Parabolas – Paper Folding

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

7 8 9 10 11 12

TI-Nspire™ | Investigation | Student | 50 min

Teacher Notes:

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The Teacher TNS demonstration file provides a dynamic representation of the paper folding and subsequent envelope (parabola) that students should achieve.
The QR code links to an introductory video hosted on YouTube and includes detailed instructions for the paper folding and calculator based instructions.

AC9M9A04

Identify and graph quadratic functions, solve quadratic equations graphically and numerically, and solve monic quadratic equations with integer roots algebraically, using graphing software and digital tools as appropriate.

AC9M9A06

Experiment with the effects of the variation of parameters on graphs of related functions, using digital tools, making connections between graphical and algebraic representations, and generalising emerging patterns.

AC9M10A05

Experiment with functions and relations using digital tools, making and testing conjectures and generalising emerging patterns.

AC9M10A04

Use mathematical modelling to solve applied problems … choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings.

Lesson:

Students should perform the paper folding activity and paste the result into their book/assignment. The physical paper folding will help with the geometry and subsequent coordinate geometry and algebra.

Students should be encouraged to save their TI-Nspire file and create a corresponding folder "Parabolas" to store the results from each activity.

Author: P. Fox

What do satellite dishes, car headlights, bouncing balls and waterspouts have in common? They are all related to the elegant shape and form of a parabolic curve. Satellite dishes and car headlights rely on the physical properties whilst bouncing balls and water fountains are acted upon by gravitational forces which help define their path. When it comes to parabolas, there are many instances of mistaken identity, curves that are look parabolic, but belong to a different family. Consider a chain supported at either end, is it parabolic? Can it be modelled by a parabola? What about the arch of the Sydney Harbor Bridge, is it an inverted parabola?

https://youtu.be/7JMA6p8CNNs

In this activity you will fold an A4 piece of paper repeatedly to create an envelope. The video link above provides a set of instructions for both the paper folding and digital creation.

Step 1

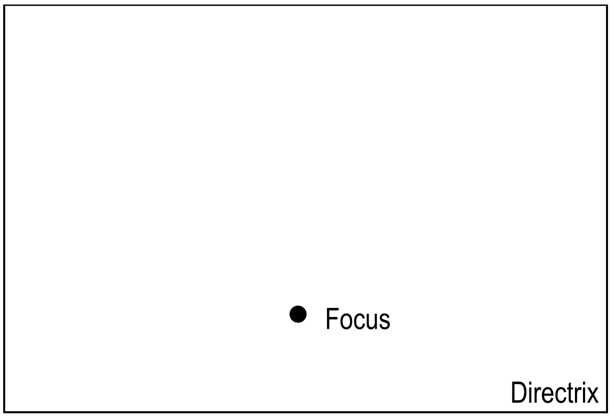
Use an A4 piece of paper in 'landscape' orientation.

Place a clearly visible dot horizontally centred somewhere between 2 to 8cm from the base of the page.

Label the dot: **Focus**

Label the bottom edge of the page: **Directrix**

As shown in the diagram opposite.

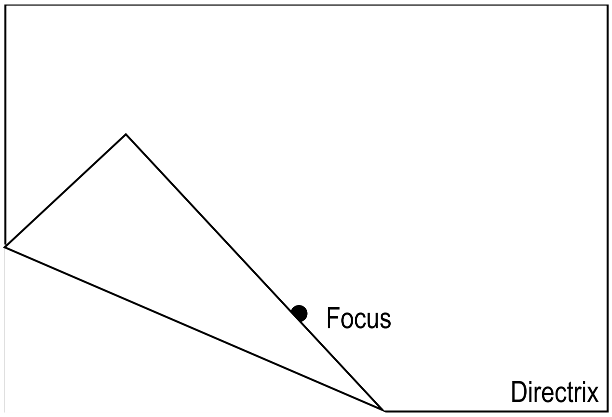


Step 2

Make a fold in the page so that the bottom edge of the paper (the directrix) just touches the dot (focus).

Press firmly on the fold so that it leaves a permanent crease in the paper.

As shown in the diagram opposite.



