TI-Nspire Activity: ***Getting to the Roots of Quadratics***

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Activity Overview

This activity uses the dynamic nature of the TI-Nspire CAS to investigate quadratics. We look at the connection between the discriminant and the nature of roots of quadratics.

Concepts

Quadratics, discriminant, quadratic formula, roots, x-intercepts, parabolas

Teacher Preparation

- Make copies of the student worksheet that accompanies this activity.
- Be sure that the RootsOfQuadratics.tns file is loaded onto all student handhelds.
- Solutions to the student worksheet and screen captures of the activity along with suggestions for implementation can be found below.

The Classroom.

Students should have some familiarity with the TI-Nspire CAS. This activity uses the following TI-Nspire CAS applications:

- *Graphs & Geometry*
- *Notes: Questions & Answers*
- *Calculator*

Background

- Have students read and complete the Background section on page 1 of their worksheet.
- Discuss the Background section to make sure that everyone is clear that the real roots of a quadratic are the values at which the graph crosses the *x*-axis.

  Student Handout Solutions:

  0 real roots – sketch a parabola that does not cross the x-axis
  1 real root – sketch a parabola that is tangent to the x-axis
  2 real roots – sketch a parabola that crosses the x-axis twice.

- It might be good to remind students to look for examples of parabolas that open down as well as up. If they do not recall that the parabola will open up when *a* is positive and down when *a* is negative, you can have them explore this on page 1.2 of the TI-Nspire CAS file before beginning the rest of the Exploration below.
- Students should be familiar with using the quadratic formula to find roots.
- Emphasize that this formula is an algebraic way of finding the values at which the graph crosses the *x*-axis.

TEACHER HANDOUT
Grade level: Pre-service/secondary
Subject: Advanced Algebra
Time required: 90 to 120 minutes

Overview
Read the Overview section on page 1 of their worksheet as a class.
- It may be helpful to remind students what it would mean for a value to be integer, rational, irrational, or real.
- Students should know that the square root of a negative number is not a real number.

STEPS
Exploration
- Have the students turn to page 2 of their worksheet and open the RootsOfQuadratics.tns file and hit ctrl ▸ to go to page 1.2 of the file.

- Ideally, the students would read all the directions on page 2 of their worksheet and get right to work exploring on page 1.2 of the TI-Nspire file. However, you may find it easier to read the directions together. Pause after the first bullet and make sure that they can edit the values of the variables *a*, *b*, and *c*. To test this, see if they can make parabolas that open up and down. Remind them not to let *a* = 0.

- Be sure they see that the function, the graph, the x-intercepts and the value of *d* change as they change the values of the variables *a*, *b*, and *c*.

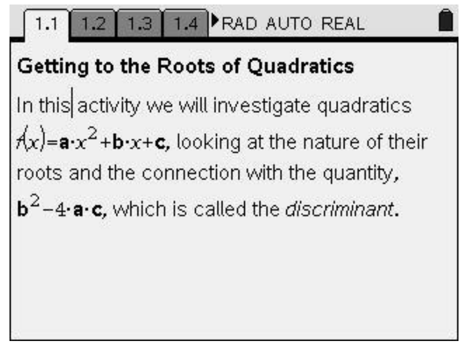
- For now we really only need to enter integer values for the variables *a*, *b*, and *c* to discover the pattern.

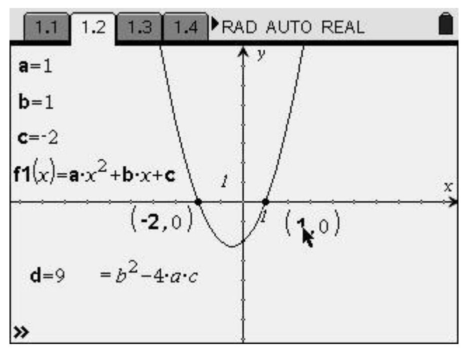
- Be sure to emphasize the **GOAL**: To find 3 examples of each category: quadratics with one (repeated/double) real root, two real roots and no real roots.

- Make sure students are recording their work on the table on the worksheet.

