Transformations 1

Translations - a Graphical Approach

Teacher Notes and Answers

7 8 9 10 **11** **12**

TI-Nspire™ Activity Student 1 hour

Introduction

The Transformations activities in this series involve using multiple functionalities of TI-Nspire to explore the concept of transformations of the plane – translations, dilations, reflections, and combinations of these – and their effect on functions. The aim of this approach is to develop a sound understanding by linking graphical and algebraic representations of transformation problems and their solution and making sense of the algebra through visualisation of the relationship between the graph of the original function and the graph of the transformed function.
This is the first activity in the series, and it will focus on **translations**.

Exploration 1. Translations

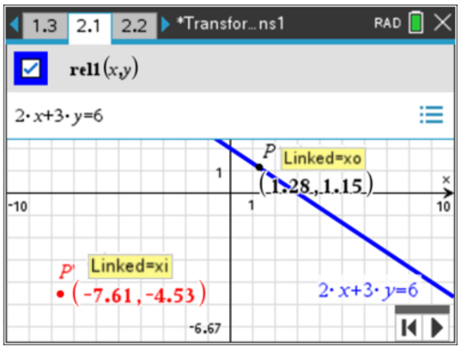
In this part of the activity, you will explore the image of the line $2x + 3y = 6$ under the set of transformations below:

- translation of 5 units in the negative direction of the x-axis (i.e. 5 units left)
- translation of 1 unit in the positive direction of the y-axis (i.e. 1 units up).

A variety of approaches will be used to determine the equation of the transformed function.

Please refer to the TI-Nspire document 'Transformation1'

Open the TI-Nspire document 'Transformation1' and select page 2.1. On page 2.1, P is a point on the graph of the functional relation with equation $2x + 3y = 6$. The equation of the graph has been entered as a relation, $\mathbf{rel1}(x, y)$ (i.e. 'Relation 1').
The coordinates of the points shown have been stored as the variables, $P(xo, yo)$ and $P'(xi, yi)$.

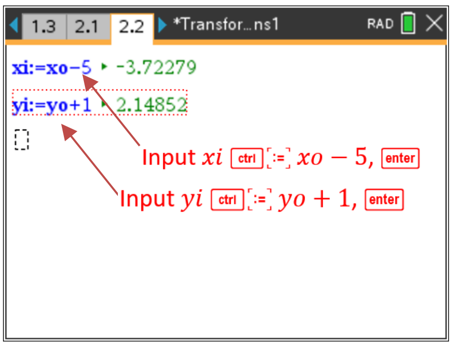


Technology Tip!

An easy way to enter (or edit) the equation of a graph as a relation:
menu > Graph Entry/Edit > Relation.

Making P' the image of P under the transformation

For P' to be the image of P, it needs to be translated:
'5 units left', therefore xi assigned the value $xo - 5$
'1 unit up', therefore yi assigned the value $yo + 5$.
What to do – select page 2.2. This page is a Notes application with two Maths Boxes. Click (☒) the Maths Box so that a red dotted border appears, then input
* In the top box: $xi := xo - 5$ and press enter.
* In the bottom box: $yi := yo + 1$ and press enter.



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The coordinates of P' are now $(xo - 5, yo + 1)$. When point P is moved along the graph on page 2.1, P' will remain the image of P under the transformation.

Question 1

Explain why the statements $xi := xo - 5$ and $yi := yo + 1$ are entered in Maths Boxes on the Notes application, rather than using the Calculator application of TI-Nspire?

Maths Boxes on the Notes application allow computations to be dynamic, so that if point P is moved and the values of (xo, yo) are changed, then the values of (xi, yi) will also be changed accordingly.

In contrast, once a computation is carried out in the Calculator application, the result becomes locked in the 'history'. If the statements $xi := xo - 5$ and $yi := yo + 1$ were entered in the Calculator application, then the values of (xi, yi) would be 'locked in' and will not be changed if point P is moved to a new position.

Question 2

Suppose that you wanted to change the set of transformations to:

• translation of 4 units in the positive direction of the x-axis (i.e. 4 units right)

• translation of $\frac{7}{2}$ units in the negative direction of the y-axis (i.e. $\frac{7}{2}$ units down).

Complete the statements below that will result in P' being the image of P under this new transformation.

$$xi := xo + 4 \text{ and } yi := yo - \frac{7}{2}\ .$$

Graph and equation of the transformed function

P can be animated using the control buttons. Alternatively, P can be grabbed (ctrl+) and dragged along the graph. Press esc to release the 'grabbed' P.

Question 3

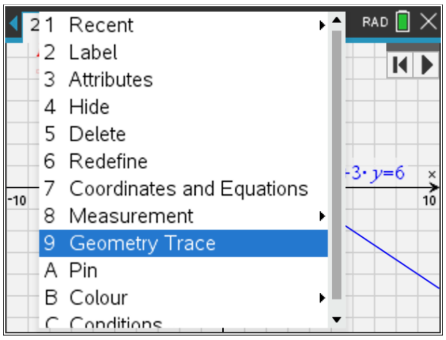
On page 2.1, use the control buttons to move point P, and observe the motion of point P'. Describe the relationship between the positions of P and P' as the points move.

