Impulse-Momentum Theorem – ID: 9900

By Irina Lyublinskaya

Time required
45 minutes

Topic: Momentum and Collisions

- *Use the impulse-momentum theorem, $F\Delta t = m\Delta v$, to solve problems involving force, time, mass, and velocity.*

Activity Overview

In this activity, students explore the relationships between momentum, force, and impulse for the linear collision of a ball with an unmovable wall. Based on these explorations, students derive the impulse-momentum theorem and then apply this theorem to solve problems.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Calculator

Teacher Preparation

Before carrying out this activity, you should review with students Newton's second law and the concepts of acceleration, momentum, and impulse.

- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on pages 8 and 9 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9900" in the search box.***

Classroom Management

- *This activity is designed to be* ***teacher-led*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

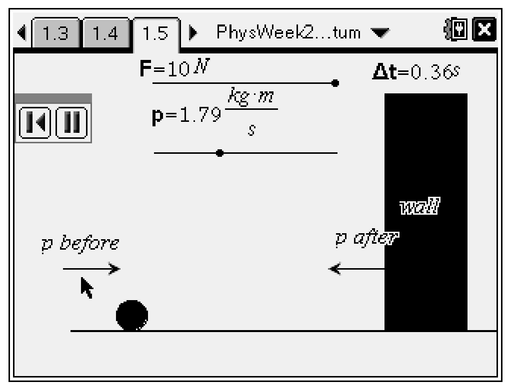
The following question will guide student exploration during this activity:

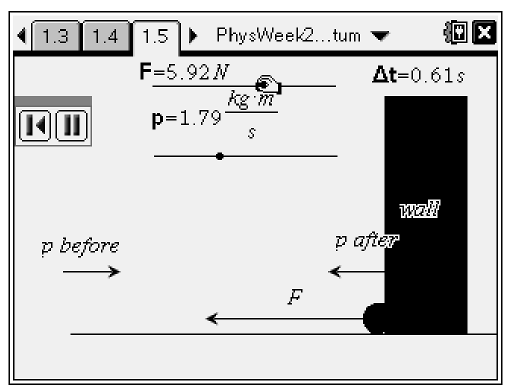
- What is the relationship between the impulse of a force acting on an object and the object's change in momentum?

The purpose of this activity is to allow students to explore the impulse-momentum theorem in the special case of the linear collision of a ball with a wall. Students observe changes in momentum, the time of the interaction, and the force the wall applies to the ball in order to develop an understanding of impulse and its effect on the momentum of the ball. Students then derive the impulse-momentum theorem and apply it to solve problems.

This activity consists of three problems. In the first problem, students explore the relationship between momentum, time of interaction, and force in order to conceptualize the cause-effect relationship between force and change in momentum in an elastic collision. They use this idea to answer conceptual questions and to derive the impulse-momentum theorem from Newton's second law. In the second problem, students apply the impulse-momentum theorem to analyze several cases of inelastic linear collisions of a ball with a wall. Two cases are presented as diagrams, and another two cases are presented as velocity vs. time graphs. In the third problem, students apply the impulse-momentum theorem to solve problems.

Problem 1 – Exploring the relationship among momentum, time of interaction, and force





Step 1: Students should open the file **PhysWeek20_Impulse_Momentum.tns**, read the first four pages, and then move to page 1.5. Page 1.5 shows an animation of an elastic collision between a ball and a wall. Students can use the animation control buttons to simulate the ball's collision with the wall, and they can observe how the wall's force acting on the ball affects the ball's motion. It is important here for students to understand that the force from the wall is exerted on the ball only when the ball is actually touching the wall (which is why the arrow for the force **F** appears only when the ball hits the wall). The students should use the sliders to vary the magnitudes of the wall's force and the ball's momentum and observe the changes in initial and final momentum and time of interaction. The momentum values range from 0.08 kg·m/s to 5 kg·m/s. The range of force values is from 0.17 N to 10 N. After students have explored the animation, they should answer questions 1–3.

Q1. What is the change in momentum, Δ*p,* of the ball, in terms of its initial momentum, *pbefore?* (Hint: Δ*p* is a vector quantity. Momentum is conserved in an elastic collision.)