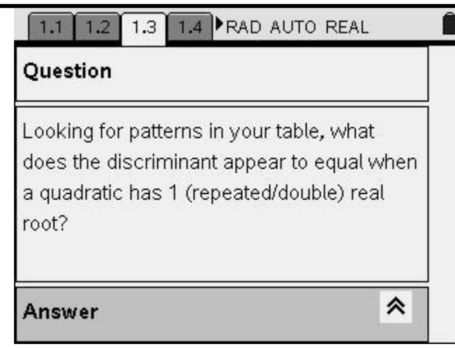
- Rows for Examples 4 & 5 are for future data collection.

SCREENSHOTS



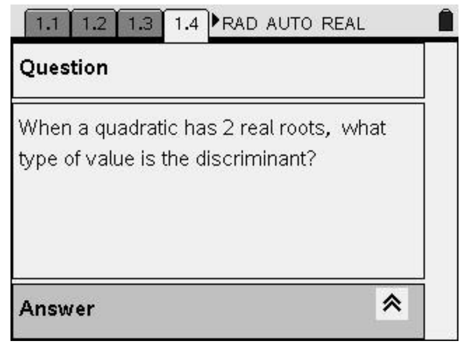


Conjectures
Students are now asked to look at their table of data and find patterns and complete conjectures.

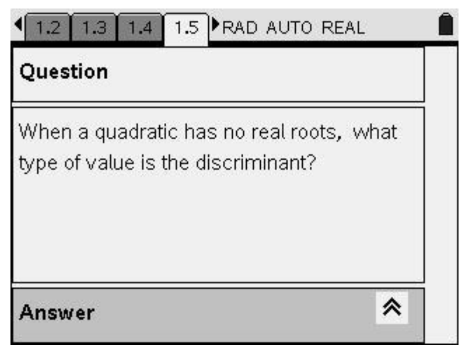


Student Worksheet Solutions:

- A quadratic has 1 real root when the discriminant $b^2 - 4ac$ is _0_.
- A quadratic has 2 real roots when the discriminant $b^2 - 4ac$ is _positive_.
- A quadratic has 0 real roots when the discriminant $b^2 - 4ac$ is _negative_.



On pages 1.3, 1.4, and 1.5 of the TI-Nspire file, students can check their answers to the conjectures by pressing tab until the icon beside the **Answer** portion of each screen is highlighted. Then press enter to reveal the answers.



Test Your Conjectures
Students now come up with three new examples of values for *a*, *b*, and c that make the discriminant 0, positive, and negative. They record their values in the table provided on the worksheet. They check the discriminant by hand and guess how many roots each quadratic will have based on their above conjectures.

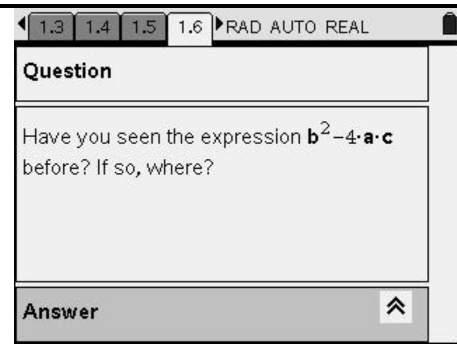
They then go back to page 1.2 in the TI-Nspire file, change the values of the variables *a*, *b*, and *c* for each of the new examples to see if the nature of the roots was what they expected.

Make sure students put these new values in the appropriate categories in their original Data Table on page 3 of their worksheet. They should also record the roots if any.

Student answers will vary based on their previous examples.

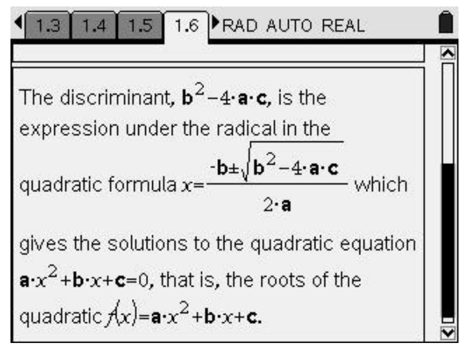
Explanation

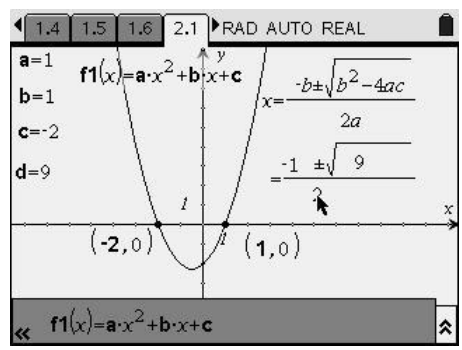
Students now explore why the discriminant determines the nature of the roots of a quadratic.



