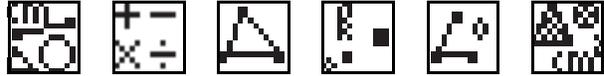


## Activity 3

### Objective

- To investigate the properties of a dilation

### Cabri® Jr. Tools



## Dilations in the Plane

### Introduction

A **dilation** is a transformation that produces a figure with a different size from that of the original figure. In this activity, you will explore the properties of dilations and the relationships between the original and image figures.

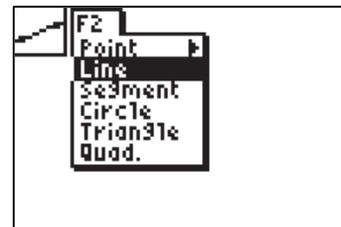
### Construction

**Part I: Construct a slider tool to generate a number in the range of approximately -4.5 to 4.5 units.**

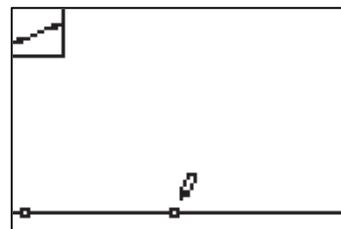
  Draw the line  $\overleftrightarrow{AC}$  horizontally across the bottom of a new screen with point A near the left edge and point C near the middle of the screen.

- Open the **Drawing Tools Menu**, and then highlight **Line**. Press **ENTER** to select it.

**Note:** The tool icon at the top left of the screen indicates that the **Line** tool is active.

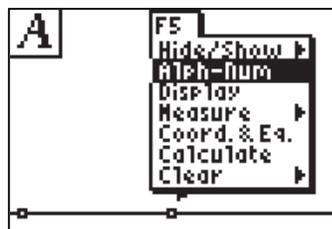


- Move the cursor to the lower left corner of the screen. Press **ENTER** to anchor one point of a line. Move the cursor to the lower middle of the screen and press **ENTER** to anchor the second point of the line.

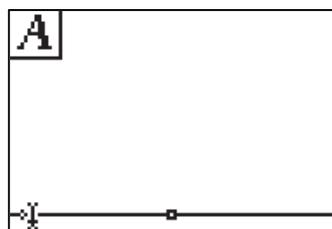


- Open the **Display Tools Menu**, and then highlight **Alpha-Num**. Press **[ENTER]** to select it.

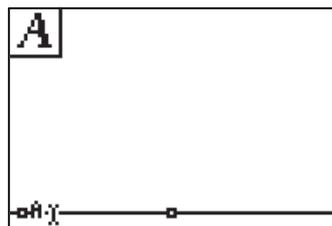
**Note:** Alpha lock is on when the **Alpha-Numeric** tool is active.



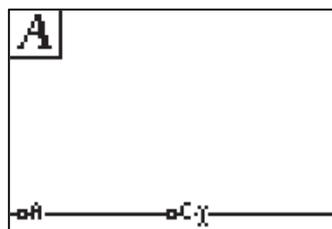
- Move the cursor to the point in the lower left corner of the screen. The point blinks when the cursor is close enough to the point to select it.



- Press **[ENTER]** to create a label for this point. Press **[MATH]** to label that point A. Press **[ENTER]** to complete the label.

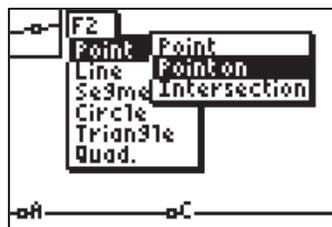


- Repeat steps 4 and 5 to label the point in the lower middle of the screen as C (**C** is located above **[PRGM]**). When finished, press **[CLEAR]** to exit the **Alpha-Numeric** tool.

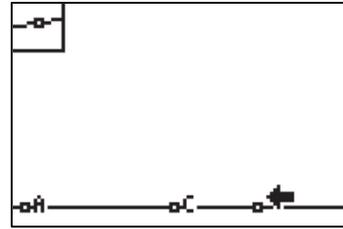


□ **A** Construct point B on line  $\overleftrightarrow{AC}$  to the right of point C.

