

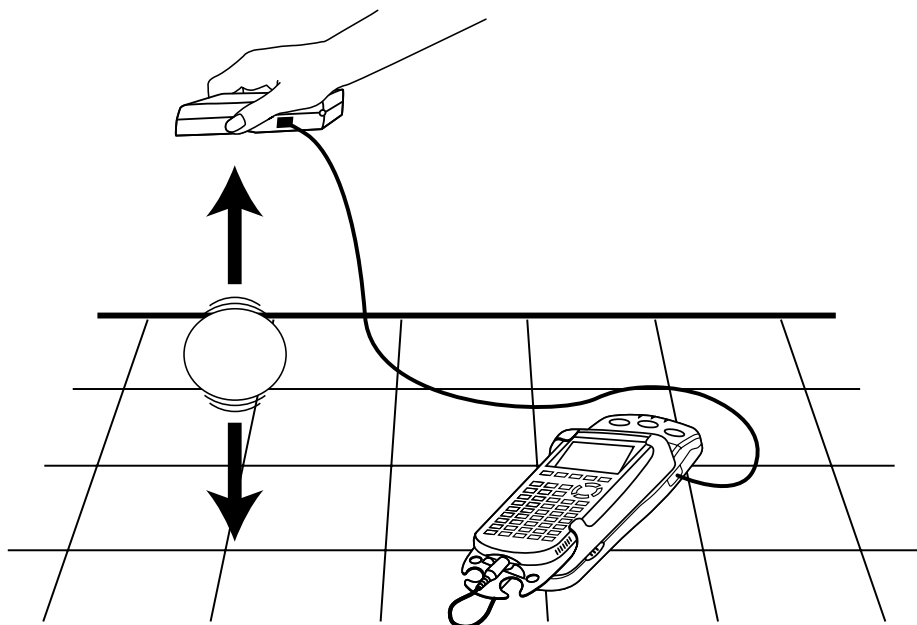
Bounce Back: The Exponential Pattern of Rebound Heights

When a ball bounces up and down on a flat surface, the maximum height it reaches decreases from bounce to bounce. In fact, the maximum height decreases in a very predictable way for most types of balls. The relationship between the maximum height attained by the ball on a given bounce (which we will call the *rebound height*) and number of bounces that have occurred since the ball was released is an exponential

$$y = hp^x$$

where y represents the rebound height, x represents the bounce number, h is the release height, and p is a constant that depends on the physical characteristics of the ball used. It's easy to see where this model comes from: Suppose that the ball is released from height h . Then on each bounce it rebounds to a fraction p of the previous maximum height. After zero, one and two bounces, the ball will attain a maximum height of h , hp , $(hp)p = hp^2$, and so forth. The relation above is generalized for any x number of bounces.

In this exercise, you will collect motion data for a bouncing ball using a Motion Detector. You will then analyze this data to test the model $y = hp^x$.



OBJECTIVES

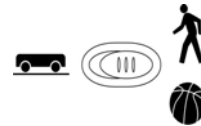
- Record the successive maximum heights for a bouncing ball.
- Model the bounce height data with an exponential function.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
CBR 2 or Go!Motion and direct calculator cable
or Motion Detector and data-collection interface
ball (a basketball works well)

PROCEDURE

1. Set up the Motion Detector and calculator.
 - a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
 - b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
 2. Position the Motion Detector about 1.5 m above the floor, so that the disc is pointing straight downward.
 3. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **(File)** from the Main screen, and then select **New** to reset the application.
 - c. Select **(Setup)** from the Main screen, and then select **Ball Bounce**.
 4. Practice dropping the ball so that it bounces straight up and down beneath the Motion Detector. Minimize the ball's sideways travel. Dropping the ball from about waist high works well. The ball must never get closer than 0.5 m from the detector. Be sure to pull your hands away from the ball after you drop it so the Motion Detector does not detect your hands.
 5. Select **(Start)** to begin data collection. The Ball Bounce instructions will appear on the calculator screen. Follow the instructions to collect data.
 6. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The distance versus time graph should contain a series of at least five smoothly changing parabolic regions.
- Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select **(Main)** and repeat Step 5.
7. Once you are satisfied with the distance versus time data, select **(Main)**. Exit EasyData by selecting **(Quit)**, and then selecting **(OK)**.

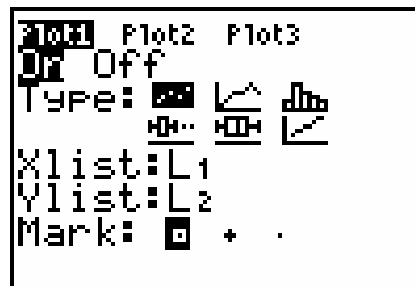


ANALYSIS

1. Display a graph of ball height above the floor versus time.
 - a. Press 2nd [STAT PLOT].
 - b. Press ENTER to select Plot1 and press ENTER again to select On.
 - c. Press ZOOM .
 - d. Press V until ZoomStat is highlighted; press ENTER to display a graph with the x and y ranges set to fill the screen with data.
 - e. Press TRACE to determine the coordinates of a point on the graph using the cursor keys.
2. To compare the distance data to the model for bounce height, read the maximum height of each bounce from your distance versus time data. Do this by tracing across the graph using the cursor keys.

Start with the initial release height, and call that bounce number zero. Record the consecutive maximum rebound heights for the next five bounces. Round these values to the nearest 0.001 m in the Data Table on the *Data Collection and Analysis* sheet.

