Gateway Arc Length

TIMATH.COM: CALCULUS

TEACHER NOTES

ID: 12439

Activity Overview

Students will investigate the arc length of the Gateway Arch. They will use the Pythagorean Theorem to approximate, and use calculus to find the exact solution. They will also use CAS capabilities, including **arcLen( )**, to solve a variety of arc length questions.

Topic: Applications of Integral, Antiderivatives and Definite Integrals

- Arc length approximation, calculus formula, and using CAS
- Find the arc length of a parametric equation

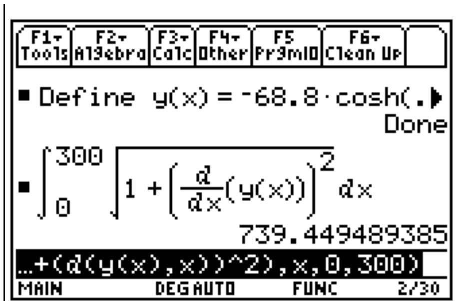
Teacher Preparation and Notes

- Arc length is a Calculus BC topic. Calculus AB teachers may enjoy using this activity after the AP* exam or using it with students in your AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP questions like multiple choice 03BC15, 98BC21, 88BC#33, and free response 04formB BC1c, 02formB BC1d&3c, 01BC1c, 97BC1e&3b. For four of these six free response questions the graph is given in parametric form. (**AP, College Board, and SAT are registered trademarks of the College Board, which was not involved in the production of and does not endorse this product.*)
- The syntax for **arcLen** is **arcLen(f(x),x,a,b)** where f(x) is the function, x is the variable, and the arc length is to be found from x = a to x = b. This activity will help students approximate arc length and use calculus to find the exact arc length.
- **To download the student worksheet, go to education.ti.com/exchange and enter "12439" in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Logistic Growth, Differential Equations, Slope Fields (TI-89 Titanium) — 5514
- Numb3rs – Season 3 – "The Mole" – Cycloid II (TI-84 Plus family) — 7511



This activity includes screen captures taken from the TI-89 Titanium.

Compatible Devices:

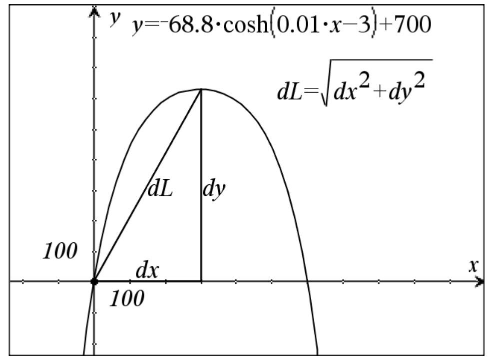
- TI-89 Titanium

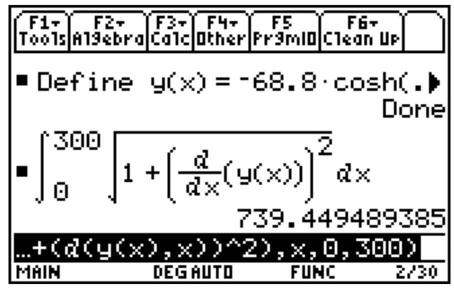
Associated Materials:

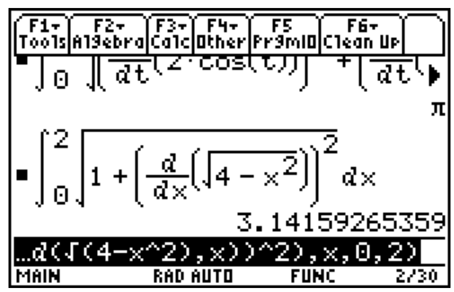
- GatewayArcLength_Student.pdf
- GatewayArcLength_Student.doc

Click HERE for Graphing Calculator Tutorials.

Part 1 – Arc Length Introduced







The first question investigates the Gateway Arch and the distance that a person would travel if he or she rode the elevator tram to reach the top. The Pythagorean Theorem is used to approximate the distance. The graph on the student worksheet helps students visually understand why those numbers were used in the solution to Exercise 1. The formula for arc length is derived from the Pythagorean Theorem. To help students understand why the integral formula approximates the arc length, compare this method to finding the area under a curve using a Riemann Sum. When the infinitesimal values of *dL* are added together from *a* to *b,* the arc length is found.