As P' moves to the left of P along a linear path parallel to the graph of $2x + 3y = 6$.

Path of P'

Return P to the original position by clicking the left control button. The 'Geometry Trace' tool will be used to obtain an outline of the path of P'.

What to do. To activate this tool, move the cursor to the point P' and open the context menu by pressing ctrl+menu. Select 'Geometry Trace', then animate P. A trace of P' will be visible. When P' is outside the window settings, return P' to the starting position, then exit the tool by pressing esc.



Question 4

Describe qualitatively some of the key features of the trace of P'. How do these key features relate to the graph of the original function?

P' traces a linear graph parallel to the graph of the original function, but the axes intercepts are to the left and below the original graph.

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Question 5

a. Use the trace of P' to determine the equation of the transformed function.

b. Test whether your equation in part b. is correct by graphing your equation on page 2.1, using the same set of axes as the graph of the original function. You should enter your equation as 'Relation 2' (menu > Graph Entry/Edit > Relation will ensure 'Relation' is selected. (The graph of your equation should contain all points traced by P').

c. Find (i) the gradient and (ii) the coordinates of the axes intercepts of the graph of the transformed function.

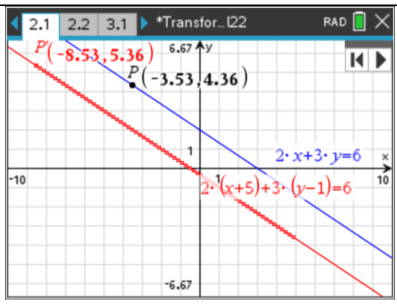
a. $2(x+5)+3(y-1)=6$ (or equivalent).

b. See screen-shot on the right

c. i. gradient $= -\frac{2}{3}$ (by using $\frac{rise}{run}$ on graph, or rearranging eqn.)

ii. When $y=0, x=-\frac{1}{2}$. When $x=0, y=-\frac{1}{3}$.

x-intercept $\left(-\frac{1}{2}, 0\right)$ y-intercept $\left(0, -\frac{1}{3}\right)$



Question 6

What is the relationship between the gradient and axes intercepts of the graph of the transformed function and the gradient and axes intercepts of the graph of the original function?

The gradient remains unchanged by the translations, but the translations change the axes intercepts as per the calculations in Question 5.c.ii. above.

Relationship between equation of the transformed and original functions

Now consider how the equation of the transformed function could have been obtained from the equation of the original function, $2x+3y=6$?

Question 7

Which one of the following is the correct equation for the transformed function? Test your answer by entering the equation as 'Relation 2' on page 2.1. The graph should contain all points on the trace of P'.

A. $2(x-5)+3(y+1)=6$

B. $\mathbf{2(x+5)+3(y-1)=6}$ correct equation

C. $2(x+5)+3(y+1)=6$

D. $2(x-5)+3(y-1)=6$

Explaining the equation of the transformed function

We saw that the point $P(x,y)$ maps to point $P'(x-5, y+1)$ under this transformation. That is, if P' has coordinates (x', y'), then

$x' = x - 5$ ...*(eqn1)*

$y' = y + 1$ ...*(eqn2)*

Question 8

Use the equations in blue above to explain why the graph of the transformed function - which contains all points on the trace of P' - has equation $2(x+5)+3(y-1)=6$.

Rearrange *eqn1* to $x = x' + 5$ and *eqn2* to $y = y' - 1$. Substitute in $2x+3y=6$.

$2(x'+5)+3(y'-1)=6$. Hence transformed function is $2(x+5)+3(y-1)=6$.

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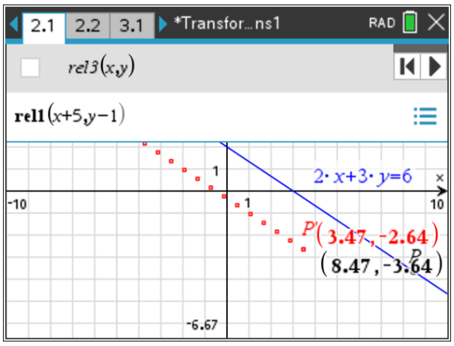
Question 9

On page 2.1, navigate to the Graph Entry (menu > Graph Entry/Edit > Relation). Turn off ‘Relation 2’ and for ‘Relation 3’ input $\text{rel1}(x+5, y-1)$, as shown on the right.



a. What is the relationship between the graph of ‘Relation 3’ and the trace of P' ? ‘Relation 3’ should contain all points on the trace of P'.

b. Write the equation of ‘Relation 3’ in the form $a(x-h)+b(y-k)=c$.

