Limits

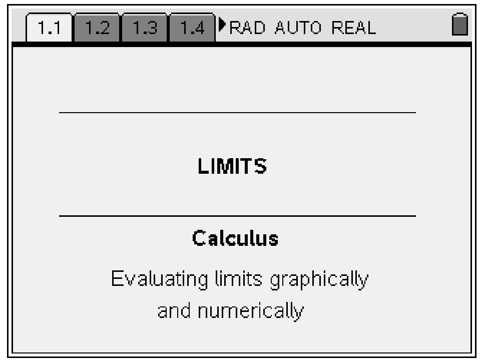
ID: 8997

Name ____________________

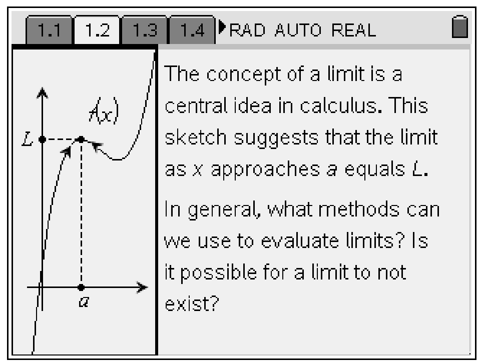
Class ____________________

In this activity, you will explore:

- *finding the limit of a function at a specified point*

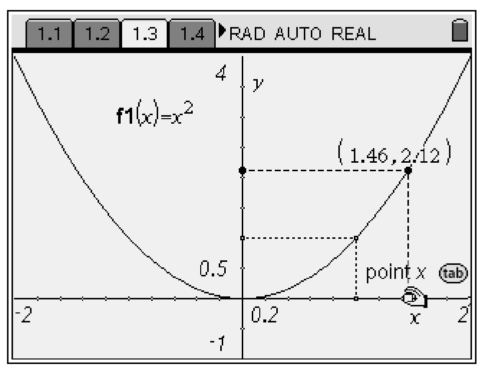


Open the file *CalcAct08_Limit_EN.tns* on your handheld and follow along with your teacher for the first two screens. Use this document as a guide to the activity and as a place to record your answers. Move to page 1.2 and wait for further instructions from your teacher.



Page 1.2 (shown at right) displays the graph of *f*(*x*). Notice that, for values of *x* near *x* = *a*, the function *f*(*x*) appears to take on a value close to *L*. This sketch provides a graphical method of "seeing" the value of a limit. In this activity, you will investigate graphical and numerical methods of finding limits. You will also look at some situations in which limits do not exist.

Problem 1 – Using graphical and numerical methods to find the value of a limit



Advance to page 1.3. You will see a screen similar to the one at right, showing the graph of $\mathbf{f1}(x) = x^2$.

You will investigate the limit of *f*(*x*) as *x* approaches 1, denoted $\lim_{x \to 1} x^2$, both graphically and numerically.

Grab point *x* and slowly move the point towards the left. Think of this action as "approaching *x* = 1 from the right." What does the value of this limit appear to be? In other words, what value does $\mathbf{f1}(x)$ appear to approach as *x* approaches 1?

Now move point x to the left of $x = 1$ and start "moving towards $x = 1$ from the left." Do you get the same limit?

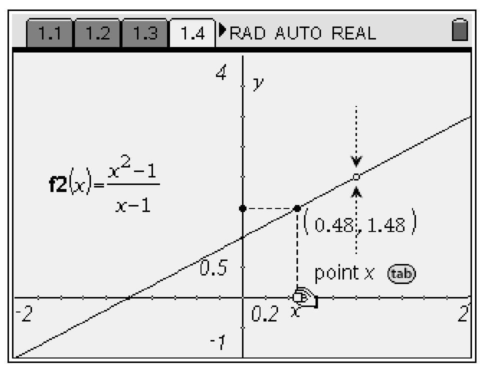
If these limits are the same, then we say the limit exists and is equal to the value of **f1**(x) as x approaches 1 from both the left and right sides.

