

Modeling Probabilities

ID: 10254

Time required

45 minutes

Activity Overview

This activity approaches probability distributions as functions taking information about a dataset as inputs and outputting a set of probabilities. This is analogous to the familiar process of choosing a function to model a set of x-y pairs. Students use simulations to explore increasing numbers of trials. The normal distribution is introduced, compared with the binomial distribution, and applied.

Topic: Statistics

- Normal distribution
- Binomial experiments with a large number of trials
- Matching probability distributions to a situation

Teacher Preparation and Notes

- This investigation is intended as an introduction to normal distribution for an Algebra 2 class. It could also be used in a Precalculus or introductory Statistics class.
- It is preferable that students have experience with the binomial probability distribution before beginning this activity. However, this is not essential, as the students can calculate both the binomial and normal distributions with the handheld. Depending on the goals of your class, you may wish to present the algebra behind these calculations in more depth or not at all.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “10254” in the keyword search box.**

Associated Materials

- ModelProb_Student.doc
- ModelProb.tns
- ModelProb_Soln.tns

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- What's Normal, Anyway? (TI-Nspire technology) — 8556
- An Approximation to the Binomial Distribution (TI-Nspire technology) — 9296
- Assessing Normalcy (TI-Nspire technology) — 11143

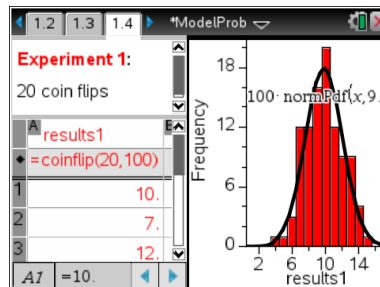
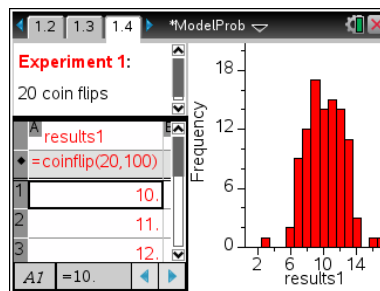
Problem 1 – Binomial Experiments with Lots of Trials

On page 1.4, students are prompted to simulate a binomial experiment using the custom function **coinflip**. This function takes two arguments: flips and number of trials. The command **coinflip(20,1)** simulates flipping a coin 20 times, for one trial.

When the simulation is running, each row of the table displays the number of heads that occurred for a trial.

Discuss what might happen if they keep running the experiment (flipping a coin 20 times) again and again. What number(s) of heads would they expect to occur most often? Least often?

Students should experiment with different numbers of trials, such as 10, 20, 100, and 1000, updating the histogram each time by reselecting **results1** as the X Variable. Discuss the shape of the graph.



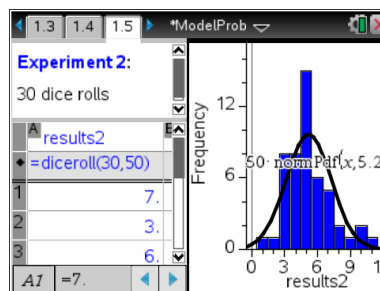
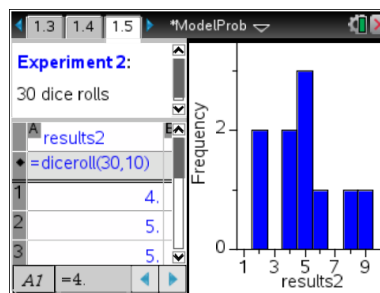
TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson

On pages 1.5 and 1.6, students should repeat this process to explore what happens when Experiment 2 (rolling a die 30 times and counting the number of 2s) and Experiment 3 (flipping a coin 10 times and counting the number of heads) are repeated many times. A custom function **diceroll(dice,numtrials)** simulates Experiment 2.

Students should notice that as the number of trials increases and the shape of the graph begins to resemble that of the graph from Experiment 1.

This is the same discovery that statisticians made years ago: that as the number of trials increase, all binomial distributions begin to resemble each other.

