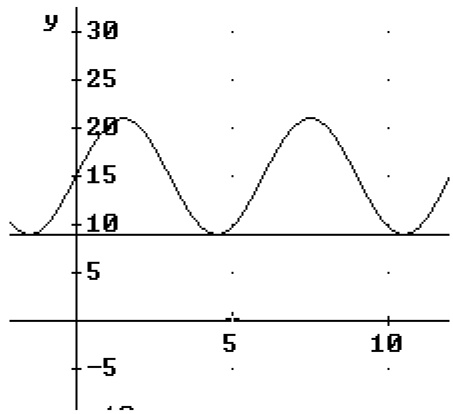
Mathematical Methods (CAS) 2002 *Examination* 2 *solutions Q 4*

Question 4

The first part of this question is conceptual, 15 + 6sin() will have a maximum value of 15 + 6*1 = 21, and a minimum value of 15 − 6*1 = 9. It is useful to draw a graph for a.ii:

#1: $15 + 6\cdot \text{SIN}\left(\frac{\pi\cdot t}{3}\right)$



#2: $\text{SOLVE}\left(15 + 6\cdot \text{SIN}\left(\frac{\pi\cdot t}{3}\right) = 9,\ t\right)$

#3: $t = \frac{9}{2} \vee t = -\frac{3}{2}$

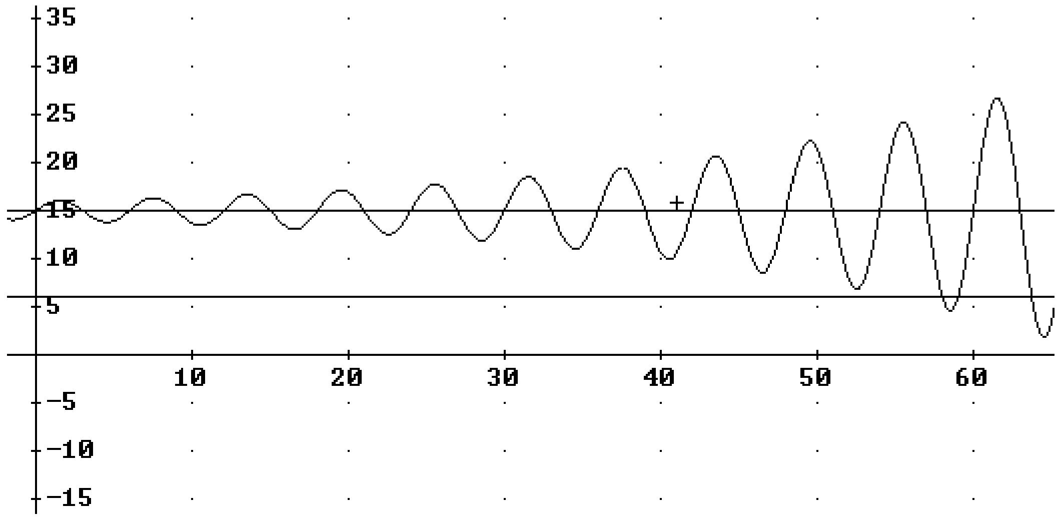
Thus, the required value is t = 9/2 = 4.5 seconds.

This value can also be found simply from a knowledge of transformations and the sin function. The first minimum value of the basic sine function for positive t occurs when t = 3π/2, sin(πt/3) is obtained from sin(t) by a horizontal dilation of factor 3/π, so the first minimum of sin(πt/3), and hence 15+6sin(πt/3),will occur at 3π/2 *3/π = 9/2 =4.5.This approach is quite general for functions of this type.

