

Activity 7

Objectives

- Use the CellSheet™ App to find the approximate slope of a tangent line of a curve
- Compare the Δx -slope relationship of parabolic and cubic curves

The Slope of the Tangent Line (Part 2)

Introduction

In Activity 6, you found the approximate slope of a line tangent to a parabolic curve by finding the slope of a secant line. You saw that the slope of the secant line changes the same amount that the value of Δx changes. In other words, there is a linear relationship between the slope of the secant line and the value of Δx . You also found that the smaller the Δx , the more accurate the approximation of the slope of the tangent line.

In this activity, you will look at the slope of a tangent line of a cubic curve to determine if the same relationship holds with the cubic curve. You will find how the slope of a tangent line of a cubic curve changes as the value of Δx changes.

Problem

How does the slope of a tangent line of a cubic curve change as the value of Δx changes? Is there a linear relationship between the two values as there is with tangent lines of parabolic curves? What is the relationship between the two sets of values?

In this activity, you will graph the function $y = x^2 + 4x - 5$ and the function $y = x^3 + 2x^2$. You will also graph the slope values of the tangent lines for each of the graphs.

Because you will approximate the slope of the tangent line using a secant line, you want the increment for x to be rather small. From Activity 6, you found that the smaller the Δx , the more accurate the approximation of the slope of the tangent line.

Exploration

1. Create a new spreadsheet in the CellSheet™ App, and name it **SLOPETAN**.
2. Enter "X" in cell A1 and "Y" in cell B1.

You want to fill column A with values of x from -5 to 7 in increments of 0.25. To do so quickly and easily, you will use the **Sequence** command.

SLOP	A	B	C
1	X	Y	
2			
3			
4			
5			
6			

B2: [Menu]

3. Select **Menu > Options > Sequence**. Enter **A2** as the 1st cell. Beside the **seq(** prompt, type $[X,T,\theta,n]$, $[X,T,\theta,n]$, **-5,7,.25** and close the parentheses. Select **Down**. Press **ENTER** three times. This will fill the column with a sequence that starts at -5 and goes to 7 with increments of 0.25.

You will enter a formula to compute the y -values. This will be done using the **Fill Range** option.

SLOP	A	B	C
1	X	Y	
2	-5		
3	-4.75		
4	-4.5		
5	-4.25		
6	-4		

A2: -5 [Menu]

4. Move to cell B2. Select **Menu > Options > Fill Range**. Enter **B2:B50** as the range. Enter the formula $=A2^2+4*A2-5$ for y at the **Formula:** prompt. Press **ENTER**.

SLOP	A	B	C
1	X	Y	
2	-5	0	
3	-4.75	-1.438	
4	-4.5	-2.75	
5	-4.25	-3.938	
6	-4	-5	

B2: =A2^2+4*A2-5 [Menu]

The result is a complete table of values. Next, you will determine the slope of the secant line through the two points (-5, 0) and (-4.75, -1.438).

5. Enter the formula $=(B3-B2)/(A3-A2)$ in cell C3.

The value in cell C3, -5.75, is an approximation of the slope of the tangent line through the point (-5, 0). As this formula is copied down column C, the slope of the curve, as approximated by a secant line at each point, is calculated.

SLOP	A	B	C
1	X	Y	
2	-5	0	
3	-4.75	-1.438	-5.75
4	-4.5	-2.75	
5	-4.25	-3.938	
6	-4	-5	

C3: =(B3-B2)/(A3-A2)

What do you notice about the values in column C? Is there a pattern? What does that tell you about the relationship between the Δx and the slope?

6. Use the **FILL RANGE** option to copy the formula through cell C50. At the **Formula** prompt, type $=(B3-B2)/(A3-A2)$.

```
FILL RANGE
Range: C3: C50
Formula: ...A3-A2)
ENTER
```

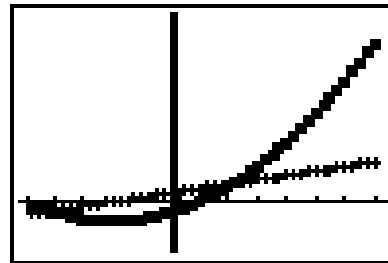
Columns A and B contain a table of values for the curve $y = x^2 + 4x - 5$. The points (x, M_T) are in columns A and C. Because there is no value in cell C2, the graphs will be plotted with points in rows 3 through 50.

