# Step-by-Step Instructions

This investigation requires students to create quadratic functions to model rocket launch data. The parabolic fight pattern assists students in recalling characteristics of a quadratic. As students create models they will need to recall the various forms of a quadratic function. Students will be representing models in function form and parametric form. Parametric form will enable students to visualize the vertical launch and understand how x and y are dependent on the third parameter, t.
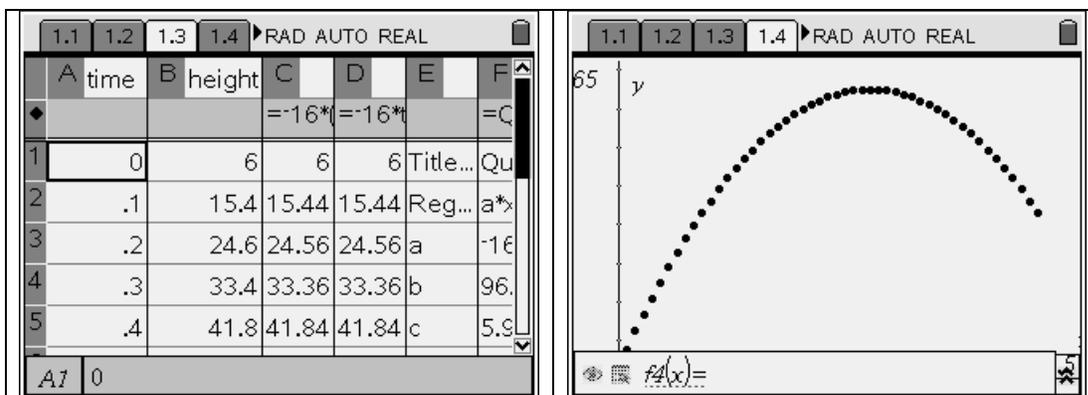
- Download the *rocketsim* document and link to student calculators.
  Note: The teacher version is *rocketsimSOL*, it included completed graphs, Spreadsheets, calculations and answers to the questions.
- The teacher should make certain students have the necessary background to complete this activity successfully. Students do need to have prior knowledge regarding the various forms of a quadratic functions and creating model functions.
- The teacher may create a hard copy worksheet to accompany this activity if it is desired.
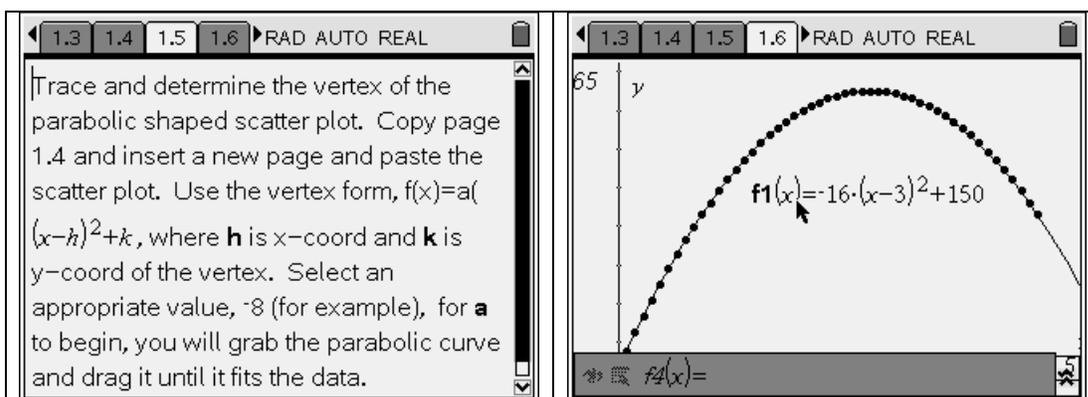
Problem 1 – Using Function Graph Type
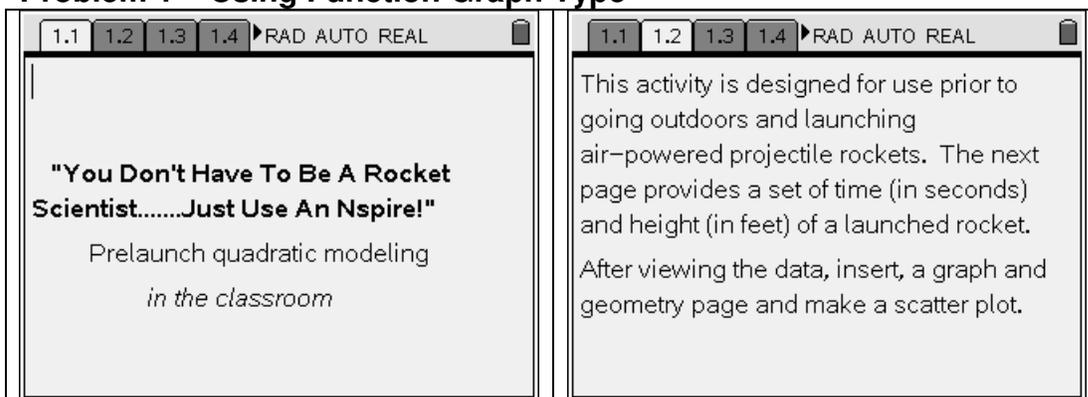
The next two pages show the data in terms of **time** and **height** and the resulting scatter plot. The verifications are shown in columns C and D; the quadratic regression is shown in columns E and F, as this is the solution document.