<u>*Discussion Questions*</u>

- What are the conditions for which the Pythagorean Theorem applies? *If students say, "It works for triangles," press them further.* What are characteristics of a triangle? *Perhaps they will see then, "Oh yeah, the Pythagorean Theorem only works for right triangles."* If you would like, you could go a bit deeper and ask, "What relationship (principle or law) applies for triangles that are not right? Explain it." *Law of Cosines* $c^2 = a^2 + b^2 - 2ab \cos \theta$ *where* θ *is the angle between a and b. The Pythagorean Theorem is a special case of this where* $\theta = 90°$. *(They may also say Law of Sines.)*

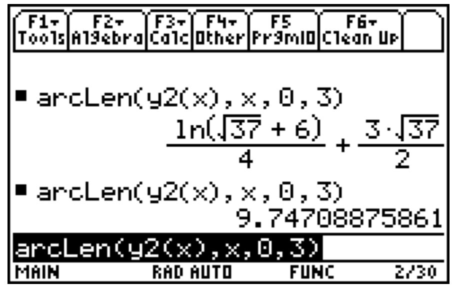
- Ask students, "Remind me, what is an integral? What does it mean?" *You may need to remind them that the definition was based on the area of rectangles or Riemann sums.* Ask again, "What does this mean? What are you doing with Riemann Sums?" *Adding infinite infinitesimals.*

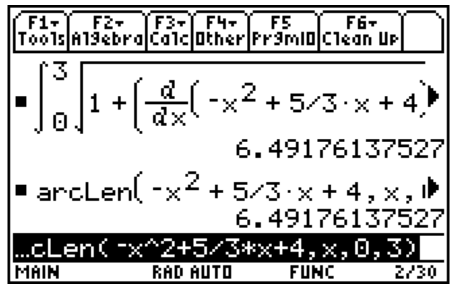
For Exercise 2, students are to use CAS to find the arc length of the Gateway Arch equation. Have students store the equation modeling the arch before entering the arc length formula. Students compare this solution with their length from Exercise 1.

Arc length for parametric equations is introduced and students are to solve this arc length by hand. For Exercise 4, students use CAS to find the arc length for the function $y = \sqrt{4 - x^2}$ from $x = 0$ to $x = 2$. Students should also make the connection that this curve is a fourth of a circle with radius 2.

The **arcLen** command is also introduced.

Student Solutions





1. The distance is at least $704 \approx \sqrt{315^2 + 630^2}$. The curve will be longer than the straight line connecting the base to the peak.

2. $\int_0^{300} \sqrt{1 + \left(\frac{d}{dx}\left(f1(x)\right)\right)^2}\, dx = 739.449$

   This is reasonable since it is a little larger than the straight line found in Exercise 1.

3. $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

   $= \int_0^{\pi/2} \sqrt{\left(\frac{d}{dt}2\cos(t)\right)^2 + \left(\frac{d}{dt}2\sin(t)\right)^2}\, dt$

   $= \int_0^{\pi/2} \sqrt{(-2\sin(t))^2 + (2\cos(t))^2}\, dt$

   $= \int_0^{\pi/2} \sqrt{4\left(\sin^2(t) + \cos^2(t)\right)}\, dt = 2t\Big|_0^{\pi/2} = \pi$

4. $\int_0^2 \sqrt{1 + \left(\frac{d}{dx}\left(\sqrt{4 - x^2}\right)\right)^2}\, dx \approx 3.14159$

5. The solution will be around 10.

   $\int_0^3 \sqrt{1 + \left(\frac{d}{dx}\left(x^2 - 9\right)\right)^2}\, dx \approx 9.747$

6. The arc length will be more than 5, because 5 = $\sqrt{3^2 + 4^2}$

   $\int_0^3 \sqrt{1 + \left(\frac{d}{dx}\left(-x^2 + \frac{5}{3}x + 4\right)\right)^2}\, dx \approx 6.492$

   This answer is larger than 5 which is expected.

Part 2 – Additional Practice

Students are expected to know the arc length formula and answer multiple-choice questions without a calculator. Question 2 is a parametric arc length question.

Student Solutions

1. d. $\int_a^b \sqrt{\frac{x^2 - 5}{x^2 - 4}}\, dx$

2. a. $\int_0^{\pi} \sqrt{\cos^2 t + 1}\, dt$

3. e. $\int_a^b \sqrt{1 + \sec^4 x}\, dx$