You will now take a look at this same situation numerically. Press ctrl + T to insert a function table. By default, the x-values start at 0 and increment by 1. You need to modify the table settings in an effort to focus on values of x that are very close to 1. Select **MENU > Function Table > Edit Function Table Settings** to access the function table settings. Change **Table Start** to **1** and **Table Step** to **0.1**, and press enter. You can navigate up and down the table using the **NavPad**. What does the limit as x approaches 1 appear to be?

Go back and edit the function table settings such that **Table Start** is **1** and **Table Step** to **0.01**. Does the limit as x approaches 1 appear to be the same?

Repeat this procedure using a **Table Step** equal to **0.001**. Are you still convinced of the limit as x approaches 1? Explain why this procedure seems to make sense when finding a limit.

Advance to page 1.4. You will see the graph of $\mathbf{f2}(x) = \dfrac{x^2 - 1}{x - 1}$. What do you notice about this graph at $x = 1$?

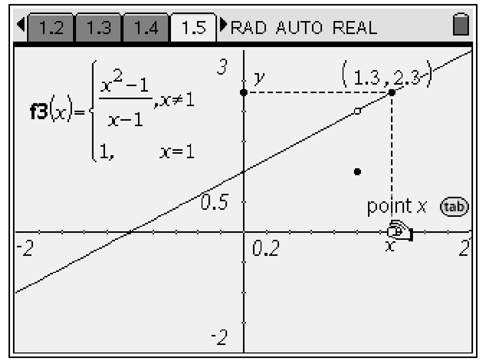


As before, drag point x towards $x = 1$ from the left and from the right. What does the value of $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$ appear to be?

Based on the value of **f2**(1), do you think that this limit exists? If so, what is its value?

Add a function table and edit the table settings to look at values of x very close to 1. From a numerical standpoint, what does the value of this limit appear to be?

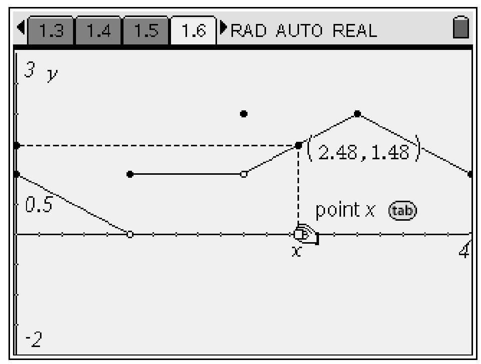
Advance to page 1.5 where you will see a graph of $\mathbf{f3}(x)=\begin{cases}\dfrac{x^2-1}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$. Drag point *x* towards *x* = 1 and note the value of $\lim\limits_{x\to 1}\mathbf{f3}(x)$.



What is the value of **f3**(1)? Does this value change what you think should be the value of the limit as *x* approaches 1?

Add a function table and edit the table settings to look at values of *x* very close to 1. From a numerical standpoint, what does the value of this limit appear to be?

Advance to page 1.6. You will see the graph of a piecewise defined function, **f4**(*x*), on the interval [0, 4]. Use your findings from the previous examples to determine the values of $\lim\limits_{x\to 1}\mathbf{f4}(x)$, $\lim\limits_{x\to 2}\mathbf{f4}(x)$, and $\lim\limits_{x\to 3}\mathbf{f4}(x)$. The values can be found *graphically* by dragging point *x* or *numerically* by adding a function table and changing the table settings.



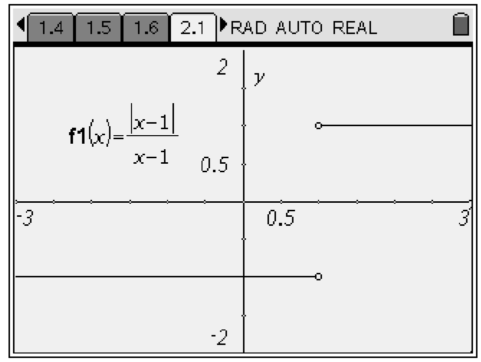
What do you think is the value of **f4**(*x*) as *x* approaches 0 “from the right,” denoted $\lim\limits_{x\to 0^+}\mathbf{f4}(x)$?