First they are asked if they have ever seen the expression $b^2 - 4ac$ for the discriminant before and where. Students record their answers on their worksheet.

They can check their answer on page 1.6 by displaying the hidden answer.





Students now go to page 2.1 to investigate why the discriminant affects the nature of the roots. Page 1.2 has the quadratic formula on it and as student change the values of the coefficients, the quadratic formula will change as well.

Students are asked to use page 2.1 to input their 3 new examples from page 4 of their worksheet. They record the quadratic formula given and then by hand they simplify the expression to find the *exact* values of the solutions. Be sure that they recognize that the ± in the formula will sometimes produce two answers. Have them leave any radicals. They can see decimal approximations on the screen.

Page 6 of the student handout asks them to give explanations. Below are some possible solutions.

Explain why the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there is 1 real root for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac = 0$. We say this is a ***double*** or ***repeated real root***. Write a formula for that one root in terms of a, b and c. Is this one root rational or irrational?

When the discriminant is 0, the square root is 0. Adding or subtracting 0 will keep just the –b in the numerator, giving only

one solution. We have
$x = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b}{2a}$. *This is a rational number. (The teacher should note that we've only had students use integer values for a, b and c. Depending on the level of your students, you can investigate why this is still rational as long as a, b and c are rational.)*

Explain why the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there are no real roots for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac < 0$.

When the discriminant is negative, the square root of a negative number is not real, which makes x not real. So there are no real roots.

Explain why the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there are 2 (distinct) real roots for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac > 0$.

When the discriminant is positive, we get two values one for the + and one for the – in the formula.

Further Exploration: Positive Discriminants
In this section students explore the case where the discriminant is positive. The goal is to determine what kind of value the discriminant must be when it is positive to produce two rational roots.

You may need to remind students as the handout does that rational numbers have decimal expansions that terminate or repeat. But since the file only shows 5 decimal places, how can we be sure.

If you want, you can have your students guess at whether the roots for these examples (as recorded in the Data Table) are rational or irrational.

Student put in the coefficients from their data table on page 3 which produce positive discriminants into page 2.1 in the file. They record the quadratic formula and simplify it by hand to find the exact roots. Students classify them as rational or irrational.

Student collect more data if their only solutions are in fact integers.

Student solutions for the table on pages 7 & 8 will vary based on their examples.

On page 8, students are asked to formulate a conjecture:

The quadratic $f(x) = ax^2 + bx + c$ will have 2 distinct RATIONAL roots when the discriminant is positive and a perfect square.

Test Your Positive Discriminant Conjecture
Students find two more examples with a discriminant which is positive and a perfect square. They enter their values in page 2.1, record the quadratic formula, simplify it by hand and confirm that the two roots are rational.

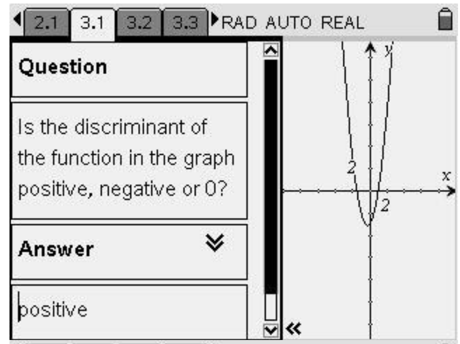
The teacher should note that again we are only using integer values for the coefficients. Depending on the level of your students, you could have them work with rational coefficients and help them to recognize rational numbers with perfect squares in both the numerator and the denominator.

