Linear Alchemy

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

7 8 **9** 10 11 12

TI-Nspire™ | Investigation | Student | 50 min

Teacher Notes:

This activity starts with the established premise that a parabola is of the form: $y = x^2$ which could be written as $y = x \times x$. We know $y = x$ is a linear function, so it follows that the product of two primitive linear functions can produce a parabola. But what happens if these linear functions are translated? Students explore products of linear factors of the form: $y = (x+a)\times(x+b)$. Does this produce a parabola? Can every parabola be written in this form? What if the two linear functions are of the form: $y = mx + c$?

In the "Code by Numbers" activity series, students see that expressing numbers in factorised form can reveal an enormous amount of information, how might the factorised form of a parabola be useful?

Australian Curriculum Standards

AC9M7SP03

Describe transformations of a set of points using coordinates in the Cartesian plane, translations and reflections on an axis, and rotations about a given point

AC9M9A06

Investigate transformations of the parabola $y = x^2$ in the Cartesian plane using digital tools to determine the relationship between graphical and algebraic representations of quadratic functions, including the completed square form, for example: $y = x^2 \rightarrow \ y = \frac{1}{3}x^2$ (vertical compression) …

AC9M10A01

Expand, factorise and simplify expressions and solve equations algebraically, applying exponent laws involving products, quotients and powers of variables, and the distributive property

AC9M10A05

Experiment with functions and relations using digital tools, making and testing conjectures and generalising emerging patterns

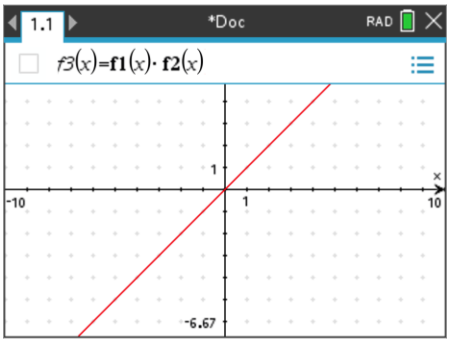
Lesson Notes

This activity is intended to create a greater understanding pertaining to quadratic functions being represented as a "product of its linear factors" including connections with the x-axis intercepts. It is not intended to supplant digital or by-hand factorisation techniques. Students should continue to expand and factorise quadratic equations by hand, but hopefully with a greater understanding of what it means.

The second part of the activity highlights that not all quadratic functions can be expressed as a product of it's linear factors. What does this look like graphically and algebraically, including concepts around completing the square, but makes no attempt to introduce the quadratic formula.

Author: P. Fox

Calculator Instructions: Linear Factors



Create a new TI-Nspire file and insert a Graphs application.

Navigate to the equation entry line and enter the following functions:

$$f_1(x) = x$$

$$f_2(x) = x$$

$$f_3(x) = f_1(x)\,f_2(x)$$

Note: The graph of $f_3(x)$ is not shown opposite.

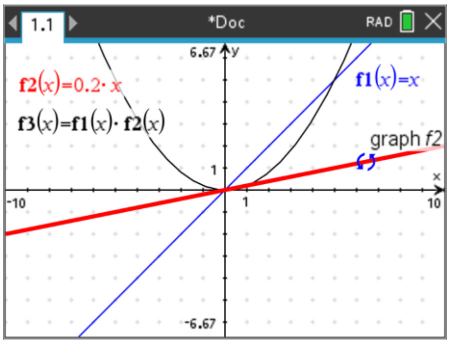
Graph 1 and 2 are directly on top of one another which is why you will only see one graph, rest assured, the second graph is there.

Question: 1.

Explain the shape of Graph $f_3(x)$.

Answer: The third graph is a parabola: $f_3(x) = f_1(x)\cdot f_2(x) = x\cdot x = x^2$.

Place the mouse over graph $f_2(x)$ and rotate the graph around the origin.



When multiple objects are in one area, a tool tip will be displayed. The 'tip' should state "graph f2". You can use the TAB key to toggle between the different layers.

Question: 2.

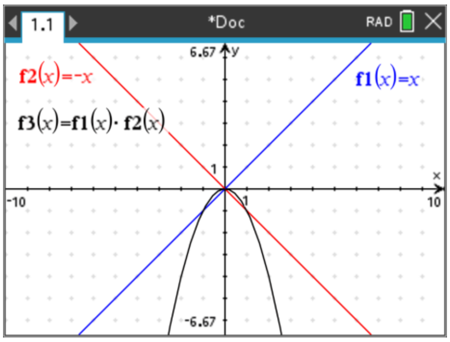
In the screen show opposite:

$f_1(x) = x$

$f_2(x) = -x$

What is the equation for $f_3(x)$?

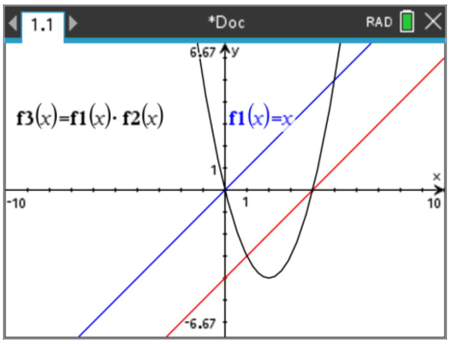
Answer: The third graph has equation: $f_3(x) = -x^2$.