What do you think is the value of **f4**(*x*) as *x* approaches 4 “from the left,” denoted $\lim\limits_{x\to 4^-}\mathbf{f4}(x)$?

Explain why these last two limits are called “one-sided limits.”

Problem 2 – Exploring limits that do not exist

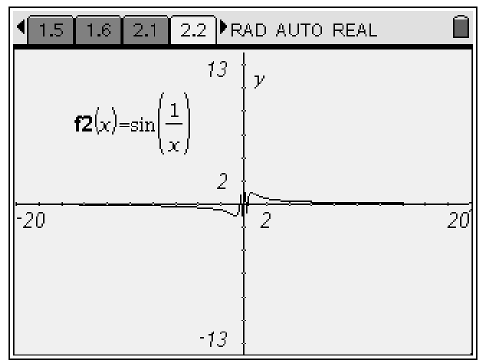
Advance to page 2.1. You will see a graph of $\mathbf{f1}(x) = \frac{|x-1|}{x-1}$.



Use **Graph Trace** (**MENU > Trace > Graph Trace**) to investigate approaching $x = 1$ from the left and the right. Explain why $\lim_{x \to 1} \mathbf{f1}(x)$ does not exist.

Do either $\lim_{x \to 1^-} \mathbf{f1}(x)$ or $\lim_{x \to 1^+} \mathbf{f1}(x)$ exist? Explain.

Advance to page 2.2. You will see a graph of $\mathbf{f2}(x) = \sin\left(\frac{1}{x}\right)$. You will be investigating $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ graphically and numerically.

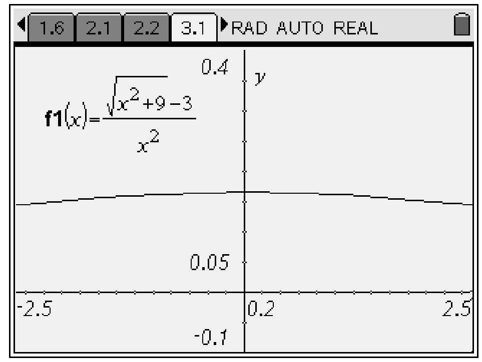


Start by selecting **MENU > Window > Zoom – In**. Move the cursor to the origin and press (click) repeatedly to zoom in on the graph at $x = 0$. What do you notice about the value of **f2**(x) near $x = 0$?

Insert a function table and set the **Table Step** to a very small number such as 0.00012. Does **f2**(x) appear to be approaching a limiting value for values of x near zero?

Problem 3 – Using other methods to find the value of a limit

Advance to page 3.1. You will see a graph of $\mathbf{f1}(x) = \dfrac{\sqrt{x^2+9}-3}{x^2}$. You will investigate the value of $\lim\limits_{x \to 0} \mathbf{f1}(x)$ graphically and numerically as was done before.



Select the **Zoom – In** function. Move the cursor to point where **f1**(x) passes through the y-axis and press four of five time to zoom in on the graph at x = 0. Use the **Graph Trace** tool to estimate the value of this limit. What does the value of the limit appear to be?

Use the **Zoom – In** function several more times to zoom in on this area of the graph until you see a change appear. What do you think is the value of the limit now?

You will now investigate this limit numerically. Insert a function table and set **Table Step** to a small number such as **0.001**. What does the value of the limit appear to be?

Change **Table Step** to a very small number such as **0.0000001**. Now what does the value of the limit appear to be?

You will resolve these inconsistencies by evaluating this limit in the *Calculator* application. Advance to page 3.2 and select **MENU > Calculus > Limit** to access the **Limit** command, and evaluate the $\lim\limits_{x \to 0} \mathbf{f1}(x)$. What is the value of this limit?

You can use the **Limit** command to evaluate any limit, including those that you explored in this activity. Evaluate each of the following:

a. $\lim\limits_{x \to 1} x^2$

b. $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1}$

c. $\lim\limits_{x \to 1} \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$

d. $\lim\limits_{x \to 1} \dfrac{|x-1|}{x-1}$