For the next parts of the question, the use of a defined function is helpful:

#4: $y(t) := 15 + e^{0.04\cdot t}\cdot \text{SIN}\left(\frac{\pi\cdot t}{3}\right)$

A graph is again useful:



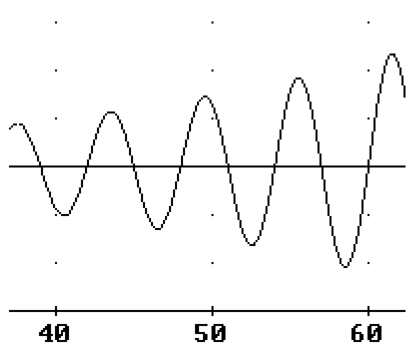
#5: $y(t) = 6$

there are two solutions between 55 and 60, which seem to be around 58 and 59 respectively:

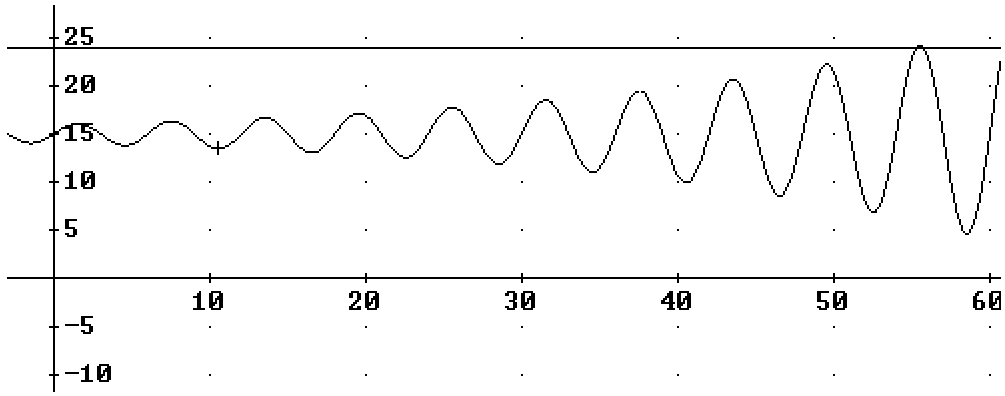
#6: NSOLVE(y(t) = 6, t, 0, 60)

#7: $t = 59.03217732$

the number of times the platform is exactly 15 metres above the ground from t = 40 to t = 59 can be determined by counting from the graph:



This occurs 6 times. To find the time from when the ride starts until the platform first reaches 24 metres above the ground consider the following graph, and solve numerically over a suitable interval:



#8:   NSOLVE(y(t) = 24, t, 50, 60)

#9:   t = 55.7419759

#10: y'(t)

#11:
$$e^{t/25}\cdot\left(\frac{\pi\cdot COS\left(\frac{\pi\cdot t}{3}\right)}{3}+\frac{SIN\left(\frac{\pi\cdot t}{3}\right)}{25}\right)$$

for part ii. the graph above shows that the platform is closest to the ground over its domain when t is between 55 and 60 seconds, and closer to 60 seconds.Hence the equation y'(t)= 0 can be solved numerically over this interval to find the corresponding value of t:the first and required time value is found by evaluating NSOLVE(y(t)=6,t,0,59)to obtain t = 58.03397161 or t = 58.03 correct to 2 decimal places.The second solution is found by evaluating:

#12: NSOLVE(y'(t) = 0, t, 55, 60)

#13: t = 58.5364579

or 58.54 seconds, correct to 2 decimal places. The corresponding distance (using the not rounded time value) is y(58.03397161) = y (58.5364579)=

#14: 4.611189273

or 4.61 metres, correct to 2 decimal places.

The final part of the modelling problem applies to the function:

#15: $h(t) := 15 + a\cdot e^{0.04\cdot t}\cdot SIN\left(\frac{\pi\cdot t}{3}\right)$

From the graph of y(t) above, it can be seen that the maximum gradient will occur at the endpoint time value, t = 60.Including the parameter, a (a positive constant), in the rule for h(t) will multiply the gradient by a scalar factor, compared with that of y(t), but otherwise leave it unchanged. Thus to ensure that h'(t) is never more than 11, it must be the case that a*y'(60) has a maximum value of 11:

#16: NSOLVE(a·y'(60) = 11, a)

#17: a = 0.9529219057

that is, a = 0.953, correct to 3 decimal places.