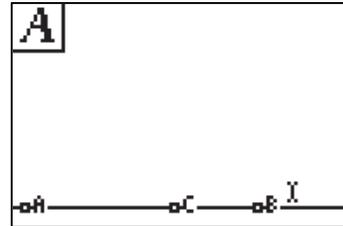
- Open the **Drawing Tools Menu**, and then highlight **Point**. Press **[▶]** to view the **Point Menu**. Highlight **Point on**, and then press **[ENTER]**.



8. Move the cursor to the right of point C on line  $\overleftrightarrow{AC}$ , and press **ENTER** to place a point on the line.

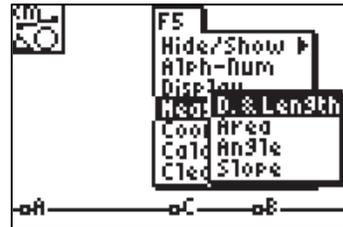


9. Use the **Alpha-Numeric** tool to label the newly created point as B (**B** is located above **APPS**). Remember to press **ENTER** once to create the label, and then again to complete the label. Press **CLEAR** to exit the **Alpha-Numeric** tool.

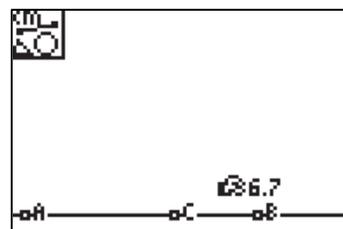


  Measure the length of segments AB and AC, and calculate the difference of segments AB and AC.

10. Open the **Display Tools Menu**, and then highlight **Measure**. Press **▶** to view the **Measure Menu**. Highlight **D. & Length**, and press **ENTER**.

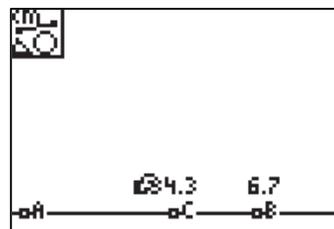


11. Use the **Distance & Length** tool to measure the length of the segment AB.
- Move the cursor to point A and press **ENTER**, and then move the cursor to point B and press **ENTER**.
  - Use the cursor keys (**◀ ▶ ▲ ▼**) to move the measurement away from the line.
  - Press **ENTER** a final time to anchor the measurement.

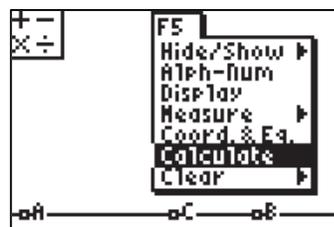


**Note:** Make sure that the point itself is blinking, not the entire line.

12. Repeat step 11 to measure the length of the segment AC. Make sure that the **D. & Length** tool is still active.

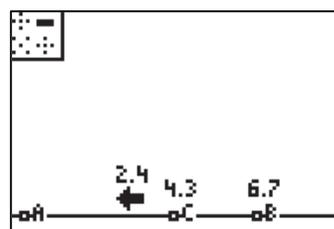


13. Open the **Display Tools**, and then highlight **Calculate**. Press **ENTER**.



14. Use the **Calculate** tool to find the difference of  $\overline{AB}$  and  $\overline{AC}$ .

- Move the cursor to the measurement of segment AB (which should be the larger number). Press **ENTER**.
- Press **-** to subtract.
- Move the cursor to the measurement of segment AC, and press **ENTER**.
- Use the cursor keys to move the calculation to a blank area of the screen and press **ENTER** a final time to anchor the calculation.
- Press **CLEAR** to exit the **Calculate** tool.

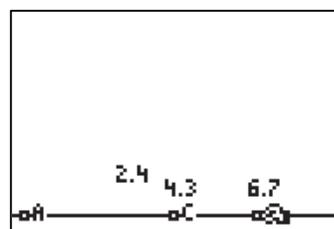


Drag point B to change the numeric value, and hide objects no longer needed.