3. Enter the information from the Data Table into lists on the calculator.
 - a. Press STAT and press ENTER to see the data lists.
 - b. Lists 1 and 2 are available for you to use. Press 2 to highlight the L1 header, and press CLEAR to clear the list.
 - c. Press ENTER to move to the first element of L1.
 - d. Enter the bounce numbers, starting from 0 and ending with 5. Press ENTER after each entry.
 - e. Use 2 and D to move to the header of L2. Press CLEAR to clear the list.
 - f. Press ENTER to move to the first element of L2.
 - g. Enter the rebound heights, starting with the release height. You must have six elements in L1 and in L2 when you are done.
4. Now that you have the data to be plotted stored in lists in the calculator, you are ready to plot the rebound height versus bounce number.
 - a. Press 2nd [STAT PLOT] and press ENTER to select Plot 1.
 - b. Change the Plot1 settings to match the screen shown here. Press ENTER to select any of the settings you change.
 - c. Press ZOOM and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the x and y ranges set to fill the screen with data.



5. The graph you see is rebound height versus bounce number. The model is an exponential function: $y = hp^x$, where y is the rebound height and x the bounce number. h and p are two parameters that you need to determine. h represents the starting height (sometimes called the zeroth bounce!), or the height before the first bounce. That is the height on the $x = 0$ line of the Data Table, which is also the y -intercept.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

Activity 19

6. To determine the parameter p , you can plot the model equation and try different values for p until one fits best. First, you need to enter the exponential model.
 - a. Press Y= .
 - b. Press CLEAR to remove any existing equation.
 - c. Enter the expression $H \cdot P^X$ in the Y_1 field.
 - d. Press 2nd [QUIT] to return to the home screen.
7. Set a value for the parameters h and p , and then look at the resulting graph. To obtain a good fit, you will need to try several values for p . Use the steps below to store different values to the parameters h and p . Start with $p = 1$. Experiment until you find one that provides a good fit for the data.
 - a. Enter the value for the parameter h , which is the starting height. Press 2nd H ENTER to store the value in the variable H.
 - b. Enter a value for the parameter p . Press 2nd P ENTER to store the value in the variable P.
 - c. Press GRAPH to see the data with the model graph superimposed.
 - d. Press 2nd [QUIT] to return to the home screen.

You will not need to change the value for h further, but the best value for p will be something smaller than 1. Try a variety of values for p until you get a good fit to the experimental data.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

8. The exponential equation $y = hp^x$ can be made into a linear equation by taking the log of both sides:

$$\begin{aligned}y &= hp^x \\ \ln y &= \ln(hp^x) \\ &= \ln h + \ln(p^x) \\ &= x \ln p + \ln h\end{aligned}$$

That is, a graph of $\ln y$ versus x is linear with a slope of $\ln p$ and y -intercept of $\ln h$. You can use the data collected so far to make this linear graph. To do this, you need to create a data list containing the natural log of the rebound heights.

- a. Press LN .
 - b. Press 2nd [L2].
 - c. Press) to close the \ln function.
 - d. Press STO , and press 2nd [L2] to complete the expression $\ln(\text{L2}) \rightarrow \text{L2}$. Press ENTER to perform the calculation.
9. Display a graph of $\ln(\text{rebound height})$ versus bounce number.
 - a. Press ZOOM .
 - b. Press $\text{}$ until ZoomStat is highlighted; press ENTER to display a graph with the x and y ranges set to fill the screen with data.
 - c. Press TRACE to determine the coordinates of a point on the graph using the cursor keys.

10. Since the graph is nearly linear, have the calculator fit a line to the data.
- Press **STAT** and use the cursor keys to highlight CALC.
 - Press the number adjacent to LinReg(ax+b) to copy the command to the home screen.
 - Press **2nd** [L1] **,** **2nd** [L2] **,** to enter the lists containing the data.
 - Press **VAR** and use the cursor keys to highlight Y-VARS.
 - Select Function by pressing **ENTER**.
 - Press **ENTER** to copy Y1 to the expression.
 - On the home screen, you will now see the entry LinReg(ax+b) L1, L2, Y1. This command will perform a linear regression with L1 as the x and L2 as the y values. The resulting regression line will be stored in equation variable Y1. Press **ENTER** to perform the linear regression.
- ⇒ Answer Question 3 on the *Data Collection and Analysis* sheet.
- Press **GRAPH** to see the graph.
- ⇒ Answer Questions 4-7 on the *Data Collection and Analysis* sheet.

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

Record the consecutive maximum rebound heights for the five bounces from Step 3 of the *Analysis* section. Round to the nearest 0.001 m.

Bounce number	Maximum height (m)
0	
1	
2	
3	
4	
5	

QUESTIONS

1. Use the function $y = hp^x$ to explain why h is equal to the y -intercept.

2. Use the parameters from Step 8 to write the optimized model equation.

3. Use the parameters a and b that appear on the calculator screen to write the fitted linear equation.

- How well does the linear regression fit the data of $\ln(\text{rebound height})$ versus bounce number?
- Use the values of h and p you determined by optimizing the fit of the model to find $\ln h$ and $\ln p$. Record these values in the table below.

$\ln p$	
$\ln h$	

How do these values compare with the linear regression constants a and b ? Do they match? Explain why they should match.

Hint: Compare the equations $\ln y = (\ln p)x + \ln h$ and $y = ax + b$.

- Suppose that you repeat this activity using a ball that is not as resilient as the ball you used. For example, if you used a basketball, imagine that some air is let out of it, and then the exercise is repeated. Would the constants h and p in the equation $y = hp^x$ be affected by this? How?
- Use either one of the models developed in this activity to determine the smallest number of bounces required by the ball for the rebound height to be less than 10% of its starting height. Remember that the number of bounces must be an integer value. Record your answer here.