Teachers Explanatory Notes
TI-30XB MultiView™: Factor Trees

T

W2

PPT

1. Photocopy the factor tree workshops for students to use.

2. It is important that students have a few *number divisibility* techniques for the exploration component of this worksheet. (Refer calculator use below)
These number techniques include, but are not limited to:

Divisibility by 2:

The number is even.

Divisibility by 3:

Add the digits in the number together.
If the sum of these digits is divisible by 3 then the original number is also divisible by 3.

Example:

Consider the number 1278. This number is divisible by 3 since 1 + 2 + 7 + 8 = 18.
The digits in 18 are also divisible by 3.
The number 1279 is NOT divisible by 3. 1 + 2 + 7 + 9 = 19 and 19 is not divisible by 3.

Divisibility by 5:

The unit's digit of the number must be 0 or 5.

Divisibility by 7:

Double the last digit of the number and subtract if from the rest of the digits, repeat this process for larger numbers. If the result is 0 or is divisible by 7, then the original number is divisible by 7.

Example:

Consider the number 882; 2 × 2 = 4 and 88 – 4 = 84.
The number 84 is divisible by 7 since 7 × 12 = 84.

Consider the number 5922, 2 × 2 = 4 and 5922 – 4 = 588. This number is still relatively large so the process is repeated. 8 × 2 = 16, and 58 – 16 = 42; 42 is divisible by 7 therefore 5922 is also divisible by 7.

Note: This divisibility rule is more complicated than 2, 3 and 5. Students will most likely use their calculator to check divisibility in this case.

3. Work through example factor trees. Note that some factors have already been included which forces the factor tree to progress in a predefined way. Photocopying the worksheet onto an overhead transparency is a useful *shortcut*. Ensure students understand that all factors must be continued to the base line *(as per sample trees)*.

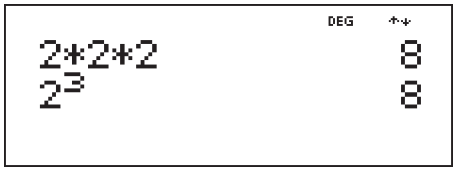
4. In question 9 students are prompted to use previous results to help in the factorisation process. Question parts have been aligned such that $294 \div 42 = 7$. The prime factorisation of 42 is $2 \times 3 \times 7$. Therefore the prime factorisation of 294 is $2 \times 3 \times 7^2$. A similar technique has been applied for question 14, relating back to questions 9 and 7.

   The final question relating to the next prime years… 2011 is a prime number, so too 2017. Students should recognise that 2010, 2012, 2014, 2015 and 2016 are clearly NOT prime numbers. Extrapolation from 19 and 23 being prime to 2019 and 2023 not being prime is not true since 2019 is divisible by 3: $2019 = 3 \times 673$ and $2023 = 7 \times 17^2$.

Using your Calculator

Students should not need to use their calculator to construct the factor tree but they may use it to check some of the products for each row. The bottom row of the factor tree often contains repeated factors. For example, the prime factorisation of 36 is $2 \times 2 \times 3 \times 3$. A more abbreviated form of this expression is: $2^2 \times 3^2$. A more significant result would be the prime factorisation of the number 3888. The prime factorisation of 3888 could be written as $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$ or in abbreviated form: $2^4 \times 3^5$.

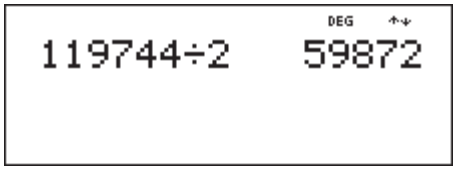
Students should be encouraged to use the exponential notation on the calculator for prime factor expressions. The two dimensional representation used on the calculator reinforces correct mathematical notation. (Shown opposite)



Question 13 from the worksheet requires students to find the prime factorisation of relatively large numbers. Students should use the common divisibility checks mentally to assist them in factorising these large numbers.

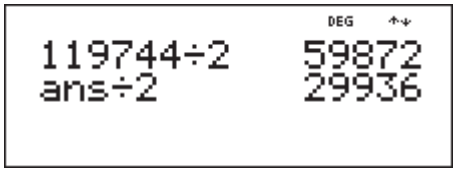
For example: 119744

- This number is even; therefore it is divisible by 2.
  Students should interpret the result as: $2 \times 59872 = 119744$



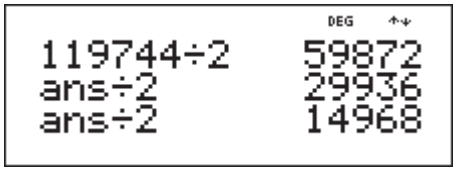
- The result of this first division is even; therefore further division by 2 is possible. Students are able to press ÷ 2 immediately, the calculator displays the expression 'ans' signifying that the previous answer 59872 is being divided by 2.

  Students should interpret the result as:
  $2 \times 2 \times 29936 = 119744$



- The result of this first division is even; therefore further division by 2 is possible. Repeating the above process results in the screen shown opposite.

