Activity 2

Differential Equations with the TI-86

The TI-86 will numerically solve systems of differential equations and graph the results. Problems in dynamics often naturally generate differential equations, but the equations are usually not directly solved in introductory physics since the mathematical sophistication required is high. Using the TI-86, you can easily explore the solutions to differential equations, such as models of experimental data.

This activity is designed to introduce you to the differential equation solving capabilities useful in physics.

Equipment Required

♦ TI-86 calculator

Program Group Required

♦ None.

Setup

1. Switch the calculator to differential equation mode. Press the [2nd] [MODE] key and highlight the **DifEq** mode. (▼ ▼ ▼ ▼ ▶ ▶ ▶ [ENTER])

2. Select the appropriate graph mode settings. Press the [GRAPH] key, then [MORE] to scroll to more of the graph menu, and press **FORMT** ([F1]). Set the modes to **CoordOn** (show coordinates of cursor), **AxesOn** (show graph axes on screen), **GridOff** (do not show a grid on screen), **LabelOn** (label the axes), **RK** (use the slower but more accurate Runge-Kutta method for solving equations), and **FldOff** (do not show slope or directional fields). Only the last setting is critical to complete this exercise.

Figure 2.1

Instructions

As an example, we will use a very simple differential equation of

$$\frac{d^2y}{dt^2} = -g .$$

Consider a one-dimensional freefall system with acceleration constant at $-g$. If we specify the initial velocity and initial position of the object, the motion is completely specified. We will use $v_{y0} = 10$ m/s, and $y_0 = 1.5$ m. These conditions correspond to a ball thrown straight upward. Although we could model this system using the equations of kinematics, it will serve as a simple example of using the differential equation method to solve the problem.

Newton's Second Law, $\Sigma F = ma$, generates second-order differential equations, since the acceleration is the second derivative of the position. The TI-86 can solve only first-order differential equations, but it can solve up to nine of them simultaneously. To solve a second-order differential equation we need only to write it as a set of two first order equations. In this example we have

$$\frac{dy}{dt} = v_y$$

or the first derivative of the y position is the y velocity, and

$$\frac{dv_y}{dt} = a_y = -g$$

or the acceleration in the y direction is the first derivative of the y velocity. In this example a_y is always just $-g$, the freefall acceleration.

To enter this pair of equations into the TI-86, we will have to re-label variables as follows. Variable **Q1** will be the y position, and **Q2** the y velocity. In that case,

Q'1 = Q2, which represents $\frac{dy}{dt} = v_y$, and

Q'2 = - g, which represents $\frac{dv_y}{dt} = a_y = -g$

The first expression, **Q'1 = Q2**, is often used in physics models with Newton's Second Law. The second expression is determined from the Second Law, solved as

$$a = \frac{\Sigma F}{m} .$$

For **Q'2,** we enter an expression for the acceleration. In the present example, the acceleration is constant, but in general the acceleration can be a function of time (t), position (**Q1**) or velocity (**Q2**). We will now enter these expressions into the TI-86.

1. Press GRAPH and **Q'(t)=** (F1). Clear any existing equations by choosing **DELf** (F4) repeatedly until the cursor is at the top near **Q'1=.** Enter **Q2**, or F2 2 ENTER. On the **Q'2=** line, you will enter your expression for the acceleration. The TI-86 has a built-in constant for freefall acceleration (9.80 m/s^2), so we can just enter **–g**, or (-) 2nd [alpha] **G**. Your screen will resemble Figure 2.2.

Figure 2.2

2. Next, the initial conditions need to be entered. Choose **INITC** (GRAPH F3) to enter the initial conditions screen. Set **tMin** = 0 to start the calculation at t = 0. Set **QI1** to 1.5, corresponding to $y_0 = 1.5$ m. Set **QI2** to 10, corresponding to $v_{y0} = 10$ m/s.

```
INITIAL CONDITIONS
 tMin=0
·QI1=1.5
·QI2=10■


Q'(t)= WIND INITC AXES GRAPH▶
```
Figure 2.3

3. The axes are set by choosing **AXES** (F4). The default of **x=t** and **y=Q** will be appropriate. That is, time (*t*) will be plotted on the horizontal axis, and both of the differential equation variables **Q1** and **Q2** will be plotted on the vertical axis. If you wanted to plot just the position variable, you could enter **Q1** in the **y=** line. For now we will plot both, so be sure **y=Q** is shown. (If the **FldOff** mode is not showing, press MORE **FORMT** and set the **FldOff** mode in the last line.)

```
AXES: FldOff
 x=t
 y=Q


Q'(t)= WIND INITC AXES GRAPH
  Q    t    Q'
```
Figure 2.4

4. The last required setting is the graph window. GRAPH **WIND** (F2) will display the window editor. **Set tMin**=0, **tMax**=2, **xMin**=0, **xMax**=2, **yMin**=–5, **yMax**=10 and leave the other settings alone. You have just set the TI-86 to solve the differential equation for time running from 0 to 2 seconds. Since *t* or time is plotted along the *x* axis, we have set the *x* range to the same interval as the *t* range. The *y* range is a guess; we can go back and change it later depending on the solution that emerges.

