EXPLORATIONS

Activity 12

Making Sense of Shapes and Sizes

Teacher Notes

Concepts

- Connections between geometry and algebra
- Triangular number patterns
- Square number patterns

Calculator Skills

- Generalizing patterns using calculator steps
- Building a table
- Developing algorithms using the constant [2nd] [K] key

Materials

- TI-30X IIS
- Student Activity pages (p.119-122)
- Centimeter cubes or square tiles
- Grid paper

Objective

- In this activity, students will develop algorithms for generating and generalizing patterns related to triangle and square geometric models.

Topics Covered

- Representing problem situations using discrete structures
- Developing and analyzing algorithms
- Relating procedures in one representation to procedures in an equivalent representation

Introduction

Five square tiles are put together in such a way that they touch only at their corners. What configuration of the 5 tiles will produce the greatest perimeter? Suppose you were allowed to arrange the 5 tiles where only their edges touched. What is the configuration of the 5 tiles if the perimeter is to be as small as possible?

Investigation

1. Use square tiles or centimeter cubes to build the geometric shapes found in the first column of Student Activity Part 1. Have the students complete the table for Area and Perimeter. As the pattern begins to build, notice that the stacks of tiles or cubes take on a *triangular* staircase shape. The values in the AREA column represent the *triangular sums* of the triangular staircase pattern.
2. Use the results to work with the class to write an algorithm (series of steps) to use on your calculator that will generate the number patterns in the Area column.

   Answer: n^2

Press:	The calculator shows:
[2nd] [K]	K = DEG
[x^2] [ENTER]	K = ^{2} DEG K
1 [ENTER]	1^2 1. DEG K
2 [ENTER]	2^2 4. DEG K
. . .	

3. Have the students work in groups to write an algorithm for the calculator for the perimeter column of the table.

   Answer: 4n

Press:	The calculator shows:
[2nd] [K] [CLEAR] [×] 4 [ENTER]	K = * 4 DEG K
1 [ENTER]	1 * 4 4. DEG K
2 [ENTER]	2 * 4 8 DEG K
. . .	

4. Use square tiles or centimeter cubes to build the shapes shown in Student Activity Part 2. Point out that, as the pattern begins to build, the stacks of tiles or cubes take on a triangular staircase shape. Therefore, the values in the Area column represent the triangular sums.
5. Use an algebraic model to generalize the pattern for perimeter and area in the table and answer the questions in Student Activity Part 1.
6. Have the students complete the table in the Student Activity Part 2 using the same techniques used for Student Activity Part 1. Discuss the answers of the questions in Student Activity Part 2.
7. Work with the students to write algorithms for the calculator to generate the values in the columns.

Answer for Area: $\frac{n \times (n + 1)}{2}$

Press:	The calculator shows:
[2nd] [K] [CLEAR] [÷] 2 [ENTER]	K = /2 DEG K
[(] 1 [×] 2 [)] [ENTER]	(1 * 2)/2 1. DEG K
[(] 2 [×] 3 [)] [ENTER]	(2 * 3)/2 3. DEG K
. . .	

Answer for Perimeter: 4n

Press:	The calculator shows:
[2nd] [K] [CLEAR] [×] 4	K = * 4 DEG K
1 [ENTER]	1* 4 4. DEG K
. . .	

Wrap-Up

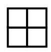
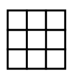
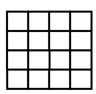
- Have the students work in groups, and extend the investigations to other figures in the plane, such as pentagons and hexagons.
- It is important to use the calculator in this activity to see how to develop an algorithm and to find the *n*th term in a sequence. It is also important to developing underpinnings for calculus and for computer programming.
- Revisit the Introduction and have students use 5 tiles to demonstrate their answers to the questions to the whole group.

Extensions

- Find the perimeter of all rectangles with whole number dimensions whose area is 72 square units. Make a table and use it to determine which rectangle has the least perimeter. Which rectangle has the greatest perimeter?
- If we allow the rectangles to be fractions, which rectangle has the least perimeter? Which rectangle has the greatest perimeter?