A. *The change in momentum is given by* $\Delta p = \left|\overrightarrow{p_{after}} - \overrightarrow{p_{before}}\right|$. *Because momentum is conserved in an elastic collision and the wall's momentum is zero after the collision, the ball's momentum must have the same magnitude before and after the collision. In other words,* $\overrightarrow{p_{after}} = -\overrightarrow{p_{before}}$. *Therefore,* $\Delta p = \left|\overrightarrow{p_{after}} - \overrightarrow{p_{before}}\right| = \left|-\overrightarrow{p_{before}} - \overrightarrow{p_{before}}\right| = 2p_{before}$.

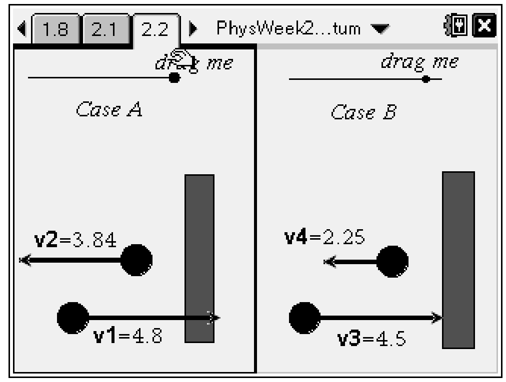
Q2. What did you observe about the time of interaction between the ball and the wall, the force the wall exerts on the ball, and the change in momentum of the ball?

A. *The force acts upon the ball only when it is in direct contact with the wall. When the magnitude of momentum increases, the time of interaction increases. When force increases, the time of interaction decreases. Encourage students to discuss their observations. Note: The time of interaction between the ball and the wall is calculated using the impulse-momentum theorem,* $\Delta t = \frac{\Delta p}{F}$, *where* $\Delta p = \left|\overrightarrow{p_{after}} - \overrightarrow{p_{before}}\right|$. *This formula is hidden, and only the numerical value of time is shown on the page.*

Q3. Use Newton's second law, *F = ma,* and the definition of acceleration, $a = \frac{\Delta v}{\Delta t}$, to find the relationship between force (*F*), time of application of the force (Δ*t*), and change in momentum (Δ*p*). (This relationship is known as the impulse-momentum theorem.)

A. $F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = \frac{\Delta p}{\Delta t}$**,** thus $F\Delta t = \Delta p$

Problem 2 – Analysis of inelastic collisions using the impulse-momentum theorem



Step 1: Next, students should read the text on page 2.1 and then move to page 2.2. Page 2.2 shows diagrams for two cases of inelastic collision between a ball and an unmovable wall. Students can use the slider to change the magnitude of the initial velocity in each case. Students should press ctrl tab to switch between case A and case B. Students should vary the initial velocities of the two balls and observe the effects. Then, they should answer questions 4–7.

Q4. Which ball has the greater velocity change during the collision, ball A or ball B?

A. *Ball B; because velocity is a vector quantity, students can use the following formulas to calculate change in velocity:*

case A: $\Delta v = v_2 - (-v_1) = v_2 + v_1$

case B: $\Delta v = v_3 - (-v_4) = v_3 + v_4$

Using the sliders, students should set up the same initial velocity in both cases and compare the final velocities.

Q5. Which ball has the greater momentum change during the collision, ball A or ball B? (Assume the two balls have the same mass.)

A. *Ball B; if students struggle with this, remind them that* $\Delta p = m\Delta v$.

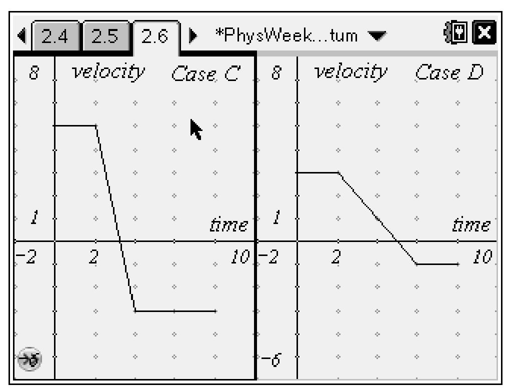
Q6. In which case does the wall produce the greater impulse? Explain your answer.

A. *Case B; if students struggle with this, remind them that* $\Delta p = F\Delta t$.

Q7. What are some factors that could explain the differences in impulse and momentum change between the two situations?