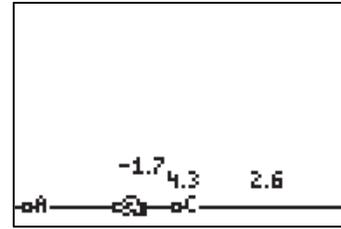
15. Move the cursor to highlight point B and press **ALPHA** to grab it.

**Note:** Make sure to grab the point itself, not the label B.

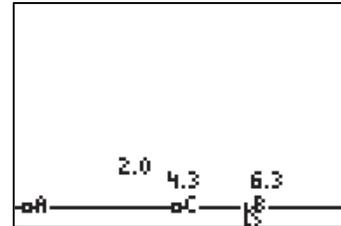
**Note:** The pointer changes to an outlined arrow when it is near an available object.



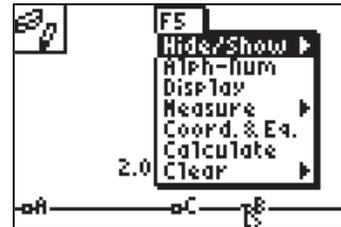
16. Use the cursor keys to drag point  $B$  along the line. Observe the difference of  $\overline{AB}$  and  $\overline{AC}$  as it changes. When point  $B$  is to the right of point  $C$ , the difference is positive. When point  $B$  is between points  $A$  and  $C$ , the difference is negative.



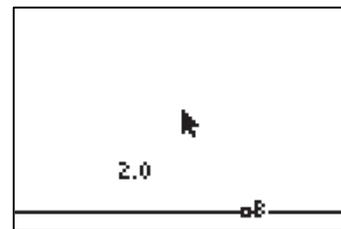
17. Move point  $B$  so that the difference is 2. Press **CLEAR** to exit the **Hand** tool.



18. Open the **Display Tools Menu**, and then highlight **Hide/Show**. Press **ENTER**.



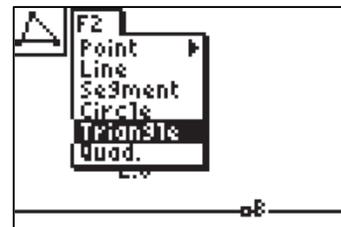
19. Move the cursor to point  $A$  on the line, and press **ENTER** to hide point  $A$ . Repeat this to hide all objects and numbers except the line, point  $B$ , and the calculated difference of  $\overline{AB}$  and  $\overline{AC}$ . Press **CLEAR** to exit the **Hide/Show** tool.



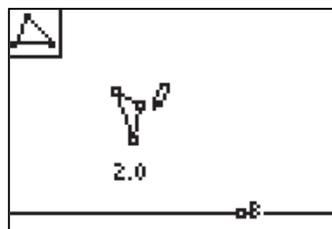
## Part II: Construct a triangle, and perform a dilation.

-    Draw a small scalene triangle on the left half of the screen and point  $P$  in the center of the screen.

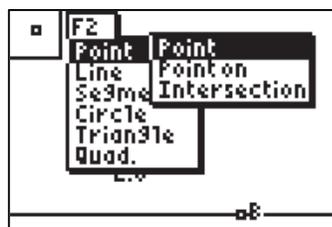
1. Open the **Drawing Tools Menu**, and then highlight **Triangle**. Press **ENTER**.



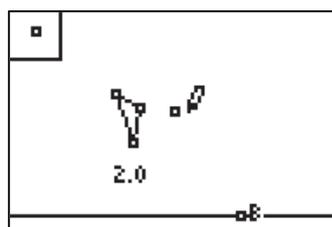
2. Move the cursor to the left half of the screen and press **ENTER** to anchor one vertex of the triangle. Repeat this two more times to anchor the other two vertices of the triangle. Press **CLEAR** to exit the **Triangle** tool.



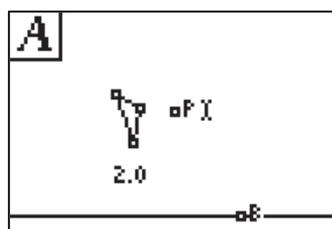
3. Open the **Drawing Tools Menu**, and then highlight **Point**. Press **▶** to view the **Point Menu**. Highlight **Point**, and press **ENTER**.