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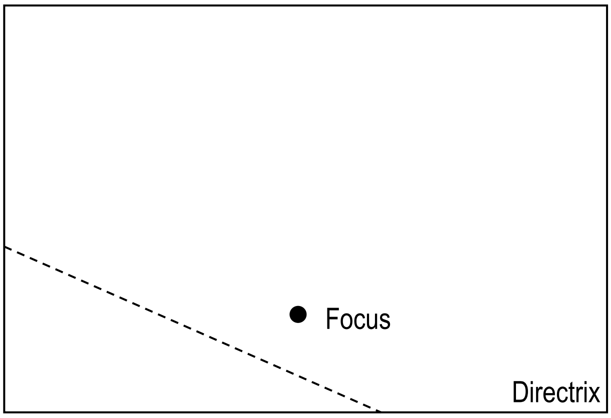
Step 3

Unfold the paper to reveal the crease.

Use a pen or pencil and a ruler to trace over the crease.

As shown in the diagram opposite.



Step 4

Repeat steps 2 and 3 until at least 20 **different** creases are formed on the page; approximately 10 on each side of the focus.

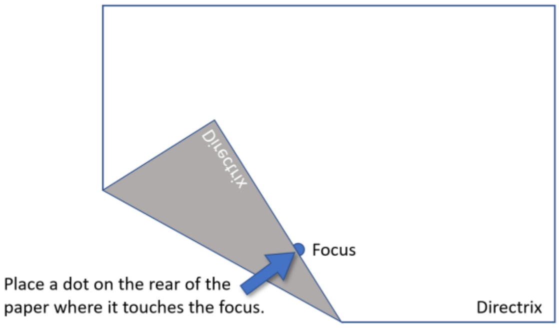
Question: 1.

The family of crease forms an 'envelope' or curve, describe the shape of the envelope.

Answer: Students may describe the envelope as a 'u-shaped' curve or a parabola. If students describe the curve as a parabola, it should be noted that this is yet to be proven.

Using a fresh piece of paper, repeat steps 1 to 3 to create a single fold. This time, when the paper is folded:

- Put a visible dot (P) on the bottom of the page (directrix) where it touches the focus (F).
- Unfold the paper and draw a line from the focus to this dot (PF).
- Measure the angle between the line (PF) and the fold.
- Measure the distances from P to the fold and from the focus (F) to the fold.



Question: 2.

What is the angle between the line (PF) and the fold?

Answer: The angle between PF and the fold is 90°.

Question: 3.

Compare the distances from the dot (P) to the fold (line) and also from the focus (F) to the fold, comment on your findings.

Answer: The two distances are equal. (The fold creates a perpendicular bisector)

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Calculator Instructions: Digital Paper Folding

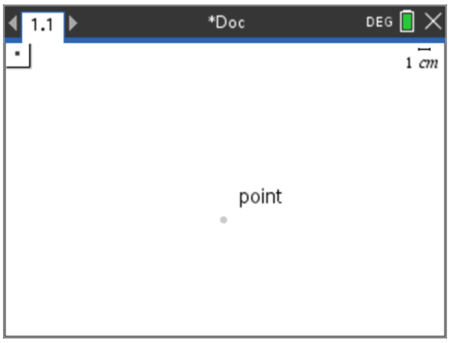
Open a new TI-Nspire document and insert a Geometry application.

Press: **P**

Place a point in the middle of the page (horizontally) and slightly towards the base of the page.

Note:

P provides a short-cut to draw a point. The point tool can also be accessed through the menu by selecting points and lines.



Press:

menu > **Points & Lines** > **Line**

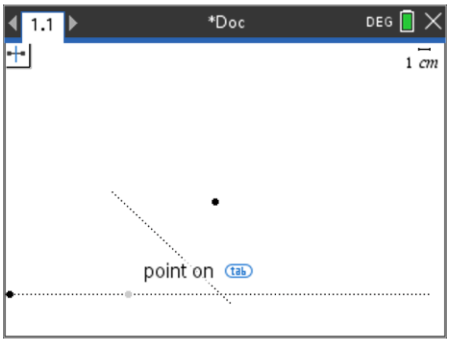
Start the line towards the bottom left of the page. Click to place the first point on the line, then, hold down the shift key and move the pen to the right. The shift key will make it easier to draw a horizontal line.

Press:

menu > **Construction** > **Perpendicular Bisector**

Click on the original point, followed by the horizontal line. A perpendicular bisector will be drawn between the original point and the newly created point on the horizontal line. Press esc to release the tool.