5. Now press GRAPH **GRAPH** (F5). You will see a graph of **Q1** and **Q2** *vs.* time, or *y* and v_y *vs.* time. To remove the menu from the screen, press CLEAR.

The line of constant negative slope is the velocity of the ball; it is initially positive but decreases uniformly through zero, continuing below the axis to higher speeds. Recall the kinematics expression $v = v_0 + at$ for velocity as a function of time; for these conditions it would read $v = 10 - 9.8t$, which corresponds to the line graphed on the TI-86.
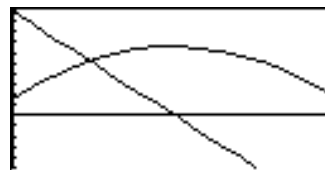

Figure 2.5

The curved line shows the ball's position as a function of time. Had we used kinematics we would have entered an expression for

$$y = 1.5 + 10t + \frac{1}{2}(-9.8)t^2 .$$

The plotted function matches this expression well.

Notice that you did not enter either kinematics expression. The TI-86 drew the graphs by numerically solving the differential equation with its initial conditions. Unlike the kinematics expressions, which can be used only when acceleration is constant, this numerical method of solving differential equations can be used whenever the force is known as a function of time, position or velocity. That is, if the force is known, then you can find the motion.

1. You have now learned the basic method for graphing the solution to a differential equation. Other functions that are useful in exploring differential equations are TABLE, **TRACE** and **ZOOM**. Each will be described briefly below. You can view a table of solutions to your differential equations.

```
TABLE SETUP
 TblStart=0
 ΔTbl=.1
 Indpnt: Auto Ask


TABLE
```
Figure 2.6

Press ⟨TABLE⟩, then **TBLST** (⟨F2⟩) to set up your table. Enter an appropriate value for the initial table entry (**TblStart**) and the increment between lines in the table (**ΔTbl**). For the freefall example, good values to enter are **0** and **0.1**. Leave the Independent variable on **Auto**. Now press **TABLE** (⟨F1⟩) to view the table. The ⟨▾⟩ key will scroll down through the table.

2. The cursor can be set to move along the graph lines when **TRACE** mode is set. Press ⟨GRAPH⟩ ⟨MORE⟩ ⟨F4⟩ to enter the mode. The cursor is now bound to a curve on the graph; switch between curves using the ⟨▾⟩ and ⟨▴⟩ keys. Move the cursor left and right with ⟨◂⟩ and ⟨▸⟩. The coordinates of the cursor will be displayed. Exit the **TRACE** mode by pressing ⟨CLEAR⟩.

Figure 2.7

3. Sometimes the range you choose for the graph window is not appropriate. You can let the calculator set the window size based on the solution to your differential equations. Press **ZOOM** (⟨GRAPH⟩ ⟨MORE⟩ ⟨F3⟩). One useful choice is **ZFIT**, ⟨MORE⟩ ⟨F1⟩, which will rescale the window to fit the solution over the time interval entered in the initial conditions. Try this now. **ZPREV** will restore the view to the previous setting.
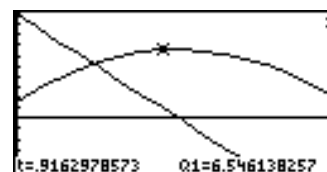

Figure 2.8

After you have finished this activity, you may want to return the calculator to function plotting mode. Press the ⟨2nd⟩ ⟨MODE⟩ key and highlight the **Func** mode, then ⟨EXIT⟩. (⟨▾⟩ ⟨▾⟩ ⟨▾⟩ ⟨▾⟩ ⟨ENTER⟩ ⟨EXIT⟩)

Extensions

♦ Change the definition of **Q'2** to include the effect of air resistance. For example, if the acceleration of a ball is

$$a_y = -g - bv_y|v_y|,$$

where the constant $b = 1$ m^{-1} sets the size of the air resistance, then we would have **Q'2 = –g – Q2 abs Q2**. Everything else would stay the same. Inspect a graph of the velocity as a function of time and see how a constant velocity is reached after some time has elapsed. This differential equation is hard to solve, but the TI-86 solves it numerically just as easily as the previous example. You may need to use times longer than 2 s.

♦ Graph two dimensional motion. A second pair of differential equations can be used to model the horizontal motion. Use **Q3** for x and **Q4** for v_x, with **Q'4** = 0 and **Q'3 = Q4** to model zero acceleration in the horizontal direction. For **AXES** choose x = **Q3** and y = **Q1**. Set appropriate window size; t and x are no longer necessarily the same, since **Q3** is plotted along the x axis. You will need to set additional initial conditions in **INITC**; try **QI3** = 0 and **QI4** = 4 to model a ball thrown from $x = 0$ m and with a horizontal velocity component of 4 m/s.