Polar Coordinates

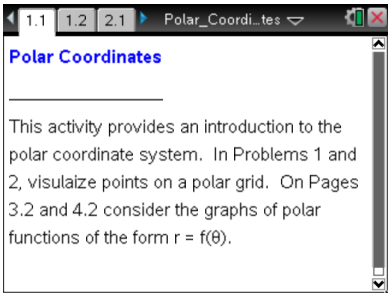
Student Activity

Name _____

Class _____

Open the TI-Nspire document *Polar_Coordinates.tns*.

In this activity, you will be introduced to the polar coordinate system. You will plot points given in polar form, convert polar coordinates to the rectangular coordinate system, and sketch the graphs of polar equations.



The polar coordinate system is a two-dimensional coordinate system defined by a point, called the pole, and a ray from the pole, called the polar axis. In a rectangular coordinate system, the **pole** is usually placed at the origin, and the **polar axis** is represented by the positive x-axis. A point in the polar coordinate system is represented by the ordered pair (r,θ) where r is the distance from the pole and θ is the angle (in radians) measured counterclockwise from the polar axis.

Problem 1 – Identifying The Quadrants in a Polar System

1. **Move to page 1.2.** On this page, the left work area contains a slider for r and a clicker for θ. The point (r,θ) is plotted in the right panel along with a position vector. Change the values of r and θ as needed to answer the following questions.
   a. Complete the following tables by finding the quadrant in which the point (r,θ) lies.

r	1.7	1.3	–0.6	–4.2	–3.2	3.1	–1.5	–2.5
θ	$\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$\frac{3\pi}{4}$	$-\frac{4\pi}{3}$	$-\frac{13\pi}{4}$	$\frac{13\pi}{12}$	$-\frac{7\pi}{4}$
Quadrant								

r	0.8	2.1	2	–2.7	4	3.5	–1.4	–3
θ	$\frac{19\pi}{6}$	$\frac{\pi}{4}$	$-\frac{17\pi}{12}$	$\frac{11\pi}{4}$	$\frac{7\pi}{6}$	$-\frac{\pi}{3}$	$\frac{11\pi}{3}$	$\frac{\pi}{3}$
Quadrant								

r	–4	2.7	1	3.9	–5	–2	–1	1.5
θ	$-\frac{\pi}{6}$	$-\frac{11\pi}{3}$	$\frac{23\pi}{12}$	$-\frac{11\pi}{6}$	$-\frac{9\pi}{4}$	$\frac{11\pi}{6}$	$-\frac{7\pi}{6}$	$\frac{9\pi}{4}$
Quadrant								

b. Describe the location of the point with the following polar coordinates:

(i) $r > 0$ and $\theta = 0$

(ii) $r < 0$ and $\theta = \frac{3\pi}{2}$

(iii) $r < 0$ and $\theta = \frac{\pi}{2}$

(iv) $r > 0$ and $\theta = -3\pi$

Problem 2 – Matching Polar Coordinates with Rectangular Coordinates

Move to page 2.1. If a point has polar coordinates (r, θ), then the rectangular coordinates are given by $x = r\cos\theta$ and $y = r\sin\theta$. Similarly, if a point has rectangular coordinates (x, y), then the polar coordinates are (r, θ) such that $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x}$, $x \neq 0$.

2. Complete each of the following tables. Use Page 2.1 to enter polar coordinates and/or rectangular coordinates, to plot the points, and to check your answers. Enter coordinates in the left work area in the appropriate Math Box. The polar coordinates are represented by the point P and the rectangular coordinates are represented by the point R. Remember that there are an infinite number of polar coordinates that can be plotted in a single location. For example, $\left(1, \frac{\pi}{6}\right)$ is equivalent to $\left(-1, \frac{7\pi}{6}\right)$.

a. For each given polar coordinate, find two different polar coordinates that represent the given point.

(r_1, θ_1)	$\left(2, \frac{\pi}{4}\right)$	$\left(3, \frac{7\pi}{4}\right)$	$\left(6, \frac{2\pi}{3}\right)$	$\left(1, \frac{7\pi}{6}\right)$	$\left(-2, \frac{5\pi}{4}\right)$	$\left(\frac{3}{4}, \frac{17\pi}{6}\right)$
(r_2, θ_2)						
(r_3, θ_3)						

b. For each point given in polar coordinates below, determine the rectangular coordinates.

