Exploring the Area Under a Curve

ExploringAreaUnderCurve.tns

Name ______________________

Class ______________________

Problem 1 – Explore and discover

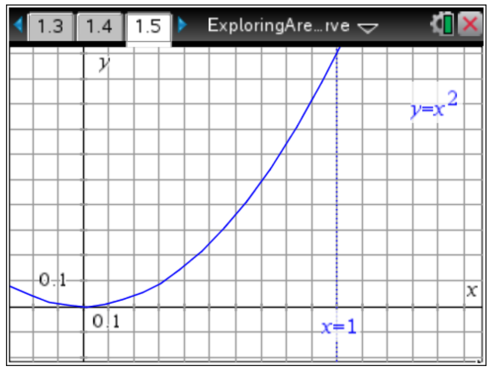
Move to page 1.2. Think of *at least* two ways to estimate the area bounded by the curve $y = x^2$ and the x-axis on the interval [0, 1] using rectangles. Furthermore, adhere to the guidelines below:

- All rectangles must have the same width
- You must build all your rectangles using the same methods
- The base of each rectangle must lie on the x-axis

For each method you identify, also consider how you could improve on your estimates.

Move to page 1.5 and test your methods. You can access the **Rectangle** tool by selecting **MENU > Geometry > Shapes > Rectangle**. To use the **Rectangle** tool:

- Move the cursor to where you want the bottom left corner of the rectangle to lie and press [enter].
- Move the cursor to where you want the bottom right corner of the rectangle to lie and press [enter].
- Move the cursor up to adjust the height of the rectangle and press [enter] once more to lock it in place.



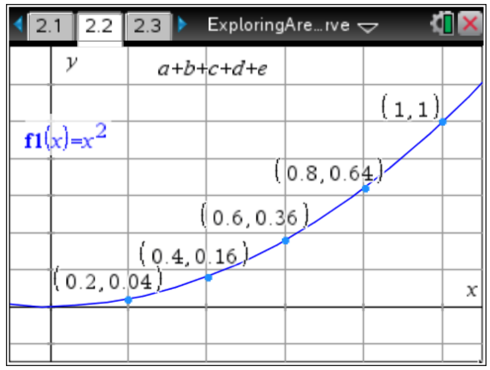
Sketch your rectangles for one of your methods on the graph to the right.

To select the **Area** tool press **MENU > Geometry > Measurement**.

- For each method, describe how you might improve upon your estimates. Keep in mind that your ultimate goal is to find the <u>*exact*</u> area under the curve.

In the following problems, you will examine three common techniques that use rectangles to find the approximate area under a curve. Perhaps you discovered some of these techniques during your exploration in Problem 1. The first problem uses rectangles whose *right-endpoints* lie on the curve $y = x^2$.

Problem 2 – Using five right-endpoint rectangles



Move to page 2.3. Using the **Rectangle** tool, draw five rectangles whose right-endpoints lie on the curve $y = x^2$. The width of each approximating rectangle will "snap" into place when you use the tick marks on the x-axis to start drawing the rectangles.

Next, use the **Area** tool to measure the area of each rectangle. Tip: Use the **Attributes** tool to show each value rounded to the nearest hundredth.

Finally, select **MENU > Actions > Calculate** to access the **Calculate** tool, click on the expression $a + b + c + d + e$, and click on each area measurement once. This will yield the sum of the five rectangles. Record your approximation for the area here.

$R_5 \approx$ ______________

The formula that can be used to express this area is:

R_5 = [0.2 • **f1**(0.2) + 0.2 • **f1**(0.4) + 0.2 • **f1**(0.6) + 0.2 • **f1**(0.8) + 0.2 • **f1**(1)]
or
R_5 = 0.2 [**f1**(0.2) + **f1**(0.4) + **f1**(0.6) + **f1**(0.8) + **f1**(1)]

Advance to page 2.6 and evaluate this expression using the *Calculator* application.

- Is the result consistent with the result you found with the rectangles?

Write and evaluate a summation expression on the *Calculator* page using a lower limit of $i = 1$. (Press [template key] to access the template palette, which contains the summation template.)

- Did you obtain the same result as before? Record the expression and result here.