Recall ⟨ctrl⟩ , ⟨i⟩ allows the student to insert a page. To create the scatter plot they will select 2. Graphs and Geometry, ⟨menu⟩ , 3. Graph Type, ▸ , 4. Scatter Plot. The entry line will show entry boxes for x and y. Press ⟨?⟩, scroll with arrow keys to select **time**, ⟨enter⟩, then ⟨tab⟩ to move to y entry box. Press ⟨?⟩, scroll with arrow keys to select **height**, ⟨enter⟩, then press ⟨menu⟩, 4. Window, ▸, 9. Zoom – Data. The scatter plot will appear. Another option is to select 1. Window settings and have students set the window according to data values. **Remind students to ⟨tab⟩ between entry boxes when entering their own settings.**

A time	B height	C	D	E	F ▲
◆		=-16*(	=-16*t		=Q
1	0	6	6	6 Title...	Qu
2	.1	15.4	15.44	15.44 Reg...	a*x
3	.2	24.6	24.56	24.56 a	-16
4	.3	33.4	33.36	33.36 b	96.
5	.4	41.8	41.84	41.84 c	5.9

| A1 | 0 |

65 y

◈ 🔳 f4(x)=

Students should recognize the parabolic curve, note it is opening down, indicating that **a,** in the y = a(x-h)^2 + k, is negative. At this time they should be aware of symmetry and vertex terminology as they view the plotted data. The next two pages will give instructions and also show the resulting function fitted to the data.

Students should recall that ctrl , ⓧ will allow them to grab the "arm" of the parabola after they have moved the ▸ to touch the curve and saw the ⤢ symbol.

Trace and determine the vertex of the parabolic shaped scatter plot. Copy page 1.4 and insert a new page and paste the scatter plot. Use the vertex form, f(x)=a(

$(x-h)^2+k$, where **h** is x-coord and **k** is y-coord of the vertex. Select an appropriate value, -8 (for example), for **a** to begin, you will grab the parabolic curve and drag it until it fits the data.

65 y

$f1(x)=-16\cdot(x-3)^2+150$

◈ 🔳 f4(x)=

The next page is a question and answer page asking for the model function and the meaning of a. The verification instruction page will direct the student back to page 1.3. Note: If the teacher has not required students to complete this verification in the spreadsheet, there may be some students who need assistance.

Question

What is the model function? What does the value of "a" represent?

Answer

The model function is

$f1(x) = -16*x^2 + 96x + 6$. a represents $\frac{1}{2} \cdot g$,

Verify how well the vertex model fits the data; go to page 1.3, column C formula entry line. Enter the appropiate model preceded by an "=" sign. Remember in this problem, x−coord values are in the "time" list.

Students will be asked to insert another graphs and geometry page and recreate the scatter plot as before. The teacher may give students the option of copying the scatter plot and pasting it to the newly inserted page. Recall that returning to page 1.4, pressing (ctrl), (K) will select the page; (ctrl), (C) will copy the page; move back to page 1.9. Press (ctrl), (I), (esc), (ctrl), (V) to paste the scatter plot on the inserted page.

Look at page 1.4, insert a new graph and geometry page and recreate the scatter plot as before. Select 3 points and create 3 equations in the standard form y=a $x^2 + bx + c$. Use matrix operations to find a, b, and c. Show matrix operations on a calculator page. Enter the function and check the fit. Verify in Column D on page 1.3 in the same manner as vertex model.

