Transformations – Graphical Dilation

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Student Activity

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TI-Nspire™ Investigation Student 50 min

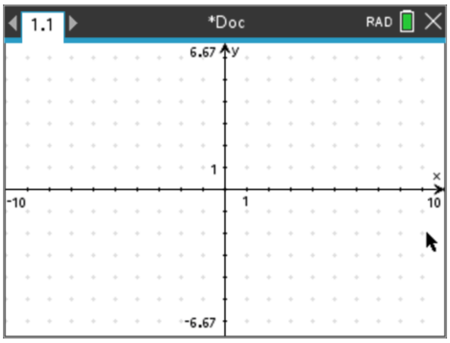
Calculator Instructions: Transformations

Create a new TI-Nspire document and insert a Graphs application.

Displaying the grid will make it easy to keep the numbers simple.

[menu] > **View** > **Grid** > **Dot Grid**

To hide the Graph entry, press: [esc]



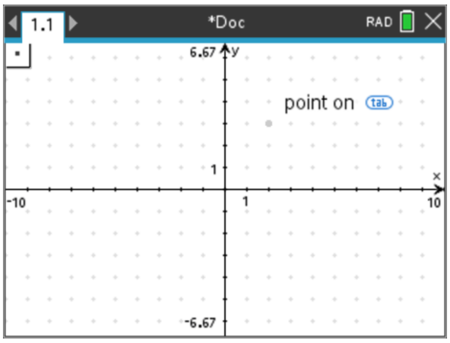
The keyboard shortcut to create a point in the Graphs application is to press **P**.
There are two options: Point or Point by Coordinates. Both options are used in this activity.

Place a point **on** the grid, "point on" appears as a prompt when the pen is close to the grid.

Once the point has been created, get the coordinates of the point:

[ctrl] + [menu] > **Coordinates & Equations**

Press [esc] to release the Point tool, then grab the point an move it around. The point should maintain integer coordinate values since the axis scale in both directions are provided in integer amounts.



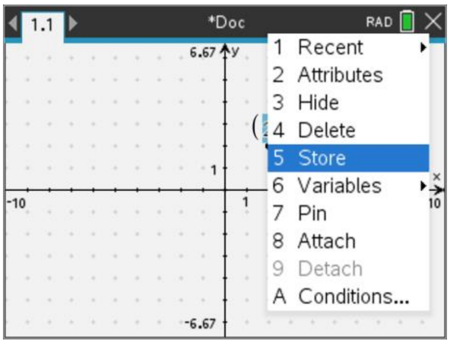
Hover the mouse over the abscissa (x – coordinate) then press:

[ctrl] + [menu] > **Store**

Store the abscissa as **xp**.

Repeat this process and store the ordinate (y – coordinate) as **yp**.

The coordinates will now appear bold. These values can be accessed and used in calculations in any other application within this problem.



Hover the mouse over the coordinate pair to see the variable link.

Author: P. Fox

Create a new point using the keyboard shortcut, select:

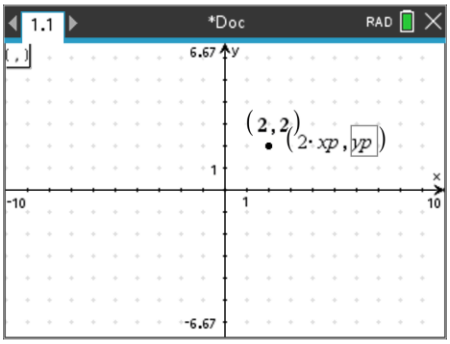
Point by Coordinates

For the abscissa, type: $2 \times xp$

For the ordinate, type: yp

Note: To navigate from the abscissa to the ordinate press: [enter]

This new point is referred to as “an image of point P”.