Place the mouse over $f_2(x)$ and rotate the graph to its original position.

The graph of $f_2(x)$ needs to be translated as shown opposite, the x and y axes intercepts are both integer values.

Translations can be performed by grabbing a point close to the centre of the graph. The mouse will appear as a double arrow: ✥ alternatively, double click on the relevant graph and type the equation directly.



Author: P. Fox

You can use ctrl + Z to "undo" previous transformations and ctrl + Y to "re-do" them.

Question: 3.

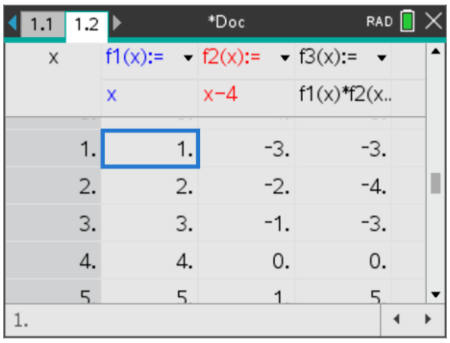
In the calculator screen shot above, given that: $f_1(x) = x$ what are the equations for $f_2(x)$ and $f_3(x)$?

Answer: $f_2(x) = x - 4$ and $f_3(x) = x(x-4)$

Press ctrl + T to generate a table of values for all three graphs.

You can also press ctrl + 6 to ungroup the Graph application from the table, this will shift the table to the next page.

Use the arrow keys to navigate up / down / left / right through the table.



x	f1(x):=	f2(x):=	f3(x):=
	x	x−4	f1(x)*f2(x..
1.	1.	-3.	-3.
2.	2.	-2.	-4.
3.	3.	-1.	-3.
4.	4.	0.	0.
5	5	1	5

Question: 4.

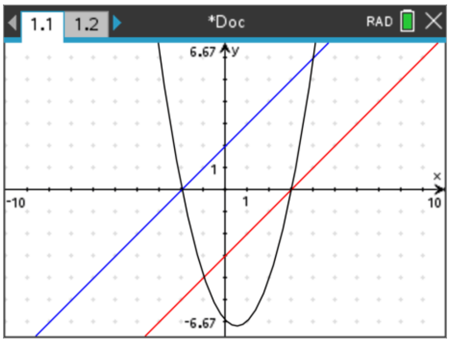
With reference to the table, what do you notice about the x axis intercepts for $f_1(x)$, $f_2(x)$ and $f_3(x)$?

Answer: $f_1(x)$ and $f_3(x)$ share their x axis intercept, similarly $f_2(x)$ and $f_3(x)$ share their x axis intercept. From the table of values, when $f_1(0) = 0$, then $f_3(x) = 0$ since $f_3(x) = f_1(x) \cdot f_2(x)$ similarly when $f_2(x) = 0$ then $f_3(x) = 0$ [Null Factor Law]

Question: 5.

In the graph shown opposite, given $f_2(x) = x - 3$, determine possible equations for $f_1(x)$ and $f_3(x)$ given they have integer axes intercepts.

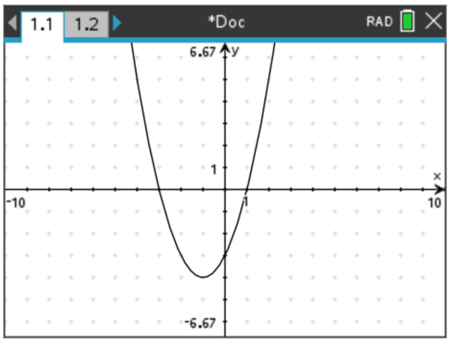
Answer: $f_1(x) = x + 2$ and $f_3(x) = (x+2)(x-3)$



Question: 6.

In the graph shown opposite $f_1(x)$ and $f_2(x)$ have been hidden, suggest a possible function for each given $f_3(x)$.

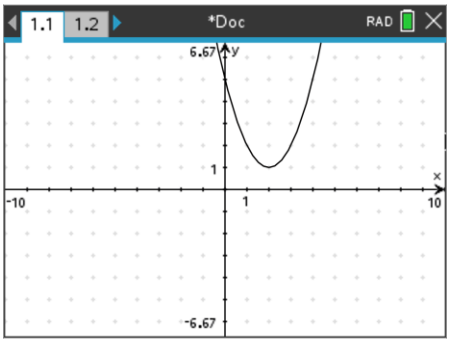
Answer: $f_3(x) = (x+3)(x-1)$



Question: 7.

Explain why $f_3(x)$ cannot be formed by two linear factors.

Answer: The graph of $f_3(x)$ does not have any x axis intercepts. A straight line that is not 'horizontal' passes through the x axis at some point, therefore the parabola would also need to pass through the axis at the same point. The graph shown has no linear factors!



