

## Topic Areas

- Periodic Functions
- Sine and Cosine
- Radian and Degree Measure
- Trigonometric Ratios and Trigonometric Functions


## Objectives

- To build from an understanding of trigonometric ratios toward basic concepts in trigonometric functions


## Give Me a Sine (and a Cosine)

## Introduction

In this lesson concept, students contribute sets of data points to build the graphs of the sine and cosine functions.

The transition from trig ratios to trig functions is a particularly difficult one, but it is also one where the social space of the TI-Navigator ${ }^{\text {TM }}$ Activity Center can provide powerful assistance. By taking ownership over different individual values of the sine and cosine functions, students can "bootstrap" their way from a point-by-point (ratio-based) understanding of trigonometry to a holistic (function-based) understanding.

## Prerequisites and Materials

Prerequisites: Ability to calculate trigonometric ratios for the "special" angles (from the $1,1, \sqrt{ } 2$ and the $1, \sqrt{ } 3,2$ triangles)
Materials: The Activity Settings file, GiveMeASine. act located on the CD.

## Setting Up the Activity

From the TI-Navigator teacher computer home screen, open the

This file sets your graph window range as: -.5 to 10 for $X$, and -3.5 to 3.5 for Y . It also configures the Activity Center for students to contribute 3 lists (" $X$ " "SINX" and "COSX") with the ability to preview the appropriate plots.

Activity Center, (\$). Then choose File>Load>Load Activity Settings to load the activity settings file, GiveMeASine. act.

Also before starting, add $\sin (X)$ and $\cos (X)$ as hidden functions in the Equation Gallery. Switch to the Graph-Equation tab, enter $\sin (X)$ in the equation entry field and click Add. Then enter $\cos (X)$ and click Add. For a visual distinction between them, turn the cosine function yellow by clicking on its green color swatch and
choosing that color in the Graphing Pen Attributes dialog box that appears. Finally, make the two equations invisible for now, by selecting both of them and clicking Hide.

## Preparing the Class for Gathering Sine and Cosine Values

Ask your students to count off 1-4 (repeating). Then describe the following special angles:

| ${ }^{*}$ | 1 a | 1 b | 1 c | 1 d | 2 a | 2 b | 2 c | 2 d |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $\pi / 6$ | $\pi / 4$ | $\pi / 3$ | $\pi / 2$ | $2 \pi / 3$ | $3 \pi / 4$ | $5 \pi / 6$ | $\pi$ |


|  | 3 a | 3 b | 3 c | 3 d | 4 a | 4 b | 4 c |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Start off the discussion by reminding your students of the definitions of $\sin (X)$ and $\cos (X)$ where $X$ is the measure of an angle in a right triangle (the "reference triangle").

Now draw a series of triangles on the board with decreasing angle sizes, such as the ones shown in the worksheet. Justify the values of $\sin (0)$ and $\cos (0)$ by virtue of the "trend" you are seeing in the triangle series. Fill in the value for $\pi / 12$, asking students to make an estimate.

The more clearly your students understand this discussion the more inventive they will be in choosing values, in their individual work, below. This will improve their understanding of the functions and will lead to a more diverse aggregate pattern. If your students are struggling, consider having them work in pairs (pairs of 1 s , of 2 s , etc.).

## Filling in the Worksheet

In the next step, students do the individual work to provide values for the functions' graphs. Each student will have the task of entering values for sine and cosine for four values of $X$, as shown above - the students in the " 1 " group should enter values for 1a, 1b, 1c, and 1d. To do this, they should use their knowledge of radian measure (radian to degree conversion) their ability to draw the reference triangle for each of these angles, and their knowledge of the trig ratios for these special angles.

Use the worksheet attached to ensure that students are working through the trig ratios, rather than simply using the calculator functionality to enter the values of the functions.

As described in the worksheet, each student must also identify a value of $X$ that is between each pair of numbers that they are responsible for. For example, a student with countoff number 1 should enter values of $X, \sin (X)$, and $\cos (X)$ for $X$ between $\pi / 6$ and $\pi / 4$, between $\pi / 4$ and $\pi / 3$, and between $\pi / 3$ and $\pi / 2$. This will give them practice in "thinking in radians" and it will provide a more fully filled-in class picture of the sine and cosine functions.

## Aggregating Sine and Cosine Value Data

Start the activity, . As students submit their data lists, patterns will emerge. Once students have submitted their lists, encourage them to work with a neighbor to check their work and compare results. The activity settings you loaded allow students to resubmit lists, so if they discover errors in their contributions, they can correct them.

In discussing the results, you may wish to turn off one of the two plots so that you can consider the functions one at a time. To do this, switch to the ListGraph tab and click Plot Options. On the plot options dialog, you can set one or the other of your plots to be showing by checking or unchecking On.


When the class has seen and discussed the pattern of the sine function, you can switch to the Graph-Equation tab and make the hidden graph visible, by selecting it and clicking Show.

After discussing the data in the shared public space of the Activity Center, you may wish for your students to be able to work on their own with the same data. To do this, stop the activity, $\quad$. Open the Contribute Lists configuration dialog by clicking Configure.


On this dialog, choose Existing activity lists. Then start the activity, . You can have students interact with the data again within the Activity Center, viewing the plots on their devices. These lists and the appropriate plots are also saved on the students calculators (the list names are X, SINX, and COSX), so that they can experiment with the data "offline").

## Next Steps

Depending on your students' comfort with the concept of trigonometric functions you may want to repeat this activity to explore the graphs of related functions such as $\sin \left(n^{*} X\right)$ and $n * \sin (X)$, and so forth.

## "Give Me a Sine" Worksheet

Name $\qquad$


| Group | * | * | 1 a |  | 1 b |  | 1 c |  | 1d |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X | 0 | $\pi / 12$ | $\pi / 6$ |  | $\pi / 4$ |  | $\pi / 3$ |  | $\pi / 2$ |
| $\sin (\mathrm{X})$ |  |  |  |  |  |  |  |  |  |
| $\cos (\mathrm{X})$ |  |  |  |  |  |  |  |  |  |


| Group | 2 a |  | 2 b |  | 2 c |  | 2d |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X | $2 \pi / 3$ |  | $3 \pi / 4$ |  | $5 \pi / 6$ |  | $\pi$ |
| $\sin (X)$ |  |  |  |  |  |  |  |
| $\cos (X)$ |  |  |  |  |  |  |  |


| Group | 3a |  | 3b |  | 3c |  | 3d |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X | $7 \pi / 6$ |  | $5 \pi / 4$ |  | $4 \pi / 3$ |  | $3 \pi / 2$ |
| $\sin (\mathrm{X})$ |  |  |  |  |  |  |  |
| $\cos (\mathrm{X})$ |  |  |  |  |  |  |  |


| Group | 4 a |  | 4 b |  | 4 c |  | 4 d |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X | $5 \pi / 3$ |  | $7 \pi / 4$ |  | $11 \pi / 6$ |  | $2 \pi$ |
| $\sin (\mathrm{X})$ |  |  |  |  |  |  |  |
| $\cos (\mathrm{X})$ |  |  |  |  |  |  |  |

