Name ______________________

Date ______________________

EXPLORATIONS

ACTIVITY 12

Sequence of Bounces

When a ball is dropped from a given height, every rebound will be shorter than the one before. The total distance traveled by the ball would be the sum of all the distances it travels in each direction.

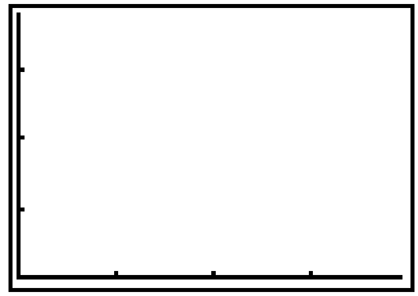
In this activity, you will explore the rebound heights of a ball and develop a sequence that will predict the rebound height of subsequent bounces. You will also find the total distance that the ball travels.

You'll Need

- 1 CBR unit
- 1 TI-83 or TI-82 Graphing Calculator
- Ball (a racquet ball works well)

Instructions

1. Run the **RANGER** program on your calculator.
2. From the **MAIN MENU** of the **RANGER** program, select **3:APPLICATIONS**.
3. Select **1:METERS**, and then select **3:BALL BOUNCE**.
4. Follow the directions on the screen of your calculator. Release the ball. Press the [TRIGGER] key on the CBR as the ball strikes the ground.
5. Your graph should have a minimum of five bounces. If you are not satisfied with the results of your experiment, press [ENTER], select **5:REPEAT SAMPLE**, and try again.
6. When you are satisfied with your data, sketch a plot of your Distance-Time.

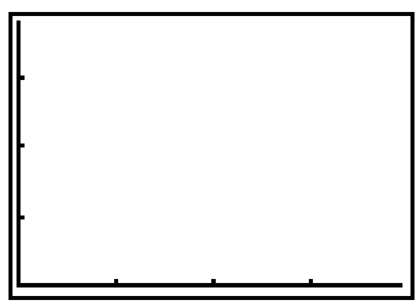


Data Collection

1. Use [▸] (the right arrow) to trace to the approximate height, y, of the first bounce. For each N, the number of the bounce, record y, the height of that bounce in the table below. Repeat for four more bounces.

N	y
1	
2	
3	
4	
5	

2. Press [ENTER] to return to the **PLOT MENU**. Select **7:QUIT** to exit the **RANGER** program.
3. Plot the data points as a scatter plot in the window at the right. Press [STAT], select **1:Edit** to enter the bounce number, N , in **L5** and the y-coordinates in **L6**. Press [2nd] **[STAT PLOT]**. Select **1:Plot1**, select for the **Type** of plot, **L5** for the **Xlist**, **L6** for the **Ylist**, and the **square** for the **Mark**. Press [ZOOM] [9]. Sketch a plot of this data.



Questions - Part One

1. The heights reached by each consecutive bounce of the ball can be thought of in terms of a geometric sequence. The model for any term of a geometric sequence is

$$a_n = a_1 r^{n-1}$$

where a_n is the n[th] term of the sequence, a_1 is the first term of the sequence, and r is the common ratio between any two terms of the sequence. Use the data from your table to determine the appropriate values for a_1 and r. Record these values.

$a_1 =$ ________________________________ $r =$ ________________________________

2. Use the values for a_1 and r to write the model for the n[th] term of this sequence.

$a_n =$ ________________________________

Rewrite this model substituting x for n.

____

Press [Y=]. Enter this equation in **Y1** and press [ENTER].

3. Press [GRAPH]. Describe how well this function fits the data.

____

4. What is the equation of the exponential function that would best fit this data?

For the TI-83: Press [STAT] [▸] and select **0:ExpReg.**
For the TI-82: Press [STAT] [▸] and select **A:ExpReg.**

This copies the command **ExpReg** on the home screen. Press [2nd] **[L5]** [,] [2nd] **[L6]** [ENTER] to find the exponential regression. Record the equation of this function:

$y =$ ________________________________

Press [Y=]. Enter this equation in **Y2** by pressing [▾], entering the equation, and pressing [ENTER].

5. Press [GRAPH]. Describe how this function compares to the function in **Y1**.

____

6. Rewrite the equation in **Y2** so that its exponent is $(x - 1)$ instead of x. To do this, rewrite the equation as

$$y = a_1 r(r)^{x-1}.$$

Then simplify $a_1 r$. Record your new equation.

____

7. Enter this equation in **Y3**. Press [GRAPH]. How does this fit compare to the fit of the functions in **Y1** and **Y2**?

____

____

8. Predict the height for the sixth bounce. Press [2nd] **[QUIT]** to return to the home screen.

> **For the TI-83:** Press [VARS], highlight **Y-VARS**, select **1:Function**, press [ENTER], select **1:Y1**.
>
> **For the TI-82:** Press [2nd] **[Y-VARS]**, select **1:Function**, select **1:Y1.**

The command to evaluate this function at $x = 6$ is **Y1 (6)**. Press [(] [6] [)]. Press [ENTER].

____

9. Predict the height for the tenth bounce.

____

Questions - Part Two

1. How far did the ball travel for each bounce? What is the total distance traveled by the ball? Copy the rebound heights from the table on page 62 and compute the total distances traveled by the ball.

Bounce (n)	Rebound Height (y)	Total Distance Traveled for this Bounce	Total Distance Traveled since $t = 0$
1			
2			
3			
4			
5			

2. The formula for the sum of a geometric series is

$$S_n = \frac{a_1(1-r^n)}{1-r}.$$

To find the total distance the ball traveled over the course of a number of bounces, this sum should be doubled. Explain why.

____

____

3. Find the sum of the total distance the ball travels for the first five bounces.

____

Confirm this answer by finding the sum of the total distance traveled in the table from question 1 above.

4. This sum can also be computed directly on the calculator using the sum and sequence commands. The command is **sum(seq(Y1,X,1,5,1)**.

For the TI-83: Press [2nd] **[LIST]** [▸] [▸] [5] [2nd] **[LIST]** [▸] [5] [VARS] [▸] [1] [1] [,] [X,T,Θ,n] [,] [1] [,] [5] [,] [1] [ENTER].

For the TI-82: Press [2nd] **[LIST]** [▸] [5] [2nd] **[LIST]** [5] [2nd] **[Y-VARS]** [1] [1] [,] [X,T,Θ,n] [,] [1] [,] [5] [,] [1] [ENTER].

5. Find the sum of the total distance the ball travels for the first six bounces.

____

6. Find the sum of the total distance the ball travels for the first ten bounces.

____

7. The formula for the sum of an infinite geometric series is

$$S_n = \frac{a_1}{1-r}.$$

Find the total distance traveled by the ball, that is, find the total distance for an infinite number of bounces.

____