Taxicab Geometry

ID: 8637

Time required
40 minutes

Activity Overview

In this activity, students begin a study of taxicab geometry by discovering the taxicab distance formula. They then use the definition of radius to draw a taxicab circle and make comparisons between a circle in Euclidean geometry and a circle in taxicab geometry. Lastly, they construct taxicab perpendicular bisectors and discover that only certain pairs of points have taxicab perpendicular bisectors.

Topic: Points, Lines & Planes

- *Construct a line segment joining two points with integral coordinates on a grid and measure its length.*
- *Given the coordinates of the ends of a line segment, calculate its length.*

Teacher Preparation and Notes

This activity is designed to be used in a high-school geometry classroom.

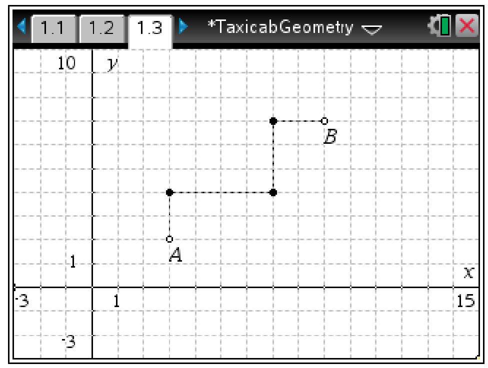
- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *Taxicab geometry is a non-Euclidean geometry. For the sake of comparisons, students are best off if they know, before starting this activity, how to find the distance between two points, as well as the definitions and characteristics of circles and perpendicular bisectors in Euclidean geometry.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8637" in the keyword search box.***

Associated Materials

- *TaxicabGeometry_Student.doc*
- *TaxicabGeometry.tns*

Begin the activity with a brief introduction to taxicab geometry. Taxicab geometry is a form of non-Euclidean geometry—meaning it has different rules and characteristics than the geometry your students are probably used to studying. There are several types of non-Euclidean geometries. Much as it sounds, taxicab geometry considers what it would be like to drive a taxi through a city in which the streets are set up on a grid of parallel and perpendicular lines. Taxicab drivers can not drive through parks and buildings; therefore, they cannot travel from one intersection to another diagonally. Likewise, no diagonal movements are allowed in taxicab geometry.

Problem 1 – Taxicab Distances

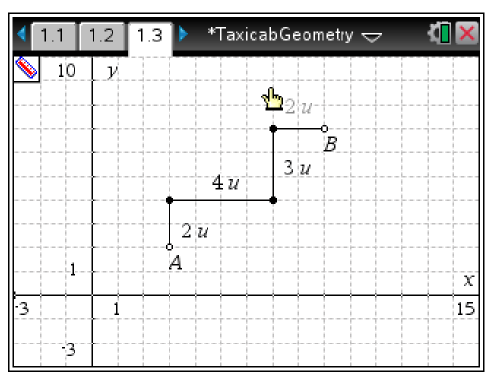


Points *A* and *B* are plotted on page 1.3. Tell students to pretend they are taxicab drivers and need to "drive" from point *A* to point *B* by drawing segments between the points. Reiterate that no diagonal movements are allowed. Tell them they can make as many, or as few, turns as they wish.

As it is difficult to construct the endpoints of segments ***directly*** on the grid points when using the handheld, have students make segments by first plotting the endpoints (**MENU > Geometry > Points & Lines > Point On**), and then drawing the segments connecting the endpoints (**MENU > Geometry > Points & Lines > Segment**).

Note: If students happen to be working with software instead of handhelds, they can easily make segments whose endpoints are the points on the grid by using just the **Segment** tool.

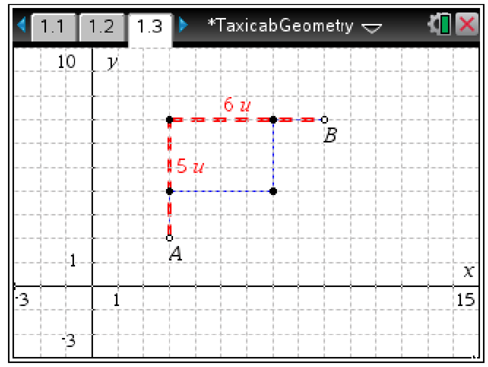
TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.