Problem 3 – Using five left-endpoint rectangles

Move to page 3.2. You will now investigate a *left-endpoint* approximation.

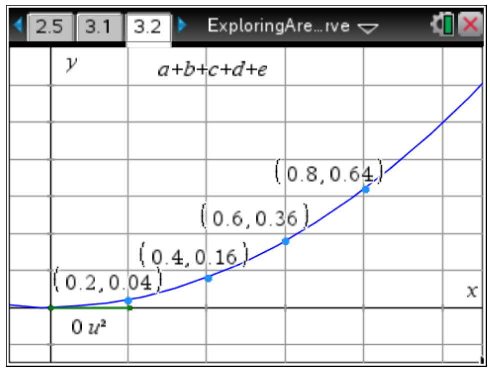
- How would you draw five rectangles, with equal width, such that their left endpoints lie on the curve $y = x^2$?

Now construct five *left-endpoint* rectangles. Use the **Area** tool to find the area of each individual rectangle and the **Calculate** tool to determine the sum of these areas.

- Record your approximation obtained for the area here.

  $L_5 \approx$ ______________

- How does this approximation compare with the one you found in Problem 2?



Note: Your first rectangle has been drawn for you. This rectangle has a height of 0 units since the curve intersects the axis at the left corner of the first rectangle.

The formula that can be used to express this area is:

L_5 = [0.2 • **f1**(0) + 0.2 • **f1**(0.2) + 0.2 • **f1**(0.4) + 0.2 • **f1**(0.6) + 0.2 • **f1**(0.8)]

or

L_5 = 0.2 [**f1**(0) + **f1**(0.2) + **f1**(0.4) + **f1**(0.6) + **f1**(0.8)]

Advance to page 3.5 and evaluate this expression using the *Calculator* application.

- Is the result consistent with the result you found with the rectangles?

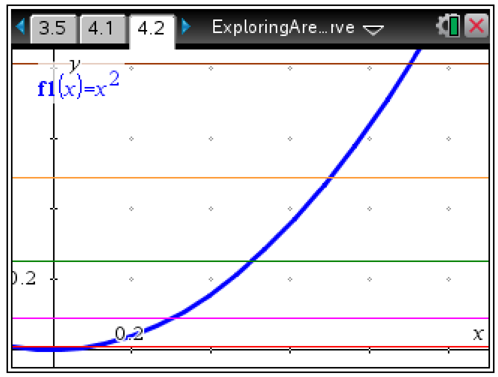
- Write and evaluate a summation expression on the *Calculator* page using a lower limit of $i = 1$.

- Did you obtain the same result as before? Record the expression and result here.

Problem 4 – Using five midpoint rectangles

Move to page 4.2. You will now investigate a *midpoint* approximation.

- How would you draw five rectangles, with equal width, such that their midpoints lie on the curve $y = x^2$?



Now construct five *midpoint* rectangles. Use [tab] to make sure to plot the base of the rectangle on the *x*-axis. Use the **Area** tool to find the area of each individual rectangle and use the **Calculate** tool to determine the sum of these areas.

- Record your approximation for the area here.

$M_5 \approx$ _______________

- How does this approximation compare with the one you found in the previous two problems?

Write an expanded expression for M_5, similar to those used for R_5 and L_5. Advance to page 4.5 and evaluate this expression.

- Is the result consistent with the result you found with the rectangles?

Try to write and evaluate a summation expression on the calculator screen using a lower limit of $i = 1$.

- Did you obtain the same result as before? Record the expression and result here.

Problem 5 – Summarize your findings

In this activity, you explored three different methods for approximating the area under a curve. The exact area under the curve $y = x^2$ on the interval [0, 1] is $\frac{1}{3}$.

- Which approximation technique produced the best estimate for the actual area under a curve?

- Describe what factors contribute to left, right, and midpoint rectangles giving *overestimates* versus *underestimates*.

- What can you do to ensure that all three of these techniques produce an answer that is very close to $\frac{1}{3}$?

- Explore your conjecture on page 5.5. How many subintervals do you need to do in order to get the midpoint to be accurate to three decimal places, that is, to the nearest thousandths?