Maclaurin Polynomials

ID: 10128

Time required
30 minutes

Activity Overview

In this activity, students will use the TI-89 to investigate Maclaurin polynomials. They produce polynomials that approximate very special functions.

Topic: Applications of integration

- *Compute the Maclaurin series for sin(x), cos(x), f(x) = e^x, and* $f(x) = \dfrac{1}{1-x}$.

Teacher Preparation

This investigation offers opportunities for students to begin their studies of power series approximations, deriving Maclaurin series expansions for common functions. This investigation offers opportunities for review and consolidation of their skills. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding. Students should already be familiar with a Taylor polynomial, that is:

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

- *The activity can serve to consolidate earlier work on differentiation, series and approximations.*
- *Students should be able to graph functions.*
- *This activity is designed to be* ***teacher-led.*** *You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. Although the majority of the ideas and concepts are only presented in this document, be sure to cover all the material necessary for students' comprehension.*
- *The students will need to be able to use commands on their own.*
- *Before starting this activity, students should go to the home screen and select* ***F6:Clean Up > 2:NewProb****, then press* [ENTER]*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10128" in the keyword search box.***

Associated Materials

- *MaclaurinPolynomials_Student.doc*

Functions that are continuous and differentiable at a center can be approximated by polynomials. One such way of doing this is to generate a Maclaurin polynomial for a function.

An approximating polynomial is to be expanded about the center *c* which is in the domain of a function *f*. If this *c* has the same value in a polynomial *P* and function *f* then *P*(*c*) = *f*(*c*). Graphically, *P*(*c*) = *f*(*c*) means that the graph of *P* passes through the point (*c*, *f*(*c*)).

A Maclaurin polynomial is a polynomial that is based upon a function's derivatives at *c* = 0.

Specifically, the *n*th Maclaurin polynomial is defined as

$$P_n(x) = \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

Problem 1 – Maclaurin polynomial for *f*(*x*) = sin(*x*)

In generating the third degree Maclaurin polynomial for *f*(*x*) = sin(*x*), we compute (Have students compute and fill the missing answers.):

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3$$

$f(0) = \underline{0}$ $f''(0) = \underline{0}$

$f'(0) = \underline{1}$ $f'''(0) = \underline{-1}$

After the derivatives are computed and values are substituted, the resulting Maclaurin polynomial is:

$$P_3(x) = 0 + (1)x + (0)x^2 - \frac{(1)}{3!}x^3$$

$$P_3(x) = x - \frac{1}{6}x^3$$

Have students plot the graphs of *f*(*x*) and $P_3(x)$. It is important to point to students that the polynomial begins its approximation from (0, *f*(0)). Students use the Taylor function, ***taylor(sin(x), x, degree)***. Students change the value of degree to view different graphs of polynomials approximated by sin(*x*).

What do you notice when the degree is 1 and 2? Why do you think this is?

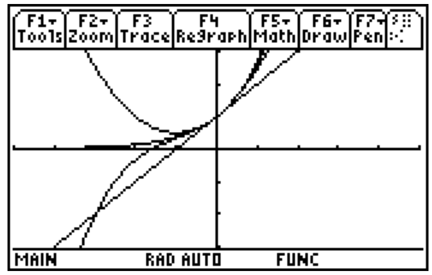
Their graphs are the same because the polynomials P_1 and P_2 are equal.

What do you notice when the degree is 3 and 4? Why do you think this is?

Their graphs are the same because the polynomials P_3 and P_4 are equal.

Students should also notice that because every sine value is 0, every even derivative will be missing.

Problem 2 – Maclaurin polynomial for $f(x) = e^x$



1. Write $P_1(x)$, $P_2(x)$, and $P_3(x)$ for $f(x) = e^x$

$$P_1(x) = 1 + x$$

$$P_2(x) = 1 + x + \frac{x^2}{2!}$$

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

2. Graph $f(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$

What do you notice?

All the approximations of e^x and the function $f(x)$ all have the value 1 at $x = 0$.

Problem 3 – Maclaurin polynomial for $f(x) = \cos(x)$

1. Find $P_8(x)$ for $f(x) = \cos(x)$

$$P_8(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

2. What do you notice about the value of each derivative after 0 has been substituted?

The cosine values are either 1 or –1 and the sine values are 0.

3. What do you notice about the approximated polynomial?

Every odd derivative is missing. For example, $f'(x)$, $f'''(x)$, $f^{(5)}(x)$, and $f^{(7)}(x)$

4. Write two expressions to describe you findings in the previous question when differentiating $\cos(x)$ in terms of n.

$$f^{(2n)}(0) = (-1)^n$$

$$f^{(2n+1)}(0) = 0$$

5. Graph $P_8(x)$ and $f(x) = \cos(x)$.

What do you notice?

The graph begins to take the shape of the cosine then goes to infinity.