A. *Encourage student discussion of the factors that can affect the elasticity of a collision (and, by extension, impulse and momentum change). Remind them to consider events they observe in their everyday lives (e.g., the difference in bounciness between a flat basketball and a well-inflated one). In this case, the most likely explanation for the difference in elasticity is a difference in composition of the walls or the balls. Softer materials will absorb more of the energy of the ball and result in a greater loss of momentum and a lower final velocity of the ball. Other environmental factors, such as temperature, can also affect the elasticity of the collisions. If you wish, you may also discuss with students the possible reasons that momentum is not conserved in these situations. One possible reason is that the wall could have anchors that hold it in place (nails or simple friction connecting it to the ground). These anchors would generate the external force necessary to cause a violation in momentum conservation.*

Step 2: Next, students should read the text on page 2.5 and then move to page 2.6. Page 2.6 shows velocity vs. time graphs for two inelastic collisions. Students can press ctrl tab to switch between case C and case D. Students should examine the graphs and then answer questions 8–11.



Q8. Which ball has the greater velocity change?

A. *Ball C; students may be able to answer this question without doing any calculations if they understand that the scale on both graphs is the same. They can confirm their answers with calculations. In each case,* $\Delta v = v_{final} - v_{initial}$. *For case C,* $v_{final} = -3$ *m/s and* $v_{initial} = 5$ *m/s, so* $\Delta v = -3 - 5 = -8$ *m/s. In case D,* $v_{final} = -1$ *m/s and* $v_{initial} = 3$ *m/s, so* $\Delta v = -1 - 3 = -4$ *m/s.*

Q9. Which ball has the greater momentum change? (Assume the balls have the same mass.)

A. *Ball C; if students struggle with this, remind them that* $\Delta p = m\Delta v$.

Q10. In which case does the wall produce the greater impulse?

A. *Case C; if students struggle with this, remind them that* $\Delta p = F\Delta t$.

Q11. In which case is the force on the ball greater? Explain your answer.

A. *Case C; if students struggle with this, remind them of Newton's second law (F = ma). The acceleration (change in velocity) in case C is greater than that in case D. Therefore, if the balls have the same mass, the force acting on ball C must be greater than the force acting on ball D. Students can confirm this by comparing the slopes of the velocity vs. time graphs.*

Problem 3 – Applications of the impulse-momentum theorem to problem solving

Step 1: Next, students should read page 3.1. In this part of the activity, students solve problems using the impulse-momentum theorem. A *Calculator* application is provided with each problem for them to use for calculations. Students can press ctrl tab to switch between each question and the calculator application.

Q12. Cart 1 is pulled with a force of 1.0 N for 1 sec. Cart 2 is pulled with a force of 2.0 N for 0.50 sec. Which cart experiences the greater impulse? Explain your answer.

A. *The impulse experienced by each cart is equal to the force exerted on the cart multiplied by the time over which the force is exerted. The calculations are shown below:*

Cart 1: $F\Delta t = 1.0\ N \cdot 1\ sec = 1\ N \cdot sec$

Cart 2: $F\Delta t = 2.0\ N \cdot 0.50\ sec = 1\ N \cdot sec$

So, both carts experience the same impulse.

Q13. A person with a mass of 50.0 kg is riding at 35.0 m/s in her car when she must suddenly slam on the brakes to avoid hitting a deer crossing the road. Her air bag brings her body to a stop in 0.500 sec. What average force does the air bag exert on her?

A. *From the impulse-momentum theorem, FΔt = mΔv. Therefore,* $F = \frac{m\Delta v}{\Delta t}$. *To solve, substitute the given values, as shown below:*

$$F = \frac{m\Delta v}{\Delta t} = \frac{(50\ kg)(35.0\ m/s)}{(0.500\ sec)} = 3{,}500\ N$$

Q14. The driver in question 13 placed her 5 kg backpack on the front seat next to her. When she hit the brakes, the backpack was stopped by the dashboard in 0.002 sec. Use the Scratchpad to calculate the average force that the air bag exerts on her.

A. $$F = \frac{m\Delta v}{\Delta t} = \frac{(5\ kg)(35.0\ m/s)}{(0.002\ sec)} = 8.75 \times 10^4\ N$$

Q15. A hockey player applies an average force of 80.0 N to a 0.25 kg hockey puck for a time of 0.10 sec. Determine the impulse experienced by the hockey puck. Assuming the puck's direction does not change, by how much does the puck's velocity change? Use the Scratchpad to calculate your answer.