Think about numbers that have factors (composite) and numbers that don't (prime). Just like prime numbers, we cannot express the graph in Question 7 as the product of two linear factors. A prime number (p) however could be expressed as a composite number (n) squared plus some constant (c).

Example: $29 = 5^2 + 4$.

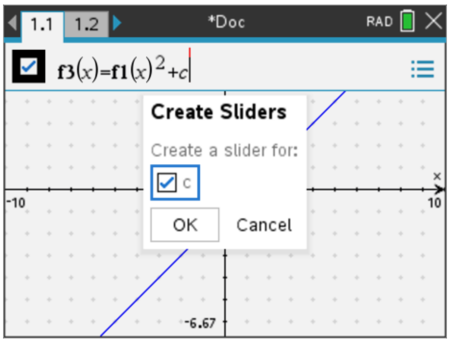
We can do the same for the graph in Question 7.

Re-write $f_3(x)$ as shown opposite:

$$f_3(x) = f_1(x)^2 + c$$

As 'c' is not defined you will be prompted to add a slider.

In this section we are not using $f_2(x)$ so it can be hidden or deleted.



The default version of the slider can be changed to a 'toggle' by minimising. Move the mouse over the top of the slider and press:

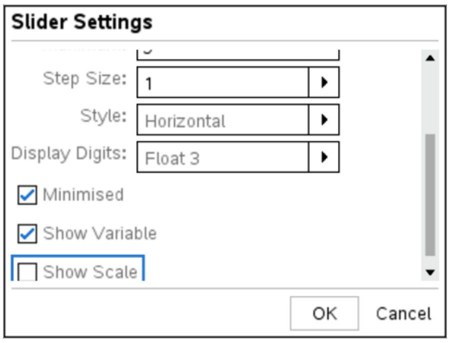
ctrl + **menu**

Change the following:

Minimised = checked

Step size = 1

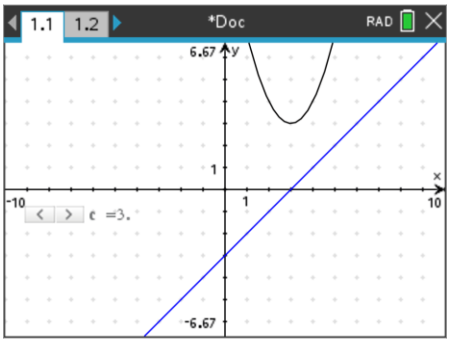
Show Scale = unchecked



Experiment with the following:

- Rotate the linear function
- Translate the linear function
- Increase and decrease the slider value (c)

For each 'experiment', observe closely what is happening with the parabola.



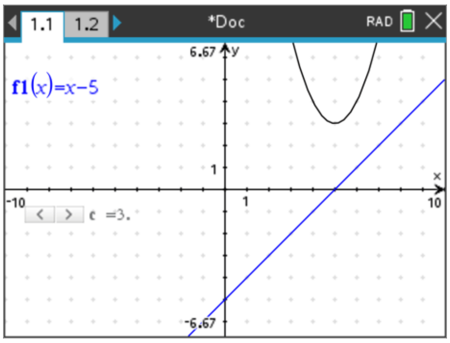
Author: P. Fox

Question: 8.

The linear function: $f_1(x) = x - 5$ and the constant (slider) are used to generate the graph of $f_3(x)$ such that: $f_3(x) = f_1(x)^2 + c$.

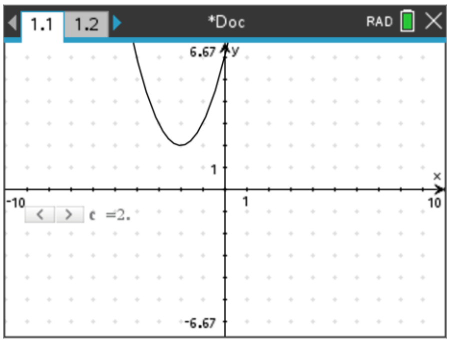
Determine the equation for $f_3(x)$.

Answer: $f_3(x) = (x-5)^2 + 3$



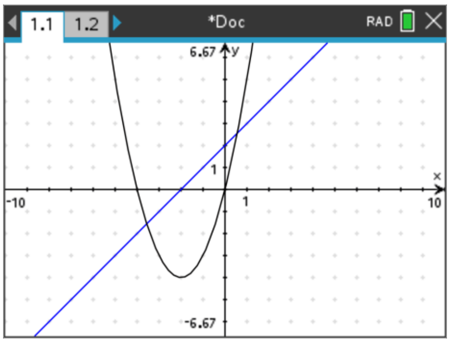
Question: 9.

In the screen shot shown opposite, the graph of $f_1(x)$ has been hidden. Suggest a possible equation for $f_3(x)$



Question: 10.

In the screen shot shown opposite, the slider has been hidden. Suggest two possible equations for $f_3(x)$



Author: P. Fox