7. Select **Menu > Charts > Line**. You can graph up to 3 curves on the same coordinate axis. Enter the values as shown on the screen.

```
LINE CHART
XRange: A3: A50
YRange1: B3: B50
YRange2: C3: C50
YRange3:
Title:
AxesOn AxesOff
DrawF1 Draw
```

8. Press **ENTER** until the graph appears.

The graph of the parabola and the graph of the slope of the curve appear. You will notice that the graph of the slope is a straight line, representing a linear function. This result agrees with what you found in the Activity 6. The slope of the secant changed in direct proportion to the change in x .



Now you want to determine if the same Δx -slope relationship holds for cubic curves. You can use the same spreadsheet, but will change the function to a cubic one, $y = x^3 + 2x^2$.

9. Use the **FILL RANGE** option to enter the function $=A2^3+2*A2^2$ for y in cells B2 through B50. It may take some time for the graphing handheld to carry out all the changes to columns B and C.

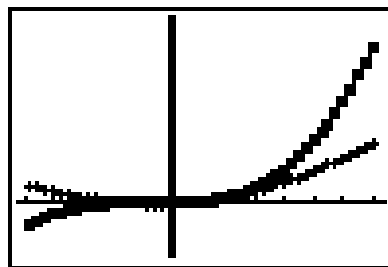
SLOP	A	B	C
1	X	Y	
2	-5	-75	
3	-4.75	-62.05	51.813
4	-4.5	-50.63	45.688
5	-4.25	-40.64	39.938
6	-4	-32	34.563
B2: =A2^3+2*A2^2 [Menu]			

10. Is there a constant difference between the values in column C as there was with the parabolic curve? Looking at the first four values listed, what are the differences? _____, _____, _____.

Is the Δx -slope relationship a linear one with a cubic curve? _____

Explain. _____

11. Graph these two curves using the **LINE CHART** option, as before. The appropriate **XRange**, **YRange1**, and **YRange2** are still present.

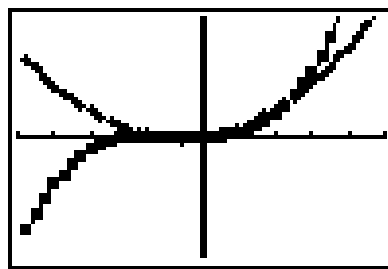


12. Press **WINDOW** to access the LINE WINDOW settings. Change the settings to those shown.

```

LINE WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-80
Ymax=80
Yscl=1
Draw Save
  
```

Look at the graphs shown. The cubic graph shows the function of the cubic equation, $y = x^3 + 2x^2$.



What is the shape of the second graph? _____

What does the graph tell you about the relationship between the slope of the tangent line and the Δx ? Is the relationship the same as with the quadratic function?

Student Worksheet

Name _____

Date _____

Solving the Problem

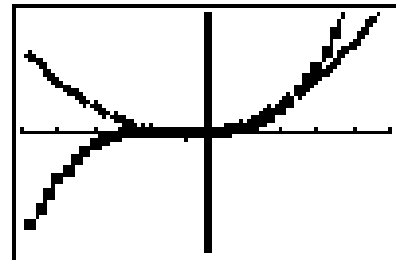
- Complete the table using data from this activity.

Slope of the Line Tangent to the Curve (estimated by the slope of the secant line)				
x-coordinate of the Point	Slope of the Cubic $y = x^3 + 2x^2$	Change in the Slope (from one point to the next)	Slope of the Quadratic $y = x^2 + 4x - 5$	Change in the Slope (from one point to the next)
$x = 0$	-0.4375		3.75	
$x = 1$	5.8125		5.75	
$x = 2$	18.063		7.75	
$x = 3$	36.313		9.75	
$x = 4$	60.563		11.75	

Analyzing the Data

- The data show that the slope of the quadratic changes _____, that is, for each change of 1 in the x-direction, there is a change of _____ in the y-direction.
- The data show that the slope of a cubic does not change _____. For each change of one in the x-direction, the change in the y-direction is _____. For example, the change in the slope from $x = 0$ to $x = 1$ is _____, and the change in the slope from $x = 1$ to $x = 2$ is _____.

