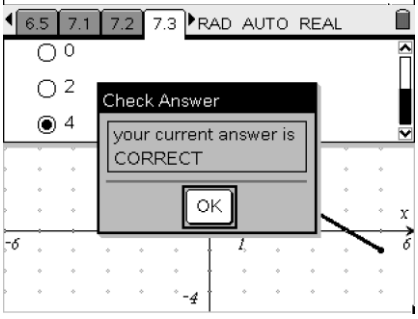

6.2 6.3 6.4 6.5 RAD AUTO REAL
Is the function shown below continuous?

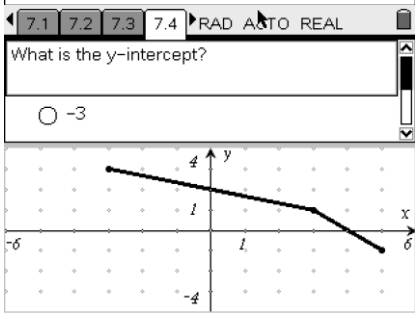

6.2 6.3 6.4 6.5 RAD AUTO REAL
Yes
Suggested Response:
Yes, it is continuous.

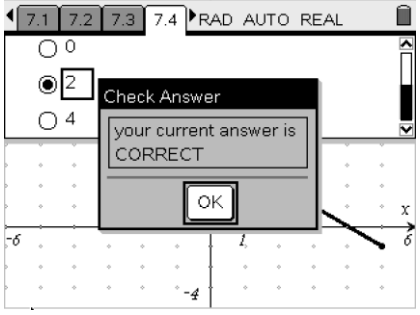

6.2 6.3 6.4 7.1 RAD AUTO REAL
Intercepts

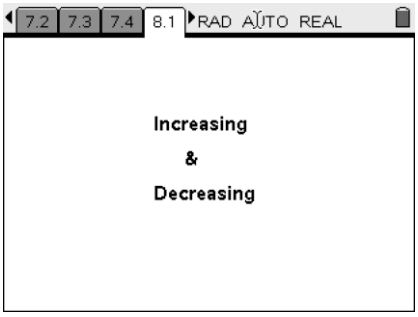

6.3 6.4 7.1 7.2 RAD AUTO REAL
The x−intercept of a relation occurs when the graph intersects the x−axis (meaning that the y−coordinate is zero).
The y−intercept occurs when the graph intersects the y−axis (meaning the x−coordinate is 0).

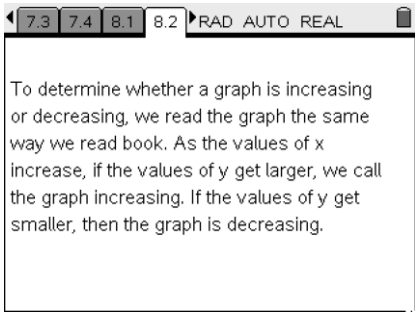

6.4 7.1 7.2 7.3 RAD AUTO REAL
What is the x−intercept?
−3

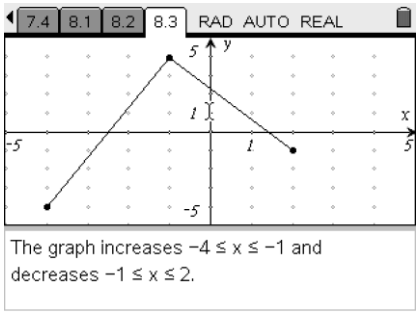

6.5 7.1 7.2 7.3 RAD AUTO REAL
0
2
4
Check Answer
your current answer is CORRECT
OK


7.1 7.2 7.3 7.4 RAD AUTO REAL
What is the y−intercept?
−3


7.1 7.2 7.3 7.4 RAD AUTO REAL
0
2
4
Check Answer
your current answer is CORRECT
OK


7.2 7.3 7.4 8.1 RAD AUTO REAL
Increasing
&
Decreasing


7.3 7.4 8.1 8.2 RAD AUTO REAL
To determine whether a graph is increasing or decreasing, we read the graph the same way we read book. As the values of x increase, if the values of y get larger, we call the graph increasing. If the values of y get smaller, then the graph is decreasing.


7.4 8.1 8.2 8.3 RAD AUTO REAL
y
x
5
-5
1
1
5
-5
The graph increases −4 ≤ x ≤ −1 and decreases −1 ≤ x ≤ 2.