Indicate the coordinates of the three points selected: (1,86),(2,134),(4,134)

The three equations formed in the form :

$$a*x^2 + b \cdot x + c = y$$

are as follows:

$$a*1^2 + b \cdot 1 + 1c = 86$$

$$a*2^2 + b*2 + 1c = 134$$

Students were asked on page 1.10 to tell which three points they selected and write the three equations formed. Page 1.11 shows the scatter plot with the equation that was created using the matrix operations shown on page 1.12.

$f2(x) = -16 \cdot x^2 + 96 \cdot x + 6$

$f4(x) =$

$$\begin{bmatrix} 1^2 & 1 & 1 \\ 2^2 & 2 & 1 \\ 4^2 & 4 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 86 \\ 134 \\ 134 \end{bmatrix} \qquad \begin{bmatrix} -16 \\ 96 \\ 6 \end{bmatrix}$$

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Go to page 1.3, highlight the time and height lists and complete a quadratic

regression. Graph and check the fit.

165 y

$f3(x) = -16.0001 \cdot x^2 + 96.$

※ ▨ $f4(x) =$
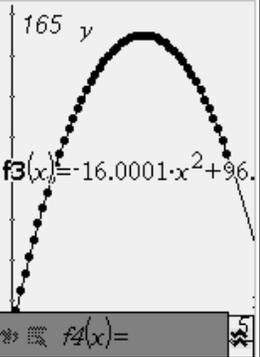
This page was created for the student using a split page layout. Press (ctrl),(tab) to move to the right side, press (menu) select 2. Graphs and Geometry, (enter) .Press (menu), 3. Graph Type, ▶ , 4. Scatter Plot. Proceed as before clicking on the x box, selecting **time** , (tab) , click on the y box, selecting **height**. Press (menu), 4. Window, 9. Zoom-Data. Scroll up in the entry line to f3(x), press (enter) to graph the function.

The last page in problem 1 is a question-answer page. Students will need to explain their reasoning.
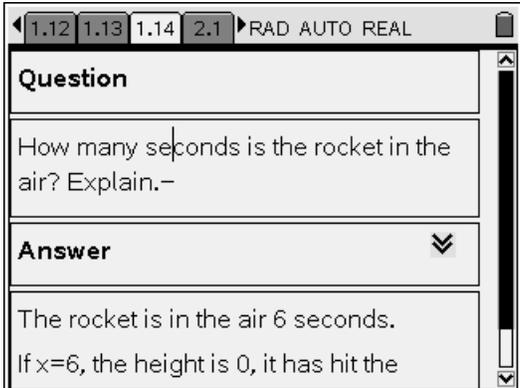
Question

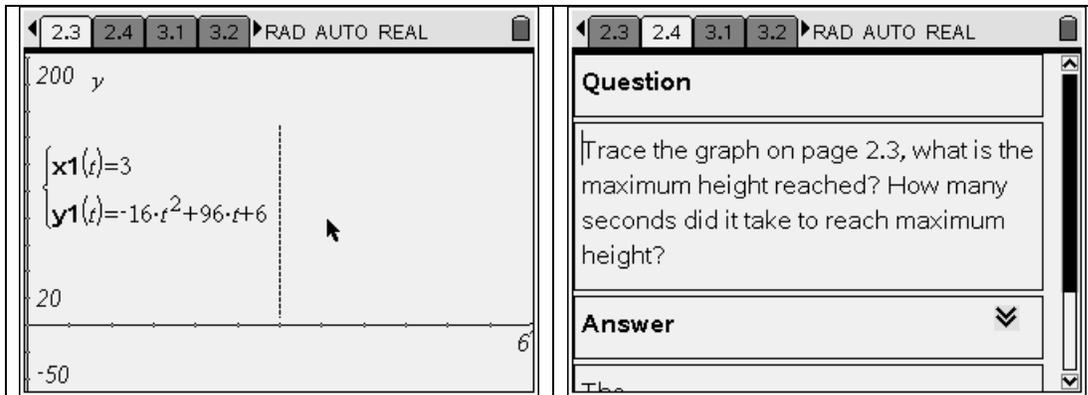How many seconds is the rocket in the air? Explain.–