4. Move the cursor to the center of the screen. Press **ENTER** to anchor a point there.



5. Use the **Alpha-Numeric** tool to label the newly created point as **P** (**P** is above the **8** button). Remember to press **ENTER** once to create the label and then again to complete the label.

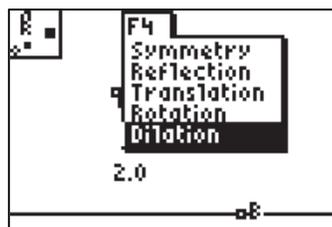


 *Dilate the triangle.*

To use the **Dilation** tool, you must do the following:

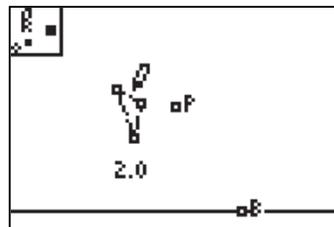
- Select what is to be dilated.
- Select a point to be the center of dilation.
- Select the numeric value to be used as a dilation factor.

6. Open the **Transformation Tools Menu**, and then highlight **Dilation**. Press **ENTER**.

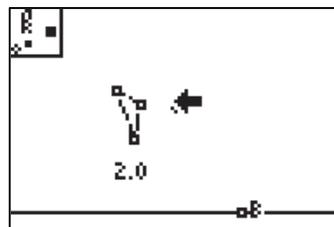


7. Move the cursor to highlight the triangle and then press **ENTER** to select it.

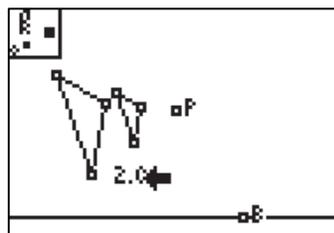
**Note:** Make sure to highlight the whole triangle, not just one side.



8. Move the cursor to highlight point  $P$  and then press **ENTER** to select it.



9. Move the cursor to the numeric difference of  $\overline{AB}$  and  $\overline{AC}$ , and press **ENTER** to select it. A new triangle (the image) is created. Press **CLEAR** to exit the **Dilation** tool.

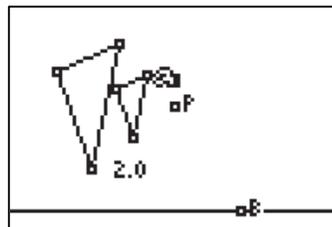


## Exploration

Investigate the relationship between the original triangle (the pre-image) and the new triangle (the image).

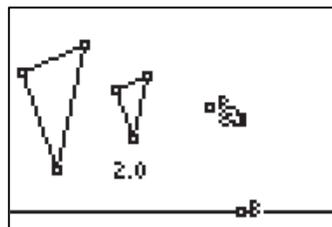
 Drag a vertex of the pre-image.

1. Move the cursor to highlight the desired vertex and press **[ALPHA]** to grab it. Use the cursor keys to drag the vertex around the screen. Observe what happens to the image. When finished, press **[CLEAR]** to exit the tool.



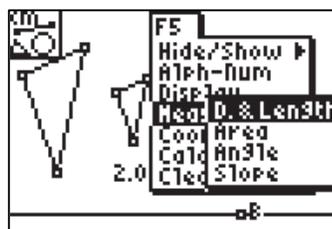
 Drag the center of dilation.

2. Move the cursor to highlight point *P*, and press **[ALPHA]** to grab it. Use the cursor keys to drag the point around the screen. Observe what happens to the image. When finished, press **[CLEAR]** to exit the tool.

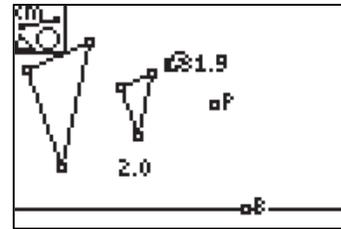


   Measure the lengths of corresponding sides, angles, and areas of the pre-image and image.