A. *First, calculate the change in momentum of the puck using* $\Delta p = F\Delta t = 80.0\ N \cdot 0.10\ sec = 8\ N \cdot sec$. *The puck's change in momentum is 8 N·sec. Therefore, the puck's change in velocity can be calculated as shown below:*

$$\Delta p = m\Delta v$$

$$\Delta v = \frac{\Delta p}{m} = \frac{8\ N \cdot sec}{0.25\ kg} = 32\ m/s$$

Q16. If a 5 kg object experiences a 10 N force for a duration of 0.10 sec, what is the momentum change of the object? Assuming the object was initially at rest, by how much does its velocity change?

A. $$\Delta p = F\Delta t = 10\ N \cdot 0.10\ sec = 1\ N \cdot sec$$

$$\Delta v = \frac{\Delta p}{m} = \frac{1\ N \cdot sec}{5\ kg} = 0.2\ m/s$$

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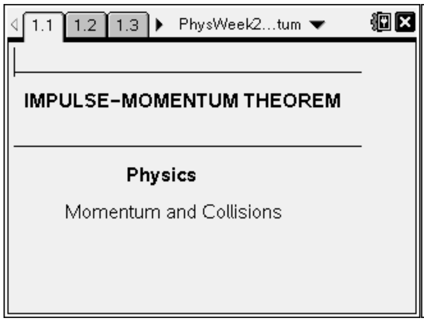
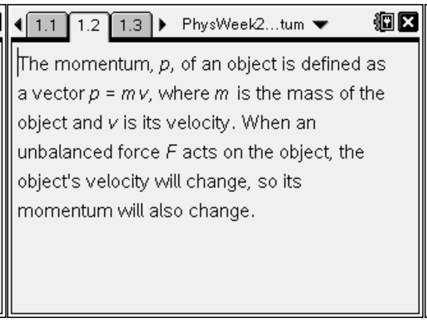
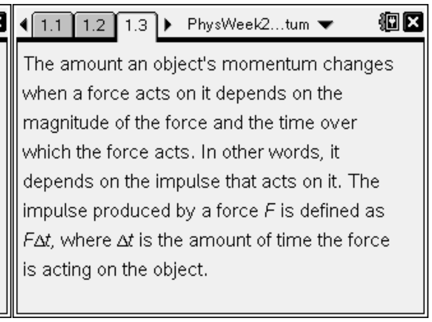
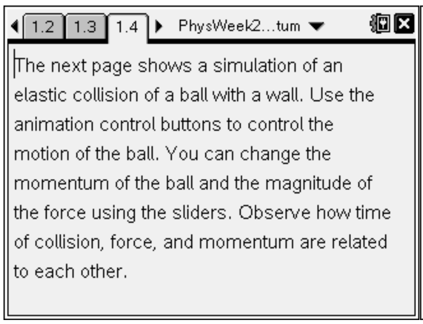
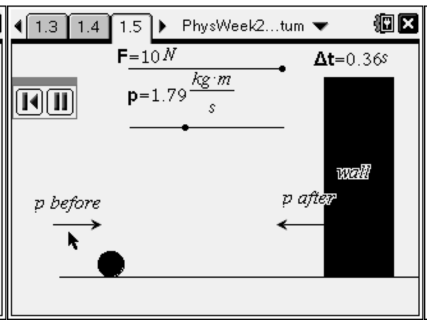
(Student)TI-Nspire File: *PhysWeek20_Impulse_Momentum.tns*

