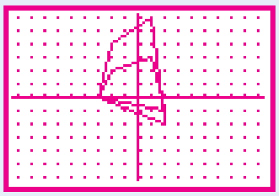
Guiding the Investigation

One step Display the transparency or pass out the Stretching a Polygon worksheet. Ask students how to transform the shape of the original picture to represent the transformed picture. As you watch, suggest as needed that they experiment with specific points.

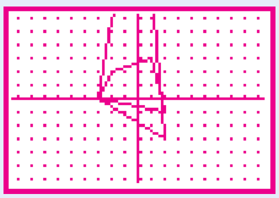
Step 2 Students may work on graph paper instead of calculators. They should graph both the original quadrilateral and the transformed one.

Step 3 If a group has fewer than four members, the first student to finish with one number should repeat the procedure with an unused number. If there are more than four group members, they can make up additional numbers.

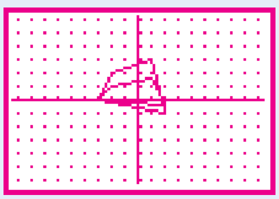
Step 3 For $a = 2$:



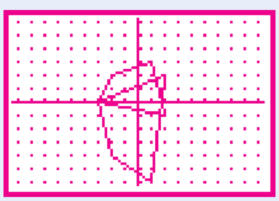
For $a = 3$:



For $a = 0.5$:



For $a = -2$:



Step 4 Some students may want to use a table to show their results. Sharing can be within groups or with the entire class.

The German painter Hans Holbein II (1497–1543) used a technique called anamorphosis to hide a stretched skull in his portrait *The Ambassadors* (1533). You can see the skull in the original painting if you look across the page from the lower-left. The painting was originally hung above a doorway so people would notice the skull as they walked through the door. Holbein may have been making a political statement about these two French ambassadors who were members of England's court of King Henry VIII.

Investigation
Changing the Shape of a Graph

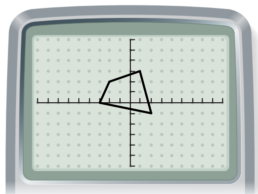
In this investigation you will learn how to stretch or shrink a graph vertically.

Procedure Note

For this investigation, use a friendly window with a factor of 2.

Step 1 $(1, 3)$, $(2, -1)$, $(-3, 0)$, and $(-2, 2)$

Step 1 Name the coordinates of the vertices of this quadrilateral.



Step 2 Graph the quadrilateral on your calculator. Use list L1 for the x-coordinates of the vertices and list L2 for the y-coordinates of the vertices.

Step 3 Each member of your group should choose one of these values of a: 2, 3, 0.5, or -2. Use your value of a to define list L3 and list L4 as follows

$L_3 = L_1$
$L_4 = a \cdot L_2$

Graph a second quadrilateral using list L3 for the x-coordinates of the vertices and list L4 for the y-coordinates of the vertices.

Step 4 The y-coordinates of each vertex are multiplied by the factor a; points above, below, and on the x-axis will behave differently as summarized in the table for Step 5.

Step 4 Share your results from Step 3. For each value of a, describe the transformation of the quadrilateral in Step 2. What was the result for each vertex?

Step 5 Organize your results from this first part of the investigation.

Step 5 Possible answer:

	The whole graph	Points above the x-axis	Points below the x-axis	Points on the x-axis
Factor greater than 1	A vertical stretch	Go farther up, away from the x-axis	Go farther down, away from the x-axis	Unchanged
Factor between 0 and 1	A vertical shrink	Go down closer to the x-axis	Go up closer to the x-axis	Unchanged
Factor less than 0	A stretch or a shrink reflected across the x-axis	As above but reflected across the x-axis	As above but reflected across the x-axis	Unchanged

Step 6 Graph should look like that shown on the student page; $L_1 = \{2, -2, 0, 2\}$, $L_2 = \{-2, -2, 1, -2\}$.

Step 8a $L_3 = L_1$ $L_4 = 3 \cdot L_2$

Step 8b $L_3 = L_1$ $L_4 = 2 \cdot L_2 + 3$

Steps 9 and 10 Graphs depend on equation chosen.

Step 11 The y-values for Y_2 will be twice the y-values for Y_1. This results in a vertical stretch so that the positive range is two times higher and the negative range is two times lower.

Step 12a The y-values for Y_2 will be one-half the y-values for Y_1; a vertical shrink.

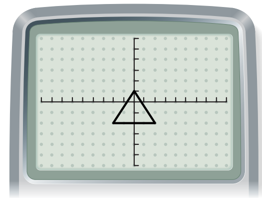
Step 12b The y-values for Y_2 will be three times the y-values for Y_1; a vertical stretch.

Step 12c The values for Y_2 will be two times the y-values for Y_1 and then negated; a vertical stretch and reflection.

Step 13a $R(x) = 3 \cdot B(x)$; $B(x) = \frac{1}{3} \cdot R(x)$

Step 13b $R(x) = -\frac{1}{2} \cdot B(x)$; $B(x) = -2 \cdot R(x)$

Step 6 Graph this triangle on your calculator. Use list L_1 for the x-coordinates of the vertices and list L_2 for the y-coordinates of the vertices.