Direct students to add a normal curve to each of their histograms on pages 1.4 through 1.6 by selecting **MENU > Analyze > Show Normal PDF**. (PDF stands for Probability Density Function.) As the number of trials increases, the histogram approaches this smooth, bell-shaped curve. The normal probability distribution calculates probabilities based on this curve.

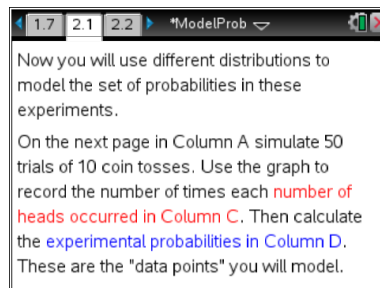


TI-Nspire Navigator Opportunity: *Class Capture*
 See Note 2 at the end of this lesson

Problem 2 – Probability Distributions

Discuss with students the analogy between matching a function $f(x)$ with a set of $(x, f(x))$ pairs and matching a probability distribution $P(X)$ with set of $(outcome, P(outcome))$ pairs.

Just as there are many different families of functions (linear, quadratic, rational), there are many different families of probability distributions, and the best one must be chosen to model each situation. The next few screens walk students through a comparison of the binomial and normal distributions as models for tossing a coin 10 times for various numbers of trials.

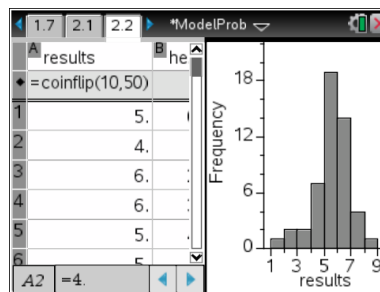


Students may be surprised to know that they have already worked with the **discrete uniform distribution**, which assigns the same (uniform) probability to each one of a set of events. An example of the discrete uniform distribution is rolling a die one time and looks like this:

$$\left\{ \left(1, \frac{1}{6}\right), \left(2, \frac{1}{6}\right), \left(3, \frac{1}{6}\right), \left(4, \frac{1}{6}\right), \left(5, \frac{1}{6}\right), \left(6, \frac{1}{6}\right) \right\}$$

Discuss other examples of the discrete uniform distribution and why it would be a not be a good choice to model Experiments 1, 2, or 3. You may also wish to discuss discrete versus continuous distributions. Experiment 3 will be used to explore these distributions.

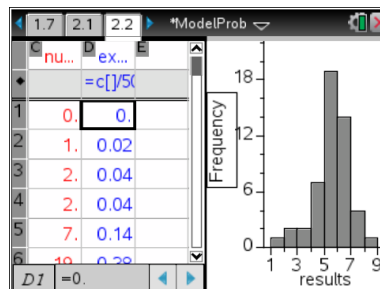
Students are to simulate 10 flips, for 50 trials in Column A. Using the histogram, students can record the number of times an outcome occurred in the experiment in Column C.



Demonstrate how to do so by adjusting the bin width to 1 and the alignment to 0 (**MENU > Plot Properties > Histogram Properties > Bin Settings > Equal Bin Width**).

Now, each bar represents an outcome: the number of times that heads occurred in 10 coin flips.

After completing the experimental probability in Column C by entering the formula $c[]/(\text{number of trials})$, students need to rewrite the results of this experiment in decimal form on their worksheets. This will make it easier to compare them with the binomial and normal distributions.



Different probability distributions are described in different ways. Again, draw the analogy to families of functions.

A linear function is described by its slope and intercept, while a rational function is described by its asymptotes.

The binomial distribution is described by the probability of success and the number of trials per experiment (i.e. 10 in this case). The normal distribution, on the other hand, is described by the mean and standard deviation of the data it is modeling.

Students should calculate these values on page 2.3.

On page 2.4, students are prompted to calculate the theoretical probability of flipping exactly 4 heads in 10 coin flips predicted by the binomial and normal distributions using the **binomPdf** and **normPdf** commands (**MENU > Statistics > Distributions**).