$\text{rel1}(x, y)$ is $2x+3y=6$ therefore
$\text{rel1}(x+5, y-1)$ is $2(x+5)+3(y-1)=6$

Generalising the equation of transformed functions involving translations

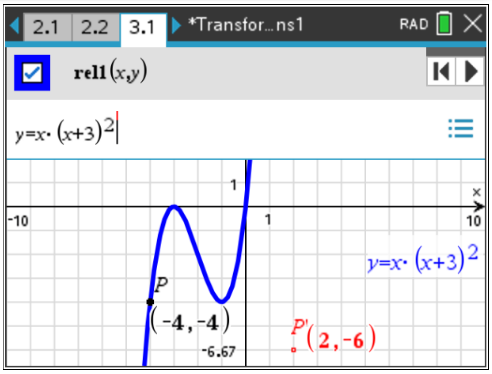
You will now consider the form of the equation of the transformed function for any functional relationship, whether linear or non-linear.
You will explore the equations of the transformed functions for all the cases in **Table 1** below.

What to do. Step 1. Erase old Geometry Trace. Navigate to page 3.1. Erase the old geometry trace before starting the next problem from Table 1.

Technology Tip!

Erasing the ‘Geometry Trace’: move the cursor to a blank part of the graphing window and open the context menu (ctrl+menu). Select ‘Erase Geometry Trace’.

Step 2. Edit Equation. On page 3.1, replace the equation of ‘Relation 1’ with the equation of the original function for the next problem in Table 1.



For example, second problem from Table 1: Edit ‘Relation 1’ to $y = x\cdot(x+3)^2$, as shown on the right.

Step 3. Edit translations. On page 3.2, edit the Maths Boxes to match the set of translations listed for that problem in Table 1.

Step 4. Obtain a trace of P' .

Return P to the original position by clicking the left control button.
Activate the Geometry Trace tool: move the cursor to the point P' and open the context menu by pressing ctrl+menu. Select ‘Geometry Trace’, then animate P by clicking the right control button.
A trace of P' will be visible. When P' is outside the window settings, return P' to the starting position, then exit the tool by pressing esc.

Step 5. Equation of transformed function: its graph should contain all points on the trace of P'.

- Write down an equation which you believe corresponds with the transformed function.

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- Test whether your equation contain all points on the trace of P'. On page 3.1, enter your equation for the transformed function as 'Relation 2'.
- On page 3.1, also enter the transformed function as 'Relation 3' in the form: $\mathrm{rel1}(x-a, y-b)$, as was shown in Question 9 above. The graph of 'Relation 3' should also contain all points on the trace of P'.

Step 6. Use algebra to confirm the equation of the transformed function.
Point $P(x, y)$ maps to point $P'(x-p, y-q)$ under a set of translations (where p, q can be positive or negative real numbers). That is, if P' has coordinates (x', y'), then

$x' = x - p$
$y' = y - q$

Use a method like the one used in Question 8 above to confirm the equation of the transformed function. Repeat Steps 1 – 6 for the next problem in Table 1.

Question 10
Follow Steps 1 – 6 above for each original equation below and complete the table.

Original equation	Original equation **TABLE 1**	Equation of transformed function
a. $y = 2 - \left(\frac{x+1}{2}\right)^3$	3 units left, 4 units down	$(y+4) = 2 - \left(\frac{(x+3)+1}{2}\right)^3$ $y = -6 - \left(\frac{x+4}{2}\right)^3$ (or equivalent)
b. $y = x \cdot (x+3)^2$	6 units right, 2 units down	$(y+2) = (x-6) \cdot ((x-6)+3)^2$ $y = (x-6) \cdot (x-3)^2$ (or equivalent)
c. $2x - 5y = 12$	4 units left, 1 unit up	$2(x+4) - 5(y-1) = 12$ $2x - 5y = -1$ (or equivalent)
d. $y = 3 - \sqrt{4-x}$	3 units right, 2 units up	$(y-2) = 3 - \sqrt{4-(x-3)}$ $y = 5 - \sqrt{7-x}$ (or equivalent)

Question 11
Show all working used in Step 6 above to obtain the equation of the transformed function using algebra. Graph your equation to verify that it is an equivalent form of the equation from Step 5.

a. $x' = x - 3$, $y' = y - 4$ therefore $x = x' + 3$, $y = y' + 4$. Substitute into equation of original function, as shown in third column of Table 1.
b. $x' = x + 6$, $y' = y - 2$ therefore $x = x' - 6$, $y = y' + 2$. Substitute into equation of original function, as shown in third column of Table 1.
c. $x' = x - 4$, $y' = y + 1$ therefore $x = x' + 4$, $y = y' - 1$. Substitute into equation of original function, as shown in third column of Table 1.
d. $x' = x + 3$, $y' = y + 2$ therefore $x = x' - 3$, $y = y' - 2$. Substitute into equation of original function, as shown in third column of Table 1.

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