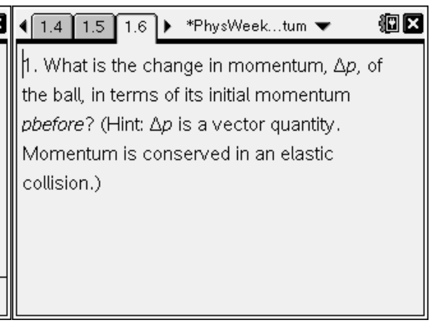


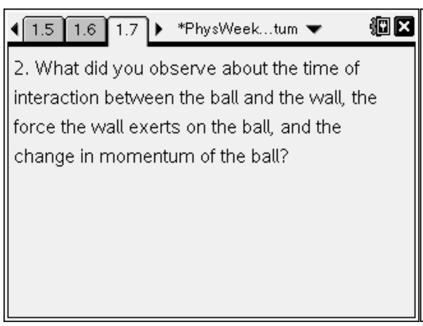


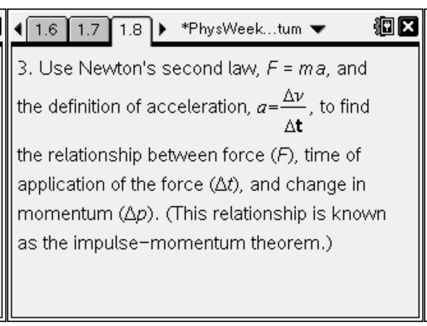


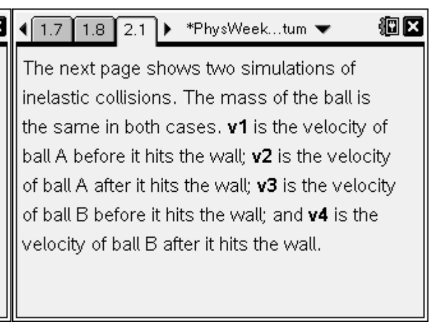


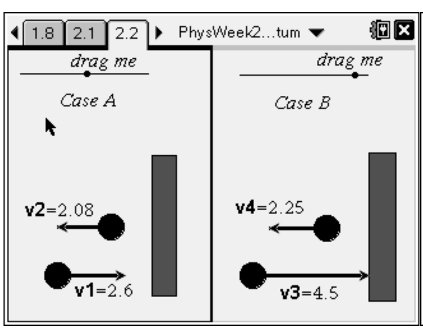


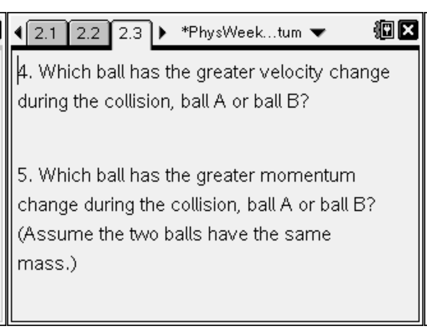


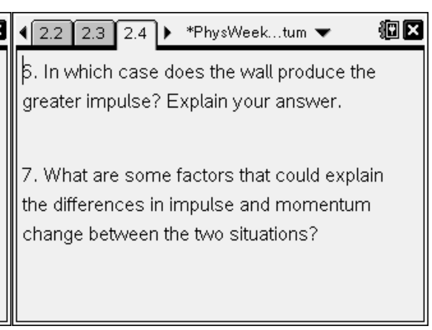




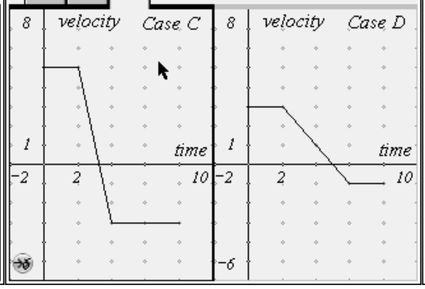








The next page shows velocity vs. time graphs for the collisions of two balls with two walls. Assume the balls have the same mass and the graphs have the same scale. Examine the graphs, and then answer the questions that follow.



8. Which ball has the greater velocity change?

9. Which ball has the greater momentum change? (Assume the balls have the same mass.)

10. In which case does the wall produce the greater impulse?

11. In which case is the force on the ball greater? Explain your answer.

In this part of the activity, you will use the impulse–momentum theorem to solve several problems.

12. Cart 1 is pulled with a force of 1.0 N for 1 sec. Cart 2 is pulled with a force of 2.0 N for 0.50 sec. Which cart experiences the greater impulse? Explain your answer.

13. A person with a mass of 50.0 kg is riding at 35.0 m/s in her car when she must suddenly slam on the brakes to avoid hitting a deer crossing the road. Her air bag brings her body to a stop in 0.500 sec. Use the Scratchpad to calculate the average force that the air bag exerts on her.

14. The driver in question 13 placed her 5 kg backpack on the front seat next to her. When she hit the brakes, the backpack was stopped by the dashboard in 0.002 sec. What average force did the dashboard exert on the backpack?

15. A hockey player applies an average force of 80.0 N to a 0.25 kg hockey puck for a time of 0.10 sec. Determine the impulse experienced by the hockey puck. Assuming the puck's direction does not change, by how much does the puck's velocity change? Use the Scratchpad to calculate your answer.

16. If a 5 kg object experiences a 10 N force for a duration of 0.10 sec, what is the momentum change of the object? Assuming the object was initially at rest, by how much does its velocity change?