3. Open the **Display Tools Menu**, and then highlight **Measure**. Press **[▶]** to view the **Measure Menu**. Highlight **D. & Length**. Press **[ENTER]**.



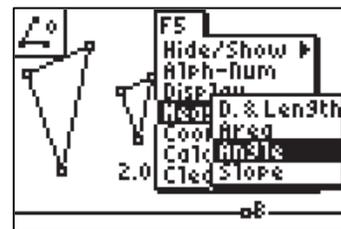
4. Move the cursor to highlight the desired side of the triangle. Press **ENTER** to measure the side, and press **ENTER** again to anchor the measurement on the screen. Repeat this for all desired length measurements. When finished, press **CLEAR** to exit the **Distance & Length** tool.



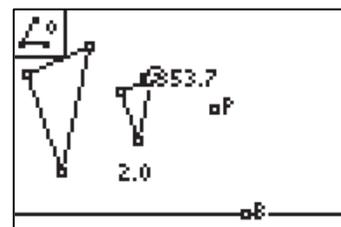
**Note:** Be careful to highlight the side of the triangle and not the whole triangle. Wait for only the side to highlight, or press **2nd** to switch between full object and side only.

**Note:** You could also measure the side length as a point-to-point distance. Highlight one endpoint, and press **ENTER**. Highlight a second endpoint, and press **ENTER** again.

5. Open the **Display Tools Menu**, and then highlight **Measure**. Press **▶** to view the **Measure Menu**. Highlight **Angle**. Press **ENTER**.

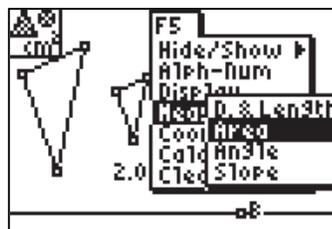


6. Angles are measured by selecting three points. Select the vertex point of the angle *second*.
- Move the cursor to highlight one point on the side of the desired angle, and press **ENTER**.
  - Move the cursor to the vertex of the angle, and press **ENTER**.
  - Move the cursor to the third point on the angle, and press **ENTER**.
  - Finally, move the measurement to a desired location, and press **ENTER** to anchor it on the screen.

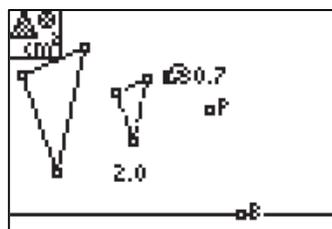


Repeat this for all desired angle measurements. When finished, press **CLEAR** to exit the **Measure Angle** tool.

7. Open the **Display Tools Menu**, and then highlight **Measure**. Press  $\rightarrow$  to view the **Measure Menu**. Highlight **Area**. Press  $\boxed{\text{ENTER}}$ .



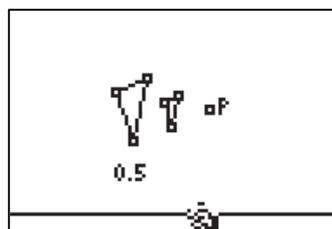
8. Move the cursor to highlight the desired triangle. Press  $\boxed{\text{ENTER}}$  to measure the area, and press  $\boxed{\text{ENTER}}$  again to anchor the measurement on the screen. Repeat this for all desired area measurements. When finished, press  $\boxed{\text{CLEAR}}$  to exit the **Measure Area** tool.



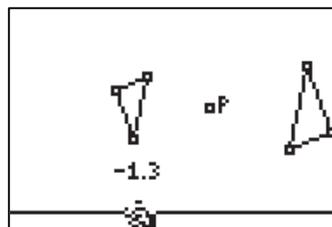
 Drag point B to explore different dilation factors.

9. Move the cursor to highlight point B, and press  $\boxed{\text{ALPHA}}$  to grab it.

**Note:** Make sure to grab the point itself, not the label B.



10. Use the cursor keys to drag point B along the line and observe the dilation factor as it changes. When point B is to the right of the original point C, the difference is positive. When point B is between the original points A and C, the difference is negative. Observe what happens to the image triangle when the dilation factor changes. When finished, press  $\boxed{\text{CLEAR}}$  to exit the tool.