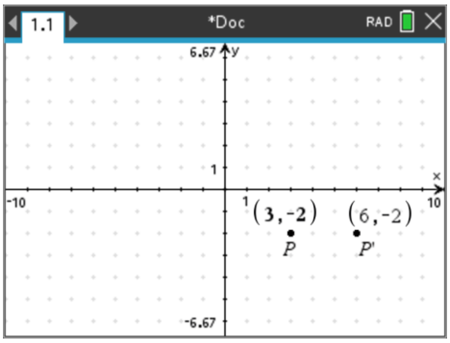
Think about the image of point P just like you would a photographic image. The image can be enlarged or reduced in size stretching uniformly in both directions. A stretch in the horizontal or vertical direction only will change the proportion or form of objects in the image making them look taller or shorter, wider or narrower.

Label the points: P and P′ where P′, pronounced “P prime” is the dilation of the original point P.

Move the mouse over the point then:

[ctrl] + [menu] > **Label**

The ‘prime’ notation can be accessed from: [?!▸]



The coordinates of points P and P′ automatically move with their respective points. Sometimes this can make reading the coordinates difficult. The coordinates can be moved away from their respective points. Think of them as ‘magnetic’, once moved far enough away they will detach and remain stationary.

Question: 1.

Move point P horizontally.

a) Describe how the point P′ moves.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

Question: 2.

Move point P vertically.

a) Describe how the point P′ moves.

b) Are there any locations where the coordinates of P and P′ are the same?

Question: 3.

Edit the formula used for the abscissa of point P′ so that it is half the value of xp.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

Question: 4.

Edit the formula used for the abscissa of point P′ so that it is equal to xp.
Change the formula for the ordinate so that it is twice the value of yp.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

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Question: 5.

Edit the formula used for the abscissa of point P′ so that it is equal to: $2 \times xp$.

The formula for the ordinate remains as: $2 \times yp$.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

The calculator's touchpad can be used as a touchpad by swiping or as single directional movement by clicking in the corresponding direction. (Arrow keys) The single direction movement is useful in menus.

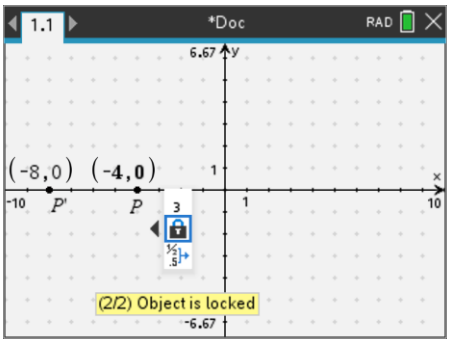
Place point P on the x axis.

Move the cursor over the ordinate for point P and press:

ctrl + menu > **Attributes**

Navigate to the padlock. Move to the right to lock the ordinate to zero.

Try and move point P off the x axis.



Our image (P′) is going to be linked or mapped to the abscissa only.

Edit the rule for P′ so that:

The abscissa is equal to: xp

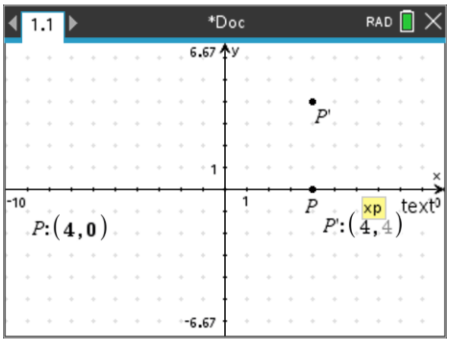
The ordinate is equal to: xp

Drag P along the x axis. Observe the movement of P′, the image of P.

It is possible to leave a temporary record of the path of P′.

menu > **Trace** > **Geometry Trace**

Click on the point P′, then drag P along the x axis.



Question: 6.

What is the equation for the path of point P′ ?

Question: 7.

Change the definition for P′: The abscissa changes to xp/2 while the ordinate remains as yp.

a) Thinking about dilations, what is happening to point P′ ?

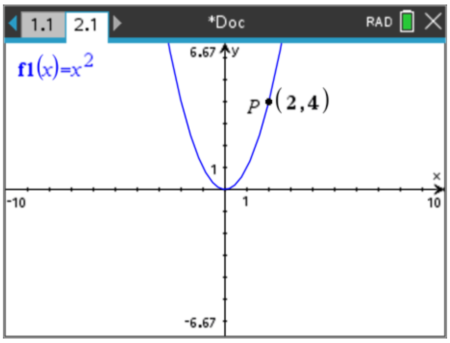
b) Change the colour of point P′ and do a geometry trace of its new path and determine the corresponding equation.