Grab the newly created point on the horizontal line and move it left and right, notice how the perpendicular bisector works.



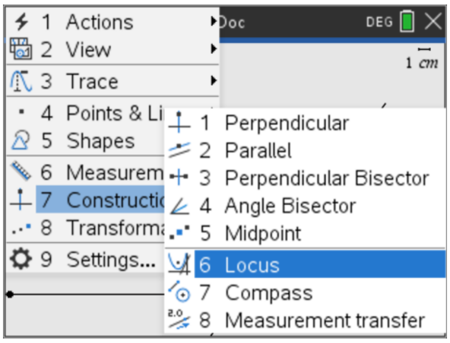
- The option to automatically label points is switched 'off' in the document shown, this is optional. It has been off here to keep the page uncluttered.
- The perpendicular bisector can be lengthened in either direction by grabbing the respective end of the line.

A locus is a curve or shape made by all the points satisfying a particular condition or set of conditions. In this situation, the perpendicular bisector is governed by the location of the line and point.

The locus can be constructed by:

menu > **Construction** > **Locus**

Once the locus tool is active, click on the original point followed by the perpendicular bisector.



The family of straight lines should resemble your paper folding outcome. The difference here is that the result is dynamic. Explore what happens when the original point is move or the horizontal line is moved.

Question: 4.

The series of lines left by the locus forms an 'envelope'. What is the shape of the envelope?

Answer: The envelope is the same shape as the paper folding result, a 'u-shaped' curve (Parabola).

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Question: 5.

Explain how this digital image (locus) relates to the one completed using pencil and paper.

Answer: This locus is the same as the paperfolding. The fold in the paper creates a perpendicular bisector, the same as the digital result.

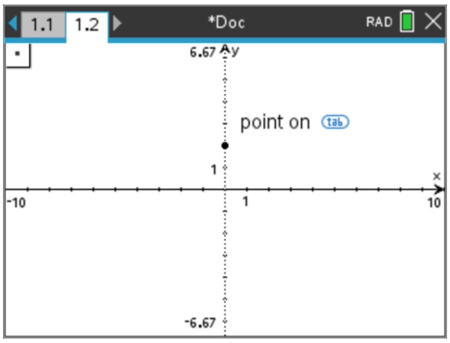
Insert a Graph application into your current document. Press esc to exit the equation entry line.

Press: **P**

Place the point on the Y axis at: (0, 2).

Note:

Placing the point over the hash mark on the y-axis will lock the point onto hash marks along y-axis.

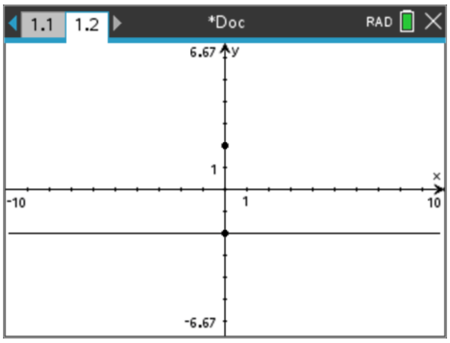


Press:

menu > **Geometry** > **Construction** > **Perpendicular**

Place the perpendicular line on the point (0, -2). (Y-axis)

The point and line can be moved later, for now, these two locations will make things easier once the envelope has been created.

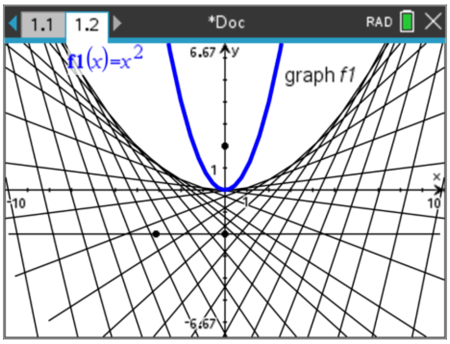


Repeat the instructions for creating a perpendicular bisector and locus.

Once the locus has been created (shown opposite), navigate to the equation entry line and enter the equation:

$$f_1(x) = x^2$$

The graph can be transformed by 'grabbing' either the vertex or the 'sides' of the parabola.



Question: 6.

Can you get the parabola to fit the curve?

Answer: The parabola is an excellent fit for the curve; it is a good *model*.

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