In this activity, you will explore:

- Using integrals to find the area between two curves.

Use this document to record your answers.


## Problem 1 - Making Pathways

While integrals can be used to find the area under a curve, they can also be used to find the area between curves through subtraction (just make sure the subtraction order is the equation of the top curve minus the equation of the bottom curve.)

Suppose you are a building contractor and need to know how much concrete to order to create a pathway that is $1 / 3$ foot deep. To find the volume, all that is needed is to multiply the area of the sidewalk by the depth of the sidewalk. The sidewalk borders can be modeled from $-2 \pi$ to $2 \pi$ by:

$$
\begin{aligned}
& f(x)=\sin (0.5 x)+3 \text { and } \\
& g(x)=\sin (0.5 x) .
\end{aligned}
$$

Graph both functions. Adjust the window settings to $-6.4 \leq x \leq 6.4$ and $-5 \leq y \leq 5$.
Now use the Integral tool from the Math menu to calculate the integrals of $f(x)$ and $g(x)$.
Enter $-2 \pi$ for the lower limit and $2 \pi$ for the upper limit.

- What is the value of the integral of $f(x)$ ? Of $g(x)$ ?

On the Home screen, define $f(x)$ and $g(x)$. Then use the Numerical Integral command (nInt) in the Calc menu to find the area of the pathway.

Hint: The area is equal to the integral of $f(x)-g(x)$ from $-2 \pi$ to $2 \pi$.
Note: The nInt command has the syntax: nInt(expression, variable, lower bound, upper bound)

- What is the formula for the volume of the sidewalk?
- How much concrete is needed for the pathway?


## Problem 2 - Finding New Pathways

The owners have changed the design of the pathway. It will now be modeled from -2 to 2 by:
$f(x)=x \cdot(x+2.5) \cdot(x-1.5)+3$
$g(x)=x \cdot(x+2) \cdot(x-2)$
(Note: Be sure to include the multiplication symbols as shown.)
Graph both functions. Adjust the window settings to $-4 \leq x \leq 4$ and $-5 \leq y \leq 9$.
Calculate the integrals of $f(x)$ and $g(x)$. Enter -2 for the lower limit and 2 for the upper limit.

- What is the value of the integral of $f(x)$ ? Of $g(x)$ ?
- On the Home screen, calculate the amount of concrete needed for the pathway. Remember to define $f(x)$ and $g(x)$.


## Problem 3 - Stepping Stones

The owners also want stepping stones, which can be modeled by

$$
\begin{aligned}
& f(x)=-(x-1) \cdot(x-2)+2 \\
& g(x)=(x-1) \cdot(x-2)+0.5 .
\end{aligned}
$$

This situation different because the starting and stopping points are not given. Assume that the stepping stones are $1 / 3$ foot thick.
(Note: Be sure to include the multiplication symbols as shown.)
Graph both functions. Adjust the window settings to $-1 \leq x \leq 7$ and $-4 \leq y \leq 4$ with a step size of 0.5 for both. Use the Intersection tool in the Math menu to find the intersection points. You can also use the Solve command on the Home screen.

- What are the coordinates for the two intersection points?

Calculate the integrals of $f(x)$ and $g(x)$. Use the $x$-values of the intersection points as the lower and upper limits.

- What is the value of the integral of $f(x)$ ? $g(x)$ ?
- On the Home screen, calculate the amount of concrete needed for the pathway. Remember to define $f(x)$ and $g(x)$.