(r,θ)	$\left(3,\frac{7\pi}{3}\right)$	$\left(1,\frac{\pi}{6}\right)$	$\left(-2,-\frac{4\pi}{3}\right)$	$\left(\sqrt{5},-\frac{3\pi}{2}\right)$	$\left(-8,\frac{3\pi}{4}\right)$	$\left(\frac{13}{4},-\frac{\pi}{3}\right)$
x						
y						

c. For each point given in rectangular coordinates below, determine two representations in polar coordinates.

(x,y)	$(3,4)$	$(-\sqrt{2},2)$	$(4,-7)$	$(-\sqrt{3},-1)$	$(-5,5)$	$(7,24)$
r_1						
θ_1						
r_2						
θ_2						

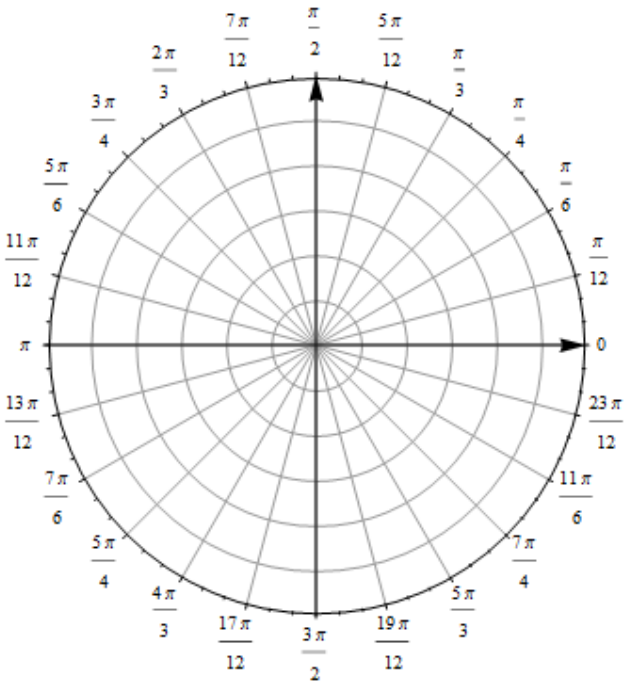
Problem 2 – Graphing Polar Functions

3. **Move to page 3.2**. Over the next two worksheet pages, carefully sketch a complete graph of each given polar equation. Create a table of values, search for patterns, and sketch the graph on the axes provided. Check your results on the handheld and sketch the graph in the right work area of Page 3.2.

Note: Make sure the Graph Type is set to Polar. Use the clicker in the left panel to step through specific points on the curve in polar and rectangular form.

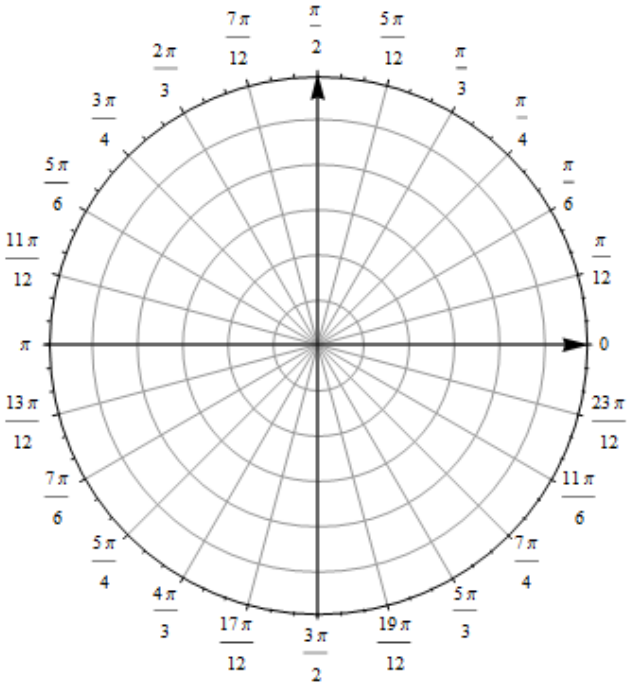
a. $r = 4\cos\theta$.

