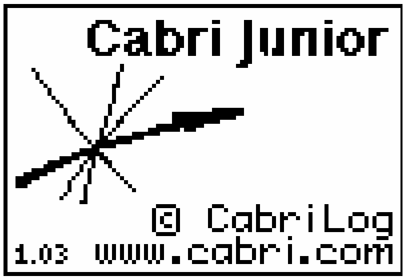
Activity 25 - The Euler Line

First, turn on your TI-84 Plus and press the APPS key. Arrow down until you see Cabri Jr and press ENTER. You should now see this introduction screen.



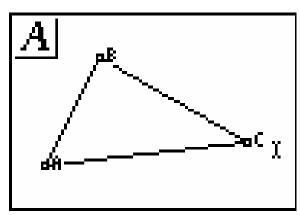
To begin the program, press any key. If a drawing comes up on the screen, press the Y= key (note the F1 above and to the right of the key – this program uses F1, F2, F3, F4, F5 names instead of the regular key names) and arrow down to NEW. It will ask you if you would like to save the changes. Press the 2nd key and then enter to not save the changes.

We are now ready to begin.

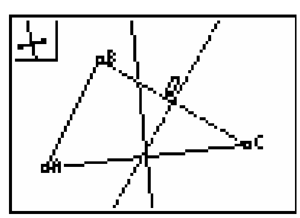
From last year's activities, you learned about four triangle centers that can be easily constructed. We will use all four centers in this construction. To review, the points are:

- The Circumcenter at the intersection of the perpendicular bisectors of the sides of a triangle – we will call this point D.
- The Incenter at the intersection of the angle bisectors – we will call this point E.
- The Orthocenter at the intersection of the altitudes – we will call this point F.
- The Centroid at the intersection of the medians – we will call this point G.

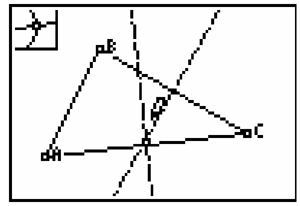
Start with any triangle ABC. In the diagram, sides BC and AC are almost congruent. This will cause a slight problem that will need to be corrected later and provide a hint for one of the quesitons posed later.



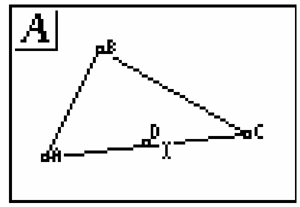
Construct the perpendicular bisectors of two of the sides. Why is it sufficient to construct only two of these lines?



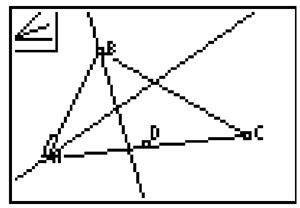
Construct the point of intersection. This point is called the Circumcenter. Using this point as the center, you could construct a circle that passes through points A, B and C.



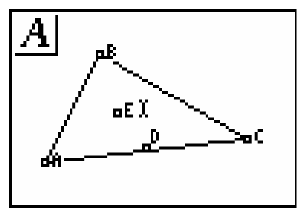
Hide the two perpendicular bisectors and label the point of intersection as D. Notice that this point is close to the side AC.



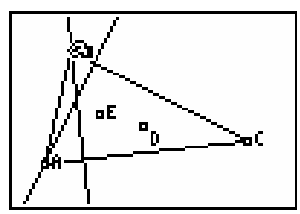
Construct two of the angle bisectors. Again, is the construction of only two sufficient or should we also construct the third angle bisector?



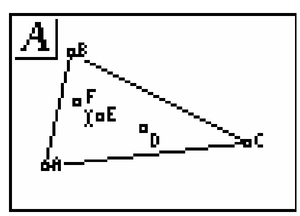
Construct the point of intersection and label it as E. E is the Incenter. With two additional steps, you could construct a circle that is tangent to AB, AC and BC.
Hide the angle bisectors.



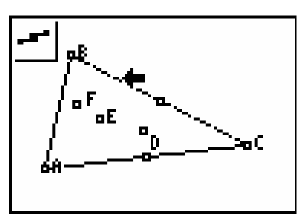
Construct a line that passes through point A and is perpendicular to BC. Construct a second line that passes through B and is perpendicular to AC. It may be necessary to drag one of the vertices of the triangle in order to clearly identify the point of intersection of these lines.



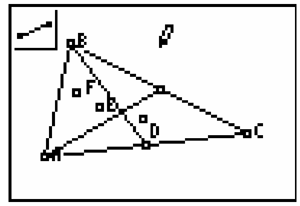
These lines could be used to form altitudes of the triangle. Construct the point of intersection of these lines and label it as point F. Hide the lines.



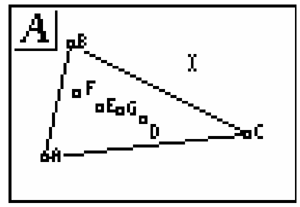
Construct the midpoints of two of the sides, say BC and AC.



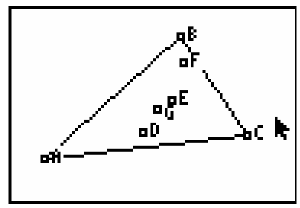
Construct a line segment connecting A to the midpoint of BC and another connecting B to the midpoint of AC. These line segments are called medians.



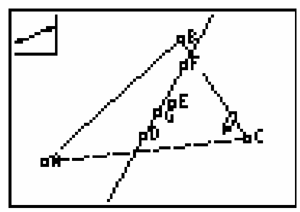
Construct the point of intersection of the medians and label it as G. This point is called the Centroid and is the center of mass of the triangle. If you cut the triangle out of cardboard and constructed this point, you could balance the triangle on a finger or the tip of a pencil.



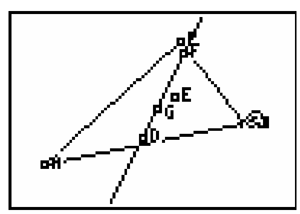
Examine the four points that you have constructed. You can construct a line, called the Euler line, that passes through three of the four points. In order to see which one should be excluded, you may need to drag one of the vertices.



Construct a line that passes through say point F and point D. This line will also pass through point G, but might not pass through point E.



Drag one of the vertices to a new location to determine if the three points remain collinear.



Continue to drag the vertices of triangle ABC. Under what conditions is the Incenter on the line? Under what conditions do the points converge?

Given the coordinates of the three vertices of a triangle, can you show that the three points are collinear? This is a challenging exercise that requires you to find the coordinates of the points of intersection of the special lines built on the triangle and then show that these three are collinear.

Other notable points that lie on the Euler line are the de Longchamps point, the Schiffler point, the Exeter point, the far-out point and the nine-point circle center.