Answer ⌄⌄

The rocket is in the air 6 seconds.
If x=6, the height is 0, it has hit the
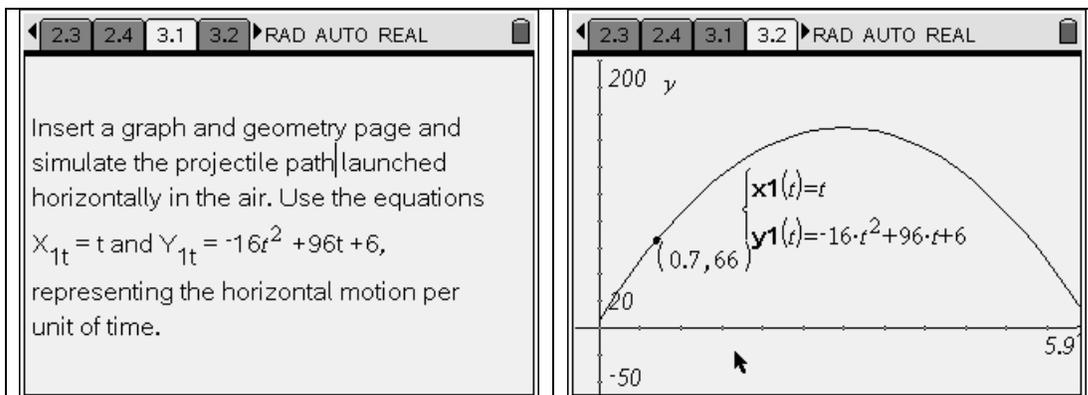
Problem 2 – Parametric Modeling

Parametric Modeling

Simulate the vertical launch of the air–powered rocket parametrically.

The equation

$Y_{1t} = -16t^2 + 96t + 6,$ *representing* vertical

motion per unit of time, t. Set $X_{1t} = 3.$

0<t<6.2   tstep 0.5

The teacher may need to remind some students to press (menu) , 3. Graph type, 2. Parametric, to enter the parametric equations and domain and step for t . The window will need to be set in order that the complete vertical launch may be seen.

NOTE: The trace currently does not work like the TI-84 for the VERTICAL LAUNCH; it will go to the maximum height, but does not trace by tenths, etc. like the TI-84 does in parametric mode. Perhaps this is due the fact that the x equation is a constant. The trace _will_ work in the parametric equations which show a parabolic path.

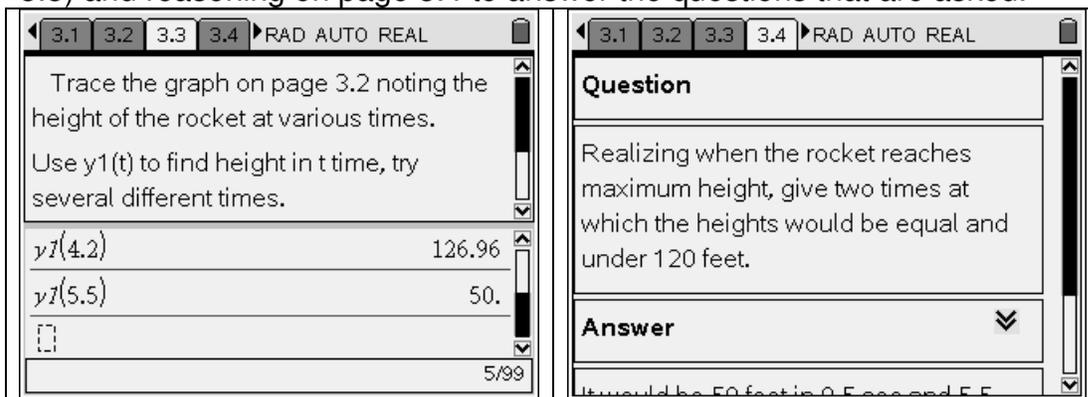| 2.3 2.4 3.1 3.2 ▶RAD AUTO REAL	2.3 2.4 3.1 3.2 ▶RAD AUTO REAL
200 y $x1(t)=3$ $y1(t)=-16 \cdot t^2 + 96 \cdot t + 6$ 20 -50	**Question** Trace the graph on page 2.3, what is the maximum height reached? How many seconds did it take to reach maximum height? **Answer**

The next two pages provide the equations and parametric graph representing horizontal motion per unit of time. Note a trace point is shown on page 3.2.

