Means with Confidence
ID: 12371

Time required
15 minutes

Activity Overview

*In this activity, students will estimate the true mean of a population when the standard deviation is known by finding the sample mean, margin of error and confidence interval. They will also use the **z Interval** command to confirm the intervals.*

Topic: Inference

- *Standardized normal curves*
- *Confidence Intervals*
- *Estimating a population mean*

Teacher Preparation and Notes

- *This activity is designed to be teacher-led. Homework problems are included. The TI-Nspire document contains self-check questions to enable students to check their understanding about the normal distribution. The activity can be paperless if desired.*
- *Depending on the level of students Problem 1 can be deleted from the TI-Nspire document and the activity can begin with Problem 2.*
- *Students should know how to calculate the mean of a sample.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12371" in the keyword search box.***

Associated Materials

- *MeansWithConfidence_Student.doc*
- *MeansWithConfidence.tns*
- *MeansWithConfidence_Soln.tns*

Suggested Related Activities

To download any TI-Nspire technology activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Estimating a Population Proportion (TI-Nspire technology) — 9986*
- *Are You Confident? (TI-Nspire technology) — 9676*
- *Hypothesis Testing: Means (TI-Nspire technology) — 10240*

Problem 1 – Review of Normal Distribution and *z*-scores

Review the normal curve and the percent of data that falls within one, two, and three standard deviations of the mean. There are self-check questions (Pages 1.3–1.6) in the TI-Nspire document to check students' knowledge of this topic.

Introduce the Central Limit Theorem: The Central Limit Theorem says that given a SRS (simple random sample) of size *n* from any population with mean *μ* and finite standard deviation *σ*, when *n* is large, the sampling distribution of the sample mean $\overline{x}$ is close to the normal distribution with mean *μ* and standard deviation $\dfrac{\sigma}{\sqrt{n}}$.

Discuss how this theorem allows one to find an interval that contains the mean of the population.

Discussion Questions:

- *If we have the mean of the sample, why would we want the mean of the population?*

- *What information do we need to use the Central Limit Theorem to approximate the mean?*

- *If we would like to be certain 95% of the time, how many standard deviations from the mean can we be?*

Self-check questions (pages 1.7–1.8) allow students to interact with this information.

Introduce the two parameters needed for a confidence interval:

- A confidence level which gives the probability that the interval will contain *μ*.

- An interval calculated from the data, generally written $\overline{x}$ ± Error.

Error = $Z \cdot \dfrac{\sigma}{\sqrt{n}}$, where *Z* is the *z*-score that corresponds to the area in the left (or right) tail.
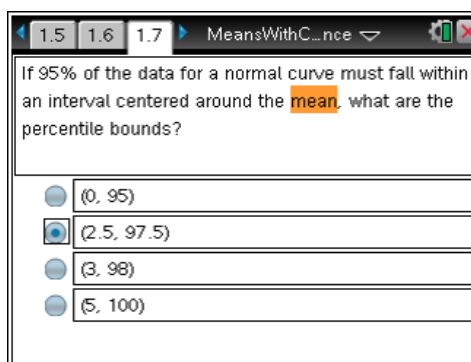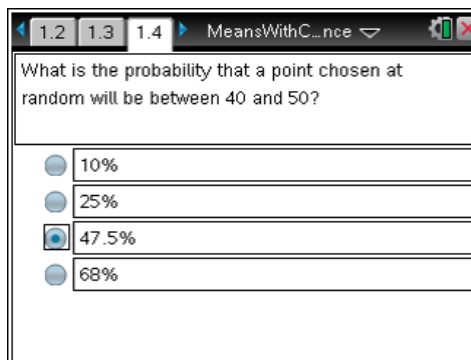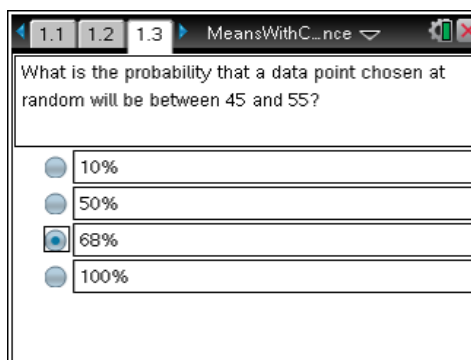
Problem 2 – Estimating the true mean

This problem can be used as either an introduction or review for estimating the population mean. If using as an introduction, you will need to guide students through each step as some more specific instructions are not given on the worksheet or the .tns file.

A SRS of 40 football receivers in a hypothetical league is given on page 2.2. Students will use Column B of this sample to estimate the true mean for the longest reception for all receivers in the league with a confidence interval of 95%.

Step 1: Calculate $\bar{x}$ for longest reception.

Enter **sum(long)/40** to the right of **x:=**. This adds all the values in Column B and divides by n (shown at right).

Students should store the value they calculate as **x** for easy calculations later. To do this, place **x:=** before the calculation.

Step 2: Find the critical value.

Students can use the **invNorm** command (**MENU > Calculations > Statistics > Distributions > Inverse Normal**). This command will return the z-score for a given percentile when given μ and σ, in this case $\mu = 0$ and $\sigma = 1$. Since students are finding a 95% confidence interval, the percentile (area) is either 2.5% or 97.5%.

Step 3: Calculate the Error. ($z = 1.96$, $\sigma = 20.39$, and $n = 40$)

Students are to use the standard deviation of population given on page 2.3 in their calculations.

$$\text{Error} = Z \cdot \frac{\sigma}{\sqrt{n}} = 6.32$$

Students should store the value they calculate for the error as **e** for easy calculations later.

Step 4: Write the solution as an interval.

Students can use *Scratchpad* or the *Calculator* application at the bottom of the page and the values they stored earlier to calculate the confidence interval.

Step 5: Interpret the result as it pertains to the problem situation.

Sample answer: I am 95% confident that the mean of the receivers' longest reception is between 22.73 and 35.37.

Alternative method:

After completing the process, show students the built-in handheld function to calculate the confidence interval using the given statistics.

From a *Calculator* screen or *Scratchpad*, students can press **MENU > Statistics > Confidence Intervals > z Interval**. From the pull down menu for Data Input Method choose **Stats**. Then they need to enter σ, xbar, n, and the confidence level.

Homework Problems

Students are given homework problems that ask them to find different confidence intervals using the sample data from Problem 2, also given on page 3.2.

1. 1.31

2. (9.62, 11.06)

3. (9.49, 11.20)

4. (25.72, 33.12)

5. (4.85, 5.90)

Problem 4 gives the complete data for the receivers in the hypothetical league. Students can check the validity of the intervals by finding the true means of the population.