Once students have traveled from *A* to *B*, they should measure the distance they traveled. They can do this by either counting the number of "blocks," or by using the **Length** tool from the Measurement menu. Students often make errors in counting the distances because they tend to count points instead of the number of spaces between the points.

Determine the greatest and shortest distances traveled within your classroom.

Students should repeat the trip from *A* to *B*, but this time choosing a different path. Once done, have them find the distance of this second trip.



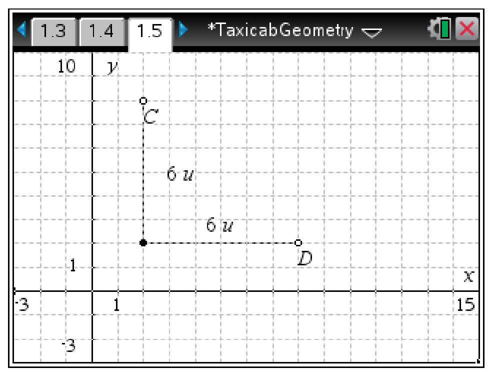
Tip: To distinguish each trip, students should change the color of each trip. To change the color of a line segment, hovering over the segment, press ctrl + , then select **Color > Line Color** and choose a new color.

Determine the shortest distance that any student made from *A* to *B*. The shortest possible distance is 11. Discuss with students how this can be achieved with just one turn.

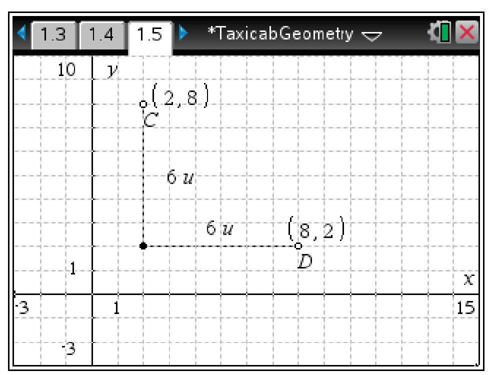
Ask students how they would normally determine the distance between two points (outside of taxicab geometry). They might discuss the distance formula or the Pythagorean Theorem. For the sake of comparison, have students measure the distance from *A* to *B* directly and then answer the first question on their worksheets.

Note: Students can use the **Length** tool without drawing a segment from *A* to *B*.

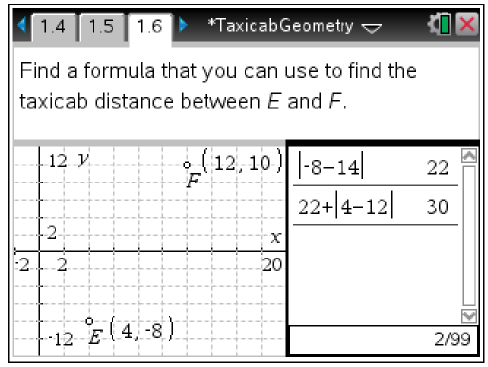
On page 1.5, students will see points *C* and *D*. Have them try drawing a path from *C* to *D* such that it is the shortest possible path, and that only one turn is made. (There are two possible ways: down then to the right, or to the right and then down.) Use the **Length** tool to find the lengths of the two segments.



The coordinates of the points should be displayed (**MENU > Actions > Coordinates and Equations**) to allow students to brainstorm a way they could find the lengths of the two segments they drew if given only these coordinates. This is the second question on the worksheet. (Subtract the *x*-values, take the absolute value, subtract the *y*-values, take the absolute value, and then add the absolute values.)



On page 1.6, students will first find the coordinates of points *E* and *F*. Then they should move to the *Calculator* application and use the coordinates to find the taxicab distance between *E* and *F*.