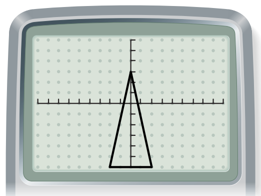
Step 7 Describe how definitions a and b below transform the triangle. Use list L_3 for the x-coordinates of the vertices of the image and list L_4 for the y-coordinates of the vertices of the image. Check your answers by graphing on your calculator.

a. $L_3 = L_1$
$L_4 = -0.5 \cdot L_2$

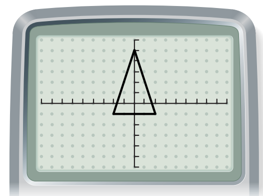
b. $L_3 = L_1$
$L_4 = 2 \cdot L_2 - 2$

Step 8 Write definitions for list L_3 and list L_4 in terms of list L_1 and list L_2 to create each image below. Check your definitions by graphing on your calculator.

a.



b.



Next, see how you can stretch and shrink the graph of a function.

Step 9 Each member of your group should choose an equation from the list below. Enter your equation into Y_1 and graph it on your calculator.

$Y_1(x) = -1 + 0.5x$ $\quad$ $Y_1(x) = |x| - 2$

$Y_1(x) = -x^2 + 1$ $\quad$ $Y_1(x) = 1.4^x$

Step 10 Enter $Y_2(x) = 2 \cdot Y_1(x)$ and graph it. [▶ See **Calculator Note 9B** for specific instructions for your calculator. ◀]

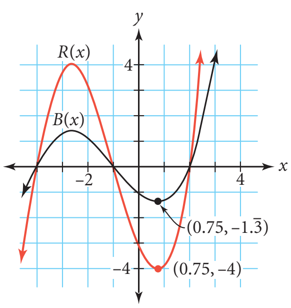
Step 11 Look at a table on your calculator and compare the y-values for Y_1 and Y_2.

Step 12 Repeat Steps 10 and 11, but use these equations for Y_2.

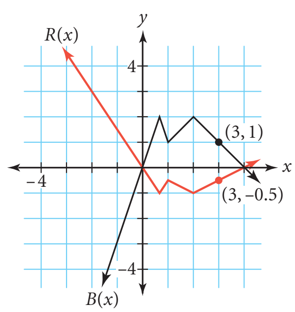
a. $Y_2(x) = 0.5 \cdot Y_1(x)$ $\quad$ **b.** $Y_2(x) = 3 \cdot Y_1(x)$ $\quad$ **c.** $Y_2(x) = -2 \cdot Y_1(x)$

Step 13 Write an equation for $R(x)$ in terms of $B(x)$. Then write an equation for $B(x)$ in terms of $R(x)$.

a.

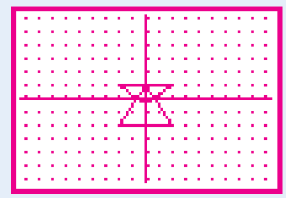


b.

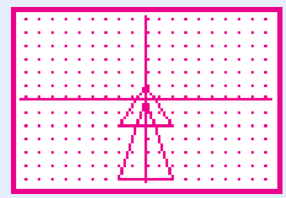


Step 7 As written, the equation in part b indicates a stretch followed by a translation. Students entering $L_3 = 2 * (L_2 - 2)$ will see a translation followed by a stretch, with a different result. If any students make this mistake, ask them to present it later so that the class can learn from the idea.

Step 7a a vertical shrink by a factor of 0.5, then a reflection across the x-axis



Step 7b a vertical stretch by a factor of 2, then a translation down 2 units



Step 8 **[Ask]** Each of these images is obtained by stretching and then perhaps translating the figure in Step 6. If students are confused, help them identify the dilation factor. **[Ask]** "How tall is the original triangle? The transformed triangle is how many times as tall?"

Step 9 If a group has fewer than four members, the first student to finish with one equation should repeat the procedure with an unused equation.

Steps 10 and 12 Calculators vary in syntax. Have Calculator Note 9B handy.

Steps 10 Students may be confused because they're transforming functions that are not parent functions.

SHARING IDEAS

Ask students to report ideas from Steps 5, 8, and 12.

Remind students that, for translations and reflections, the change to a variable in an equation was backward from the change to the corresponding coordinate of a point. **[Ask]** "Is that the case for a vertical stretch or shrink? How and why?"

As a prelude to Improving Your Reasoning Skills, ask about horizontal stretches and shrinks. **[Ask]** "How would you change the coefficients of a point? How would you change the equation?"

If you were able to generate some controversy in Lesson 9.2 about whether or not the functions $y = 33(1 + 0.30)^x$ and $y = 92(1 + 0.30)^x$ were in the same family, you might ask the question again. Elicit the idea that they are both dilations of $y = (1 + 0.30)^x$, so they're in the same family.