- From the table above and the graph shown, you can determine that the change in the slope for the quadratic equation is _____ while the change for the cubic equation is _____ in nature.



- Looking at the graph in Question 4, why is the vertex of the parabola at $(0, 0)$?

Extending the Activity

1. Recall that the degree of a polynomial in x is the value of the highest power of x present in the polynomial. The degree of a cubic is 3, the degree of a quadratic is 2, and the degree of a linear function is 1. In each of the two examples in this activity, how does the degree of the function compare with the degree of the curve made up of the points (x, M_T) ?

Teacher Notes



Activity 7

The Slope of the Tangent Line (Part 2)

Objectives

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Materials

- TI-84 Plus/TI-83 Plus

Time

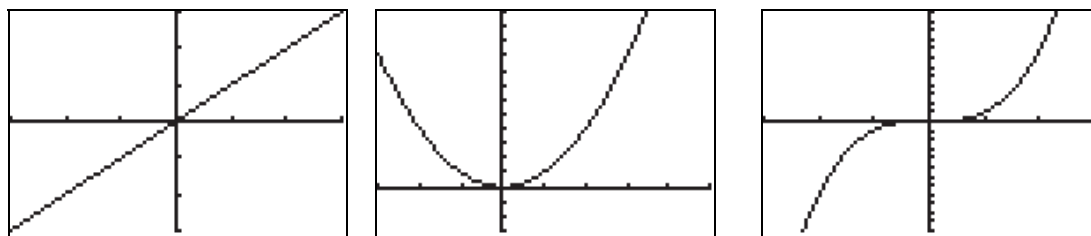
- 60 minutes

Preparation

The appropriate preparation for this activity is Activity 6. However, another activity which focuses on the same concepts can be substituted. Students should already realize the relationship between the Δx and the slope of a line tangent to a parabolic curve.

Elicit Questions

Consider how the slope of each curve shown here changes as x changes. What characteristics do you expect? What patterns might be present?



Management

As the concepts in this exploration require prior learning, it may be useful to have students work in pairs to help those students whose prior knowledge of the essential concepts is not as instantiated.

Answers to Exploration Questions

5. There is a constant difference between the values. The relationship is linear in nature.
10. 6.125, 5.75, 5.375; No. The difference between the values is not constant.

12. Parabolic; The graph shows that the relationship is quadratic in nature. It is not the same as that of the quadratic function, which is linear in nature.

Answers to the Student Worksheet

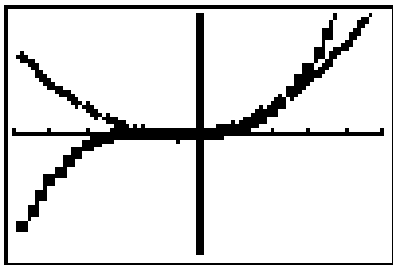
Solving the Problem

1. **Slope of the Curve (estimated by the slope of the secant line)**

x-coordinate of the Point	Slope of the Cubic $y = x^3 + 2x^2$	Change in the Slope (from one point to the next)	Slope of the Quadratic $y = x^2 + 4x - 5$	Change in the Slope (from one point to the next)
x = 0	-0.4375		3.75	
x = 1	5.8125	6.25	5.75	2
x = 2	18.063	12.25	7.75	2
x = 3	36.313	18.25	9.75	2

Analyzing the Data

- Linearly; 2
- Linearly; increasing; 6.25; 12.25
- Linear; quadratic



- The parabola is a graph of the slope of the cubic. Because the cubic flattens out (has a zero slope) at $x = 0$, the value of the quadratic at 0 must be 0.

Extending the Activity

- In each case, the degree of the curve made up of the points (x, M_7) is 1 less than the degree of the function.