Depending on the ability level of your class, either display the following formula or have students write it themselves, based on what they already know. This is the third question on the worksheet.

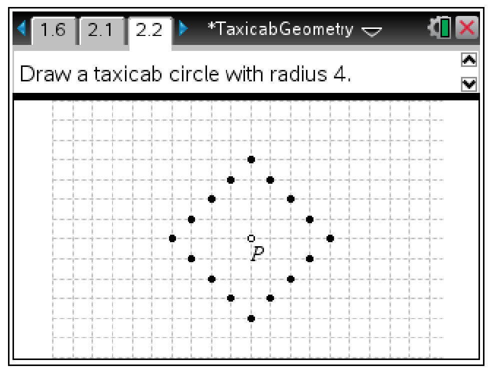
taxicab distance = $|y_2 - y_1| + |x_2 - x_1|$

Problem 2 – Taxicab Circles

On page 2.2, students create a taxicab "circle" with point *P* as the center and 4 as the radius. To do this, students will need to plot every point that is a total of 4 units from *P*. For example, one such point is 3 units up and 1 unit left.

From the Geometry > Points & Lines menu, remind them to use the **Point On** tool, rather than the **Point** tool.

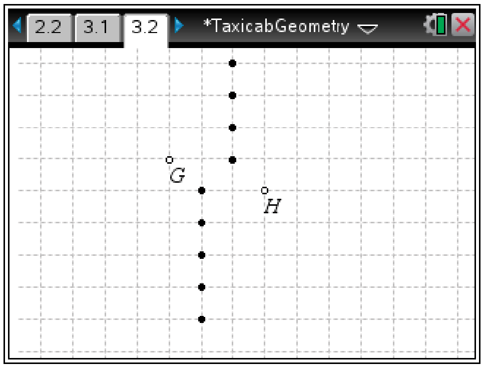
Ask students how many points make up the circle. (16) Lead a class discussion about the differences between a circle in Euclidean geometry and a circle in taxicab geometry. This is the fourth question on the worksheet. (shape, infinite v. finite number of points)



If time permits, you can have students insert a new page by pressing ctrl + I and draw one taxicab circle with a radius of 2 units and another with a radius of 3 units to make a conjecture about the relationship between the radius and number of points in the circle. (number of points increase as radius increases; the number of points is equal to 4 times the radius)

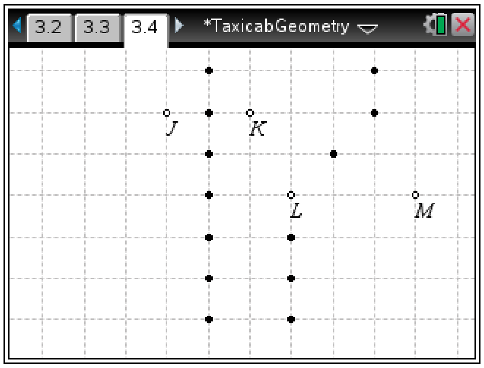
Alternatively, students can draw the taxicab circles on the same page, but use a different color to distinguish the circles.

Problem 3 – Taxicab Perpendicular Bisectors



On page 3.2, have students try to plot every point that is equidistant from points *G* and *H*.

Once they have finished, explain that they have just created a taxicab perpendicular bisector. Discuss similarities and differences between perpendicular bisectors in Euclidean and taxicab geometry. This is the fifth question on the worksheet. (equidistance, infinite v. finite)



On page 3.4, students try to create perpendicular bisectors for points *J* and *K* and then for points *K* and *L*.

Students will see that it is impossible to create equidistant points for *K* and *L*.

Ask students if they can create perpendicular bisectors for *K* and *M* (yes) and *J* and *L* (no). See if students can identify the condition required for two points to have a perpendicular bisector. This is the sixth question on the worksheet. (the minimum distance between the two points must be an even number of units)

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*

This would be a good place to do a screen capture to compare student results. The Screen Capture may be used to determine who has the shortest and longest trips.