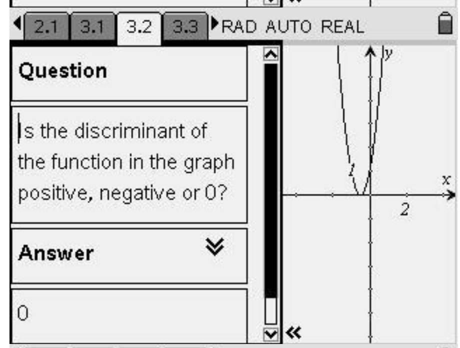
Summary
Students are asked to fill in the blanks.

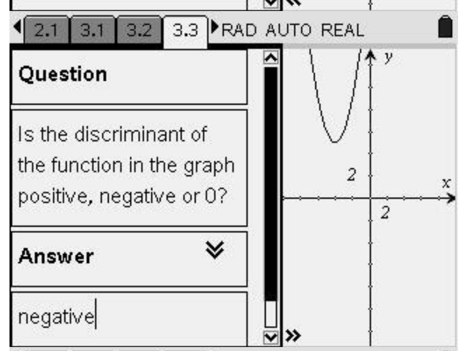
- The graph of a quadratic $f(x) = ax^2 + bx + c$ is called a *parabola*.
- The roots of $f(x) = ax^2 + bx + c$ are the *x* values where the graph *crosses the x-axis*.
- The roots of $f(x) = ax^2 + bx + c$ are solutions of the equation $\underline{ax^2 + bx + c = 0}$.
- The nature of the roots depends on the discriminant, $\underline{b^2 - 4ac}$.
- If the discriminant is 0, $f(x) = ax^2 + bx + c$ has _1_ real root(s). This root is given by $x = \frac{-b}{2a}$. This root is $\boxed{\text{rational}}$/irrational (circle the correct response).
- If the discriminant is positive, $f(x) = ax^2 + bx + c$ has _2_ real root(s).
  In addition, if the discriminant is _a perfect square_, then the roots are rational. Otherwise they are irrational.
- If the discriminant is negative, $f(x) = ax^2 + bx + c$ has _0_ real root(s).

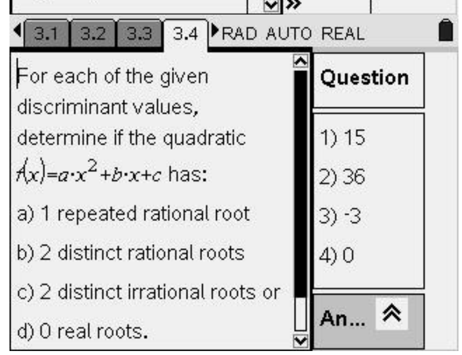
Test Your Understanding
Students answer questions from pages 3.1-3.4 on the handheld. They can check their answers.

For page 3.4, you may need to remind students that they have to press ctrl tab to navigate to the right side of the split screen to check their answers.



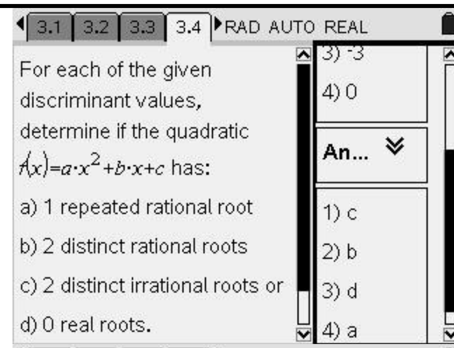


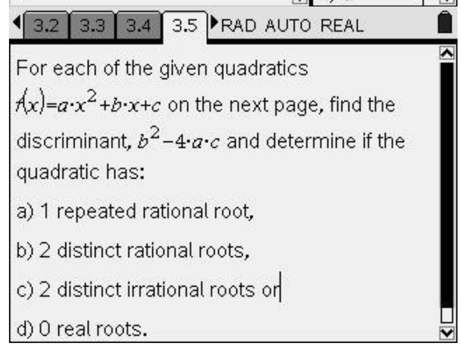




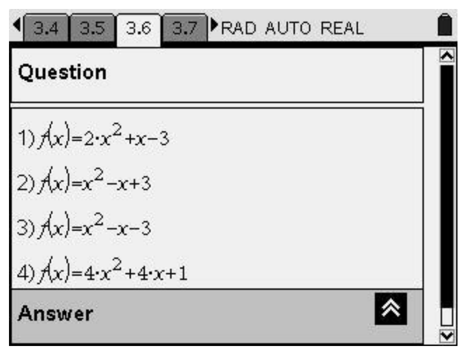
On page 3.5, they are given the directions for page 3.6. They are to find the discriminant of each of the quadratics on page 3.6 and determine the nature of their roots.