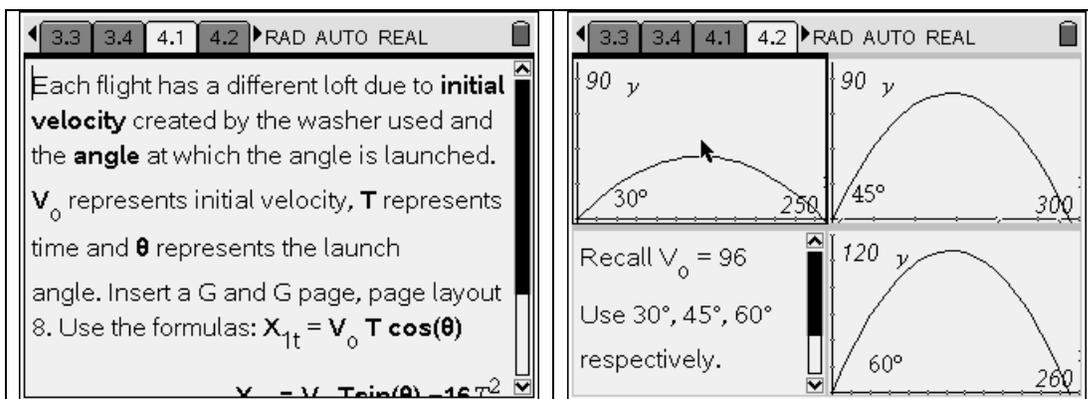| 2.3 2.4 3.1 3.2 ▶RAD AUTO REAL	2.3 2.4 3.1 3.2 ▶RAD AUTO REAL
Insert a graph and geometry page and simulate the projectile path launched horizontally in the air. Use the equations $X_{1t} = t$ and $Y_{1t} = -16t^2 + 96t + 6$, representing the horizontal motion per unit of time.	200 y $x1(t)=t$ $y1(t)=-16 \cdot t^2 + 96 \cdot t + 6$ $(0.7, 66)$ 20 -50 5.9

The trace feature worked on page 3.2, you can click on the x coordinate or y coordinate and use the – symbol to reduce the decimal places. You can **not** edit these coordinates and type in another value for x or y and result in a "jump" to that value while in parametric mode. You receive a message "cannot accept the change" and it jumps back to the point you tried to alter.

These two pages require to use the calculator app with function notation (page 3.3) and reasoning on page 3.4 to answer the questions that are asked.

| 3.1 | 3.2 | 3.3 | 3.4 | ▶ RAD AUTO REAL |

Trace the graph on page 3.2 noting the height of the rocket at various times.

Use y1(t) to find height in t time, try several different times.

$y1(4.2)$　　　　　　　126.96

$y1(5.5)$　　　　　　　50.

▢

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| 3.1 | 3.2 | 3.3 | 3.4 | ▶ RAD AUTO REAL |

Question

Realizing when the rocket reaches maximum height, give two times at which the heights would be equal and under 120 feet.

Answer

It would be 50 feet in 0.5 sec and 5.5

The next two pages deal with parametric relationships which involve the angle at which the rocket was launched and the initial velocity. Students will insert a graph and geometry page and use page layout 8 to show three launches on separate windows. The windows need to have an appropriate window set to view each graph and ⟨ctrl⟩ , ⟨G⟩ will have to be used to remove the entry line in order to view the graphs. The "blank pane" can be used to recall the initial velocity and measures of launch angles as shown.

| 3.3 | 3.4 | 4.1 | 4.2 | ▶ RAD AUTO REAL |

Each flight has a different loft due to **initial velocity** created by the washer used and the **angle** at which the angle is launched.

V_0 represents initial velocity, **T** represents time and **θ** represents the launch angle. Insert a G and G page, page layout 8. Use the formulas: $X_{1t} = V_0 \, T \cos(\theta)$

$Y = V \, T \sin(\theta) - 16\, T^2$

| 3.3 | 3.4 | 4.1 | 4.2 | ▶ RAD AUTO REAL |

90 y

30°　　　　　250

45°　　　　　300

Recall $V_0 = 96$

Use 30°, 45°, 60° respectively.

90 y

120 y

60°　　　　　260

At a glance, some students may not notice the x-axis and y-axis differences in the 45 and 60 degrees launches. To put it in a better perspective students will graph all three on the same axis.

Graph the parametric equations for the 30°, 45°, 60° launch angles on the same set of axes. Use attributes to distinquish

130 *y*

300

Question

Which launch angle results in the greatest horizontal distance traveled?

Answer ⨠

The 45° angle launch travels the greatest horizontal distance. It travels

Page 4.3 makes it very apparent the differences in launching at the three different angles. The last page requires an explanation from the students regarding the two launches that appear to travel the same horizontal distance.

Question

Explain why it appears that two of the launches travel the same horizontal distance.

Answer ⨠

One explanation could be using the

One explanation could be using the range relationship $\dfrac{V_0{}^2}{g}\left(\sin(2\theta)\right)$.
When using 30° and 60° in the formula, sin(2θ) has the same value and it travels 249.4 ft. Other students may use a different formula or explain it in tems of complementary angles.