NUMB3RS Activity: Checkbook (Mis)Calculations
Episode: "Provenance"

Topic: Algebraic expressions, linear diophantine equations **Grade Level** 9 - 11
Objective: Use algebraic expressions and equations to analyze data entry errors
Time: 30 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

A running gag in "Provenance" concerns Charlie's inability to balance his checkbook. This activity examines the three most common errors in entering data and a classic problem related to one of these errors.

Discuss with Students

Students will analyze the three most common types of errors in entering data – single-digit errors, transposition errors ($ab \leftrightarrow ba$), and jump transposition errors ($abc \leftrightarrow cba$). In the activity, students will need to identify and represent four digit whole numbers by the digit in each place of the number. That is, a number *abcd* would be represented as $1000a + 100b + 10c + d$.

A classic problem related to this topic is given below.

> *Brett cashes a check worth less than $100 for x dollars and y cents, but the teller inadvertently him pays y dollars and x cents. After Brett buys a newspaper for k cents, the remaining money is twice as much as the original value of the check. If k = 50, find the amount of the check.*

The solution to the classic problem will involve looking for integer solutions of equations such as $2y - 5x = 13$, by entering $Y_1 = \dfrac{13+5x}{2}$ into a graphing calculator and using the table feature to search for integer solutions.

Student Page Answers:
1a. *$8000* **1b.** $1 **2.** $1000a + 100b + 10c + d - (1000a + 100b + 10d + c) = 9(c - d)$;
$1000a + 100b + 10c + d - (1000a + 100c + 10b + a) = 90(b - c)$;
$1000a + 100b + 10c + d - (1000b + 100a + 10c + d) = 900(a - b)$; *all three differences are divisible by 9.*
3. *The difference between your total and the bank's total is divisible by 99.* **4.** *Use the equation* $Y_1 = (199x + 50)/98$ *to show $16.33 is the unique solution.*
5. *The solution* $x = 73$, $y = 149$ *has* $y > 99$. **6.** *$48.99*

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Name: ________________________________ Date: _______________

NUMB3RS Activity: Checkbook (Mis)Calculations

A running gag in "Provenance" concerns Charlie's inability to balance his checkbook. In this activity, you will examine the three most common errors in entering numbers such as the amount of a check and a classic problem related to one of these errors.

1. The most common error, by far, in entering numbers such as the amount of a check is a **single-digit error**; for example entering an 8 instead of a 3 somewhere in the number. For simplicity, suppose all amounts in this checkbook are in dollars (no cents) from $1 to $9999.

  a. If you make one single-digit error, what is the greatest possible difference between your total and the bank's total?

  b. What is the least possible difference?

2. Another common error is a **transposition error** where you reverse the order of two adjacent digits; for example writing "83" instead of "38." A common rule for spotting this error is ***"if the difference between your total and the bank's total is divisible by 9, look for a transposition error."*** Using algebra, explain why this rule makes sense. (Hint: First consider the case where the transposition error is in the last two digits. If the correct amount of a check is $1000a + 100b + 10c + d$, then the incorrect amount entered will be $1000a + 100b + 10d + c$. Then analyze the other two cases.)

3. A third type of error is a **jump transposition error**, in which two non-adjacent digits are switched. For example, you write "483" instead of "384" somewhere in the number. Develop a rule for detecting when two digits separated by another digit have been switched. Explain why your rule works.

A classic problem related to this topic begins as follows:

Brett cashes a check worth less than $100 for x dollars and y cents, but the teller inadvertently him pays y dollars and x cents. After Brett buys a newspaper for k cents, the remaining money is twice as much as the original value of the check.

4. If $k = 50$, find the amount of the check. (Hint: Develop an equation which begins $2(100x + y) =$ _____, solve for y, and enter the result as Y_1 in your calculator. Examine the table of values to search for integer solutions.)

5. If $k = 75$, show there is no such check.

6. What is the largest possible original value for such a check? (Hint: Generalize the method used in Question #4. The price of the paper k could be any price.)

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

1. Show that you can verify the rule for identifying a transposition error using one equation instead of three.

2. Find several other values of *k* for which the classic problem has a solution. Do you see any pattern in the values for the checks?

3. Find several other values of *k* for which the classic problem does not have a solution. Do you see any pattern in the values for the checks?

4. Prove that the classic problem has at most one solution for any given value of *k*.

Related Topics

One method of identifying data entry errors is through a **check digit** – a decimal (or alphanumeric) digit added to an identification number for the purpose of detecting the sorts of errors humans typically make on data entry. Using a check digit is now standard practice for identification numbers such as airplane ticket numbers, ISBNs, UPC barcodes, credit card numbers, checking account numbers, passport numbers, VINs (vehicle identification numbers) POSTNet codes (bar codes from mailing addresses), etc.

To learn more about check digits, look at the activity **Check That Digit** from the NCTM Illuminations site: **http://illuminations.nctm.org/LessonDetail.aspx?id=L693**.

For more information including methods of constructing check digits, you can consult one of the following resources:

- <u>For All Practical Purposes</u>, 7th edition, Freeman, 2006, Chapter 16
- Check Digits: **http://www.augustana.ab.ca/~mohrj/algorithms/checkdigit.html**
- Take a Break: **http://plus.maths.org/issue12/features/codes/**
- Identification Numbers and Check Digit Schemes: **http://www.academic.marist.edu/mwa/vin.htm**
- Check the validity of several types of identification numbers: **http://www.d.umn.edu/~jgallian/fapp5/**

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