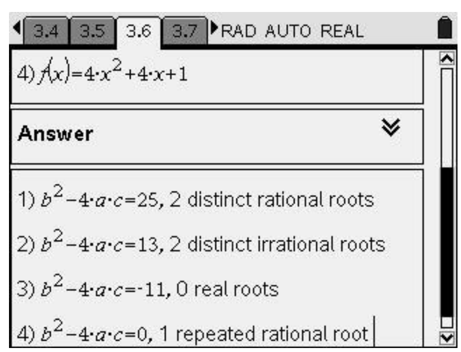
Students should record their discriminant calculations on their worksheet on page 10.





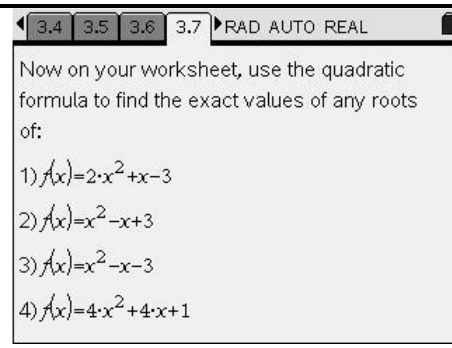
They can check their answers on page 3.6.





On page 3.7, students are asked to take those same 4 quadratics and compute their exact roots by hand using the quadratic formula. Students should record their work on page 10 of their worksheet.

Students use page 3.8 to check their solutions. This is a calculator page. They use the *solve* command to set each quadratic equal to 0 and find the roots. This command will return "*false*" if the roots are not real.



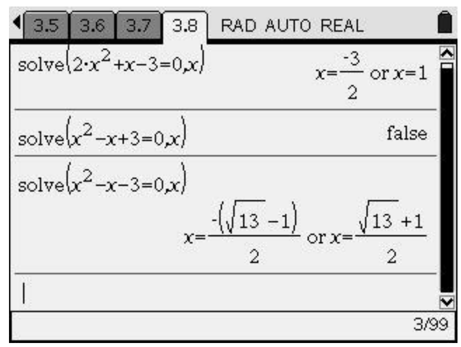
The teacher could show students the cSolve command depending on ability. That will return complex roots.

Students should check their answers. They are asked to reconcile their answers that don't seem to match.

Students should explain why their solutions to $x^2 - x - 3 = 0$ may not look exactly like the ones returned by the calculator.

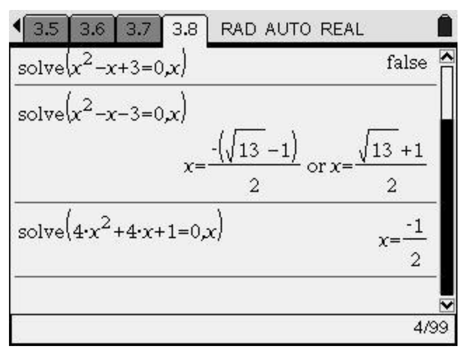
Students will get the following by hand:

$x = \frac{1+\sqrt{13}}{2}$ or $x = \frac{1-\sqrt{13}}{2}$.

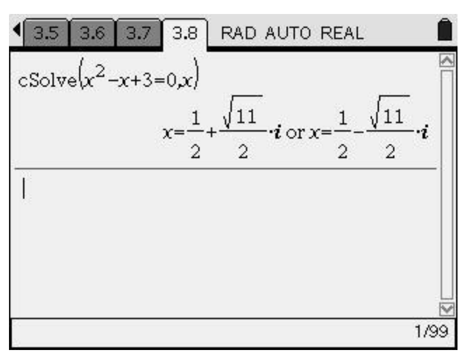


Students should explain that

$-\left(\sqrt{13} - 1\right) = -\sqrt{13} + 1 = 1 + \sqrt{13}$



to see that the calculator does return the same answers.



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