Transformation Algebra

Insert a new problem into the current TI-Nspire document.

doc▾ > **Insert** > **Problem**

- Insert a Graphs application and graph the function: $y = x^2$.
- Place a point on the graph. Display the coordinates and store them as xp and yp.
- Label the point: P



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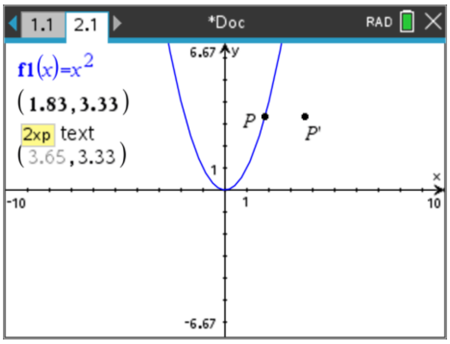
Insert a **Point by Coordinates** and set them as follows:

(2×xp, yp)

Label the point as: P′

Drag point P along the function and observe the path traced out by: P′.



Point P(xp,yp)[1] is no longer free to move anywhere; it now moves along the function: $y = x^2$. The coordinates of P′ are defined by xp and yp, so the path of P′ can also be defined, the aim here is to determine the equation for that path.

Point P′ will be defined as: P′(x',y') & Point P as P(x, y)

We need to find a rule that relates x' to y'. Our relationships, as defined on the calculator are as follows:

Equation 1	$x' = 2 \times xp$	Equation 2	$y' = yp$	Equation 3	$y = x^2$

Step 1:

Changing the notation:	$x' = 2x$	$y' = y$	$y = x^2$

Step 2:

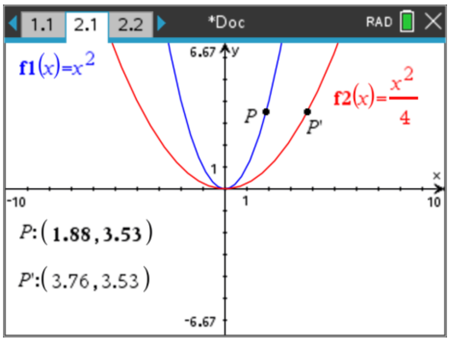
Express Eqn1 & Eqn2 in terms of x and y respectively:	$x' \div 2 = x$	$y' = y$

Step 3:

Substitute Eqn 2 into Eqn 3	$y' = x^2$
Substitute Eqn 1 into Eqn 3	$y' = \left(\frac{x}{2}\right)^2$

With the relationship established, the 'prime' notation can now be removed and the function graphed:

$$y = \frac{x^2}{4}$$



- Point P lies on the graph with equation: $\boldsymbol{y} = x^2$
- Point P′ lies on the graph with equation: $\boldsymbol{y} = \frac{\boldsymbol{x^2}}{\boldsymbol{4}}$
- The relationship between P(xp,yp) and P′(x′,y′) is such that: $x' = 2 \times xp$
- The dilation factor is 2 units away from the y axis.

Question: 8.

Change the definition for P′: The abscissa changes to xp/2 while the ordinate remains as yp.

[1] Point P is expressed in terms of (xp, yp). This notation reflects the limitation of the digital platform rather than mathematical terminology. Assigning values to variable names such as x and y on the calculator means they will no longer treated as variables, however, it is important to maintain correct mathematical notation, written notes are not bound by such limitations.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) x' and y'.

c) What is the dilation factor for the graph?

Question: 9.

Change the definition for P′: The abscissa changes to xp while the ordinate changes to: $2 \times yp$.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) x' and y'.

c) What is the dilation factor for the graph?

Question: 10.

Change the definition for P′: Leave the abscissa as xp and change the ordinate to: $yp \div 2$.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) x' and y'.

c) What is the dilation factor for the graph?

Question: 11.

If the dilation factor for a quadratic is ½ towards the y axis, what would it be equivalent to if referenced against the x axis? Explain the similarities / differences and support your answer with appropriate equation(s).