Solutions Part 1

1.

Geometric Shape	Shape Number	Area	Perimeter
	1	1	4
	2	4	8
	3	*9*	*12*
	4	*16*	*16*
	5	*25*	*20*
	6	*36*	*24*
	7	*49*	*28*
	…		
	n	n^2	$4n$

2. Describe the overall shape of each model that you constructed at each step.

   A Square

3. What is the area of the 13th shape in the pattern? What is the perimeter?

   The area is 169 square units, and the perimeter is 52 units.

4. Is it possible to build a model that fits the pattern using exactly 500 tiles or blocks? *No.*

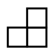
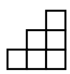
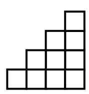
   Explain. *You could only use 484 square tiles:* $22^2 = 484$ *tiles and* $23^2 = 529$ *tiles*

5. All squares are *similar*. The number by which the length of the side of the first square is multiplied to get the length of the side of the second square is called the *scale factor*. Describe the growth of the *length*, the *perimeter*, and the *area* in terms of the scale factor.

   The length is equal to the scale factor, the perimeter is 4 times the scale factor, and the area is the scale factor squared.

Solutions Part 2

1.

Geometric Shape	Shape Number	Area	Perimeter
	1	1	4
	2	3	8
	3	*6*	*12*
	4	*10*	*16*
	5	*15*	*20*
	6	*21*	*24*
	7	*28*	*28*
	…		
	n	$\frac{n(n+1)}{2}$	$4n$

2. What would be the total number of square tiles or cubes needed to build a staircase with 10 blocks on the bottom row?

   55 tiles

3. How many blocks are on the bottom row of a staircase of made up of 276 cubes?

   $\frac{n \times (n + 1)}{2} = 276$; *inches = 23 tiles*

4. What familiar shape could you get if you "fit" a given staircase to the one immediately following it in the sequences of staircases?

   A square

5. What familiar shape can you get by "fitting" two identical staircases together?

   A rectangle

   What are its dimensions?

   n and n +1

   How is this related to the expression for the area of shape *n* in the table?

   The product of these dimensions is exactly twice that for the area of the shape n in the table.

Student Activity 12

Name ______________________________

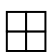
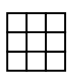
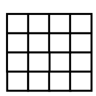
Date ______________________________

Patterns with Squares: Area and Perimeter—Making Sense of Shapes and Sizes

Objective: *In this activity, you will develop algorithms for generating and generalizing number patterns related other objective square geometric models.*

Part 1: Square Number Patterns

1.

Geometric Shape	Shape Number	Area	Perimeter
	1	1	4
	2	4	8
	3		
	4		
	5		
	6		
	7		
	. . .		
	n		

2. Describe the overall shape of each model that you constructed at each step.

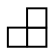
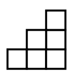
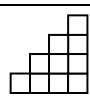
3. What is the area of the 13th shape in the pattern? What is the perimeter?

4. Is it possible to build a model that fits the pattern using exactly 500 tiles or blocks? Explain.

5. All squares are *similar*. The number by which the length of the side of the first square is multiplied to get the length of the side of the second square is called the *scale factor*. Describe the growth of the *length*, the *perimeter*, and the *area* in terms of the scale factor.

Part 2: Triangular Number Patterns

1.

Geometric Shape	Shape Number	Area	Perimeter
	1	1	4
	2	3	8
	3		
	4		
	5		
	6		
	7		
	. . .		
	n		

2. What would be the total number of square tiles or cubes needed to build a staircase with 10 blocks on the bottom row?

3. How many blocks are on the bottom row of a staircase made up of 276 cubes?

4. What familiar shape could you get if you "fit" a given staircase to the one immediately following it in the sequences of staircases?

5. What familiar shape can you get by "fitting" two identical staircases together? What are its dimensions? How is this related to the expression for the area of shape *n* in the table?