The arguments of each command are:

- **binomPdf**(number of flips per trial, probability of success, outcome)
- **normPdf**(outcome, mean, standard deviation)

Discuss the results. *Which would you expect to be a closer match to experimental probabilities calculated from performing the experiment many times? Which would you expect to be a closer match to experimental probabilities calculated from performing the experiment only a few times?*

Students should return to page 2.2 and rerun the simulation with more or fewer trials and record the new experimental probability of exactly 4 heads.

Then they should recalculate the theoretical probabilities with each distribution, noting that the mean and standard deviation have changed.

Each normalPdf curve shows a distribution with μ (mean) and σ (standard deviation). Find μ and σ for **results**.

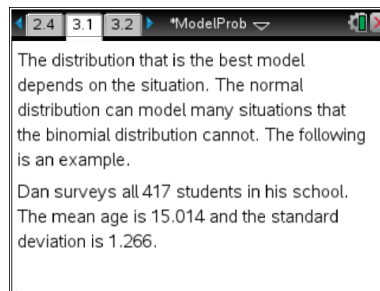
Use the mean and stDevSamp commands.	
mean(results)	5.08
stDevSamp(results)	1.33768
3/99	

Compare the experimental probability of 4 heads with the theoretical probabilities from the binomial and normal distributions.

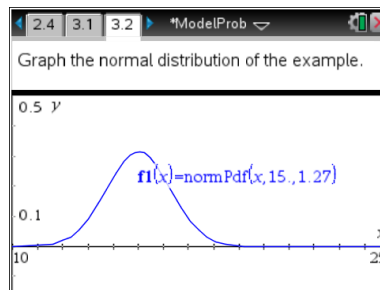
(outcome, mean, standard deviation)	
binomPdf(10, 0.5, 4)	0.205078
normPdf(4, 5.12, 1.50699)	0.200844
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Problem 3 – Application of the Normal Distribution

Page 3.1 presents a situation that can be modeled by the normal distribution, but not the binomial distribution.

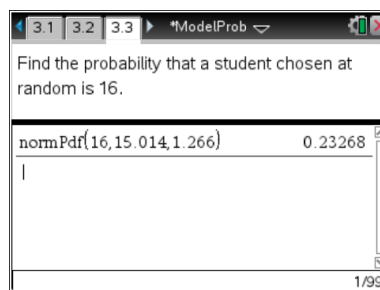


Students are to graph the normal distribution described in this problem on page 3.2 simply by typing the **normPdf** command in the formula bar and with the variable x as the first argument.



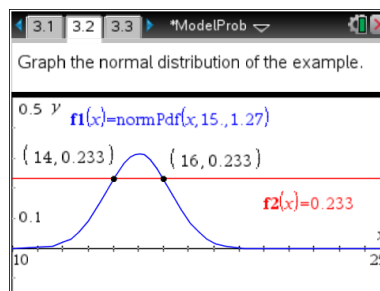
An appropriate window is $10 \leq x \leq 25$, $-0.15 \leq y \leq 0.5$.

Page 3.3 asks students to calculate the probability that a randomly chosen student is 16.



TI-Nspire Navigator Opportunity: Quick Poll and Live Presenter
See Note 3 at the end of this lesson

After calculating the probability, students should confirm that it agrees with the graph on page 3.2 by graphing it as a horizontal line and finding its intersection with the curve using the **Intersection Point(s)** tool.



TI-Nspire™ Navigator™ Opportunities**Note 1****Question 1, *Live Presenter* and *Class Capture***

Use Live Presenter to demonstrate for students how to change the number of coin flips in the list and spreadsheet as well as how to adjust the bin width on the histogram. Use Class Capture to monitor student progress and to display everyone's data and resulting histogram. Point out to students that while everyone generated random rolls of their dice, the histograms are all similar.

Note 2**Question 14, *Class Capture***

Before the students gradually increase the number of trials to 1000, ask them what affect the additional trials will have on the resulting histogram. Use Class Capture to display Student works and to facilitate discussion with the effect the increased number of trials has on the shape of the histogram.

Note 3**Problem 3, *Quick Poll* and *Live Presenter***

Send a Quick Poll asking students the probability that a person chosen at random is 16. Use Live Presenter to aide those students that send an incorrect response.