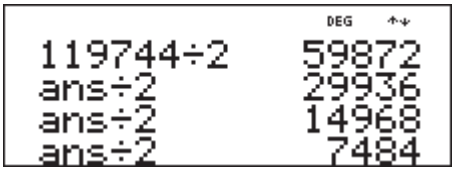
  Students should interpret the result as:
  $2 \times 2 \times 2 \times 14968 = 119744$



- Note that previous calculations are still displayed on the screen. The calculator screen clearly displays the result of 119744 divided by 2, followed by two further divisions.

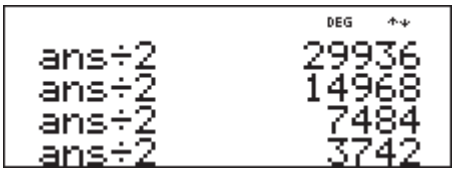
- One more division by 2 has been performed here; further division will see the original number scroll off the screen.

  Students should interpret the result as:
  $2 \times 2 \times 2 \times 2 \times 7484 = 119744$

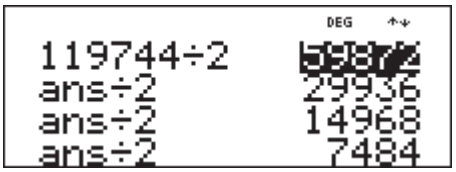


- One more division by 2 has been performed here.

  Students should interpret the result as:
  $2 \times 2 \times 2 \times 2 \times 2 \times 3742 = 119744$

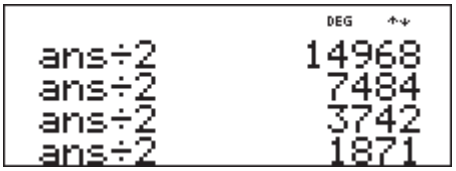


- The original number has now scrolled off the screen, it may however be returned by using the up arrow. Use the down arrow to return to the blank space ready for more calculations.



- One more division by 2 results in an odd number. No more division by 2 is possible. This is an important concept for students to understand. Regardless of what the result is divided by, no further division by 2 is possible.

  Students should interpret the result as:
  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1871 = 119744$

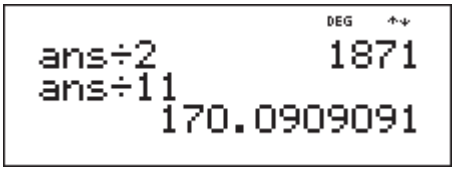


- This expression is also getting very large and should be abbreviated using index notation:

  $2^6 \times 1871 = 119744$

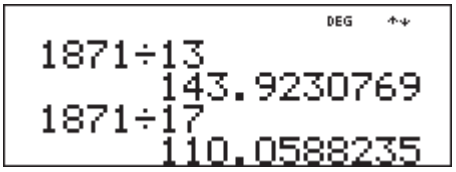
  A quick mental check for divisibility by 3 reveals that the result is NOT divisible by 3. Similarly, the result is not divisible by 5 or 7. Using this check: $187 - (2 \times 1) = 185$, further reduction yields $18 - (2 \times 5) = 8$ which is not divisible by 7.

- Students can proceed to other PRIME factors and check for divisibility.
- The number 1871 is not divisible by 11.

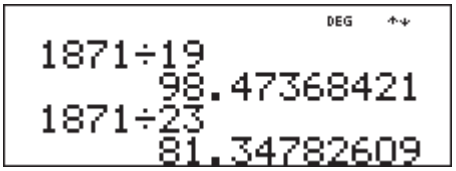


IMPORTANT

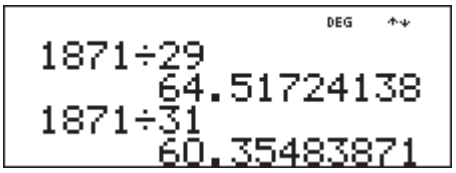
- The 'ans' is no longer 1871. Clearing this last calculation will not return the 'ans' to 1871. Student must now type in the number 1871 to check for further divisibility.



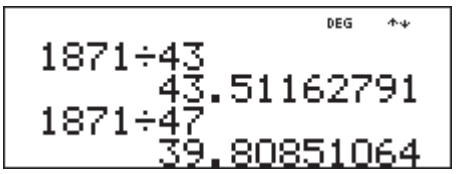
- Once again, students should recognise that ONLY prime numbers need to be tested. For example, if 15 was checked for divisibility, 1871 would have been divisible by 3 and 5.
- Students should use their answers from question 6 here to expedite this process. (Write down all the prime numbers up to 60).



- At first it would seem that this process will continue for a long time.



- The largest number that students will need to use in the divisibility check is 43. Using prime numbers larger than 43 results in divisions smaller than 43.



Please Note:

It is important to review the notion of *factor pairs*. Dividing 1871 by prime numbers larger than 43 results in answers smaller than 43. The result of this division must be a prime number, however all the prime numbers less than 43 have already been tried. The conclusion, 1871 is a prime number.

The number 119744 has been included here to challenge students understanding of factorisation, in particular prime factorisation.