θ	r



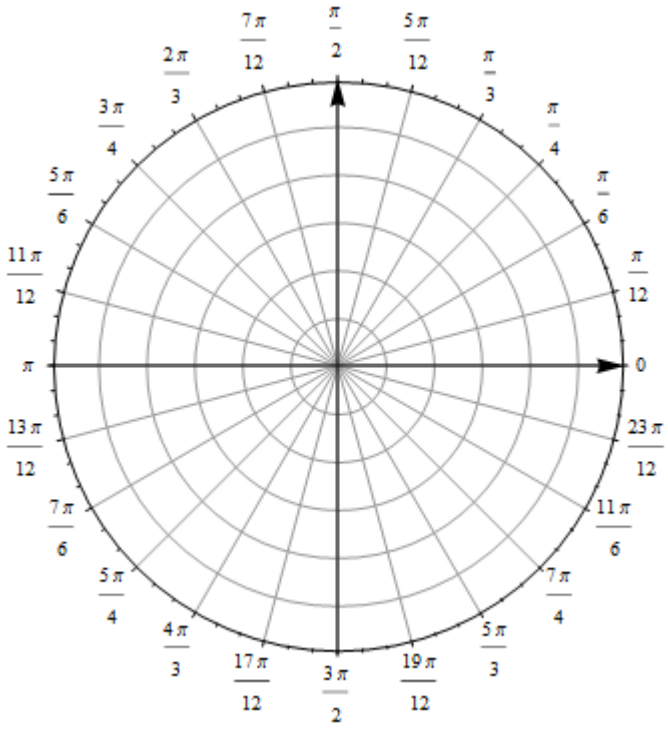
b. $r = 2 + 2\sin\theta$.

θ	r



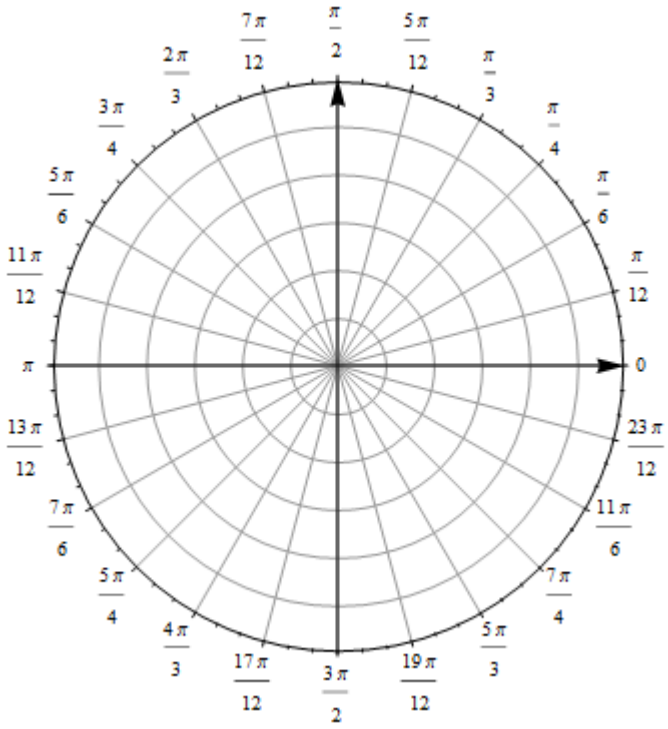
c. $r = 4\cos 3\theta$.

θ	r



d. $r = 1 + 2\cos\theta$.

θ	r



Problem 4 – Comparing Trigonometric Functions in the Polar and Rectangular Systems

In this problem, you will compare the four polar graphs from **Problem 3** with their rectangular system counterparts. **Move to page 4.1**. Graph each of the following on the **Graphs** page provided and answer each corresponding question.

4. $f(x) = 4\cos(x)$

   (a) What shape was created on the polar graph from question 3 (a)?

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

5. $f(x) = 2 + 2\sin(x)$

   (a) What shape was created on the polar graph from question 3(b)?

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude, period, and vertical shift in the explanation.

6. $f(x) = 4\cos(3x)$

   (a) What shape was created on the polar graph from question 3(c)?

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.

7. $f(x) = 1 + 2\cos(x)$

   (a) What Shape was created on the polar graph from question 3(d)?

   (b) What connection can be made between the parts of the trigonometric function in the rectangular system as compared to the polar system? Pay close attention to the amplitude and period in the explanation.