# Data Collection and Analysis

Name \_\_\_\_\_

Date \_\_\_\_\_

## *Questions and Conjectures*

1. Did the new triangle change shape compared with the original triangle?

\_\_\_\_\_

2. How are the pre-image and the image related?

\_\_\_\_\_

3. What happens when the center of dilation is moved?

\_\_\_\_\_

4. What is the relationship between lengths of corresponding sides in the triangles?

\_\_\_\_\_

5. What is the relationship between measures of corresponding angles in the triangles?

\_\_\_\_\_

\_\_\_\_\_

6. What is the relationship between the areas of the triangles?

\_\_\_\_\_

\_\_\_\_\_

7. Explain what happens when point  $B$  is dragged to make a larger dilation factor.

\_\_\_\_\_

\_\_\_\_\_

8. Explain what happens when the dilation factor is a negative number.

\_\_\_\_\_

\_\_\_\_\_

9. Did the triangle change orientation with a positive or negative dilation?

\_\_\_\_\_

\_\_\_\_\_

10. Is there another transformation that is equivalent to a dilation by a factor of  $-1$ ?

\_\_\_\_\_



## Teacher Notes



### Activity 3

#### Objective

- To investigate the properties of a dilation

#### Cabri® Jr. Tools



## Dilations in the Plane

### Answers to Questions and Conjectures

1. Did the new triangle change shape compared with the original triangle?

The new triangle and the old triangle have the same shape, but different sizes.

2. How are the pre-image and the image related?

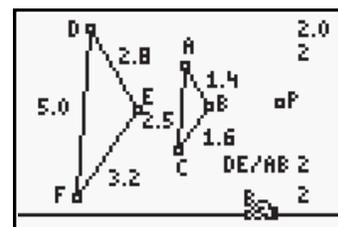
The pre-image and image are similar to each other. Their corresponding angles are congruent and their corresponding sides are in proportion. The ratio of the sides equals the dilation factor.

3. What happens when the center of dilation is moved?

The distance from the image to the center of dilation and the distance from the pre-image to the center of dilation is in proportion, and their ratio equals the dilation factor. For example, if the dilation factor is 2, then the image is twice as far away from the center of dilation as the pre-image. Therefore, if the center of dilation is moved, the image also moves. If the center moves toward the pre-image, then the image moves toward the pre-image. If the center moves away from the pre-image, then so does the image.

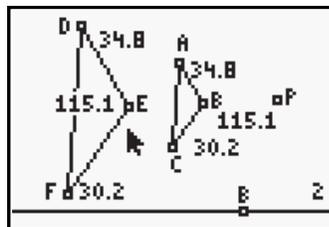
4. What is the relationship between lengths of corresponding sides in the triangles?

Corresponding sides have the same ratio as the dilation factor.



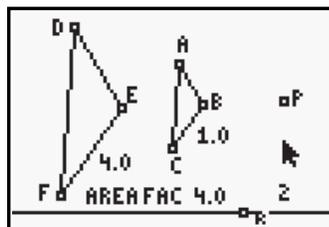
5. What is the relationship between measures of corresponding angles in the triangles?

Measures of corresponding angles are equal; their ratio is 1:1.



6. What is the relationship between the areas of the triangles?

The ratio of corresponding areas equals the square of the dilation factor. For example, if the dilation factor is 2, then the ratio of areas is 4:1.



7. Explain what happens when point  $B$  is dragged to make a larger dilation factor.

The image gets larger and moves farther away from the center of dilation.

8. Explain what happens when the dilation factor is a negative number.

A negative dilation factor causes the image to be rotated  $180^\circ$  around the center of dilation and then dilated by the absolute value of the dilation factor.

9. Did the triangle change orientation with a positive or negative dilation?

No; dilations preserve orientation whether the dilation factor is positive or negative.

10. Is there another transformation that is equivalent to a dilation by a factor of  $-1$ ?

A dilation by a factor of  $-1$  is equivalent to a rotation of  $180^\circ$  around the center of dilation.