

Math Objectives

- Students will recognize slope as a constant rate of change.
- Students will interpret units of slope in the context of a realworld problem.
- Students will graph a line determined by a point and a slope.
- Students will use appropriate tools strategically. (CCSS Mathematical Practic)

Vocabulary

- ordered pairs
- rate of change
- slope

About the Lesson

- This lesson involves the concept of interpreting slope as a rate of change.
- As a result, students will:
 - Use a rate of change to plot ordered pairs and observe a linear pattern in the data.
 - Make connections between slope and the rate of downloading songs.
 - Predict outcomes based on the rate of change.

TI-Nspire™ Navigator™ System

- Use Screen Capture to examine patterns that emerge.
- Use Quick Polls to check student understanding.
- Use Teacher Edition computer software to review student documents.

1.1 1.2 *Rate_of_Change ⊂ Image: Change Rate of Change Image: Change Image: Change Image: Change On the next page you will move a point to collect data as you explore the rate at which songs are downloaded from a website. Image: Change Image: Change

TI-Nspire[™] Technology Skills:

- Open a document
- Move from one page to another
- Grab and drag a point
- Manually capture data

Tech Tips:

- Download the TI-Nspire TNS document to your computer and to your TI-Nspire handhelds.
- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- If the function entry line appears in Graphs you can hide it again by pressing ctrl G.

Lesson Files:

Student Activity

- Rate_of_Change_Student.pdf
- Rate_of_Change_Student.doc

TI-Nspire document

• Rate_of_Change_Student.tns

Visit <u>www.mathnspired.com</u> for lesson updates and tech tip videos.



Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (\mathfrak{D}) getting ready to grab the point. Also, be sure that the word "point" appears. Then press $\mathfrak{ctrl} \cong$ to grab the point and close the hand (\mathfrak{D}) . When finished moving the point, press \mathfrak{esc} to release the point.

When capturing points using ctrl , if students make a mistake, have them simply close and reopen the document without saving in order to go back to the initial state.

Move to page 1.2.

Problem Situation: After searching for songs online, you finally find the site you want and download your first song. You look at your watch and realize it has taken you 10 minutes to find and download only 1 song. Now that you have found the right site, you realize that on average you can download 3 songs every 2 minutes.

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 What is the meaning of one unit along the horizontal axis? Vertical axis?

Sample Answers: One unit on the horizontal axis represents one minute. One unit on the vertical axis represents one download.

2. Move *P* to (10, 1), and press ctrr . once. How does this point relate to the context of this problem?

Sample Answers: The point (10, 1) is the initial condition. It represents the 10 minutes you have already spent searching for and downloading 1 song.

Teacher Tip: Discuss the importance of units. In general, x and y are arbitrary and abstract variables that can represent anything. Their units remain nameless until we assign something to go along with the numbers. The number 10 is very different than saying 10 minutes.



Likewise, in general, (10, 1) can represent anything, but we have chosen to make it represent 10 minutes and 1 song. Subsequently, every ordered pair will have the same units. The units for slope correspond to the rate of songs per minute.

- 3. Remember that the average rate at which you can download songs is 3 per every 2 minutes.
 - a. How many total songs will you have 2 minutes later (after the first 10 minutes)? Explain.

Answer: You have 1 song from the original 10 minutes and 3 more songs after the 2 additional minutes, so you will have 4 songs after 12 minutes.

b. What ordered pair would you use to represent this point?

Answer: The point (12, 4) should be plotted.

c. Move *P* to these coordinates, and press ctrl . once to mark the point.

Teacher Tip: Some students might say they moved up 3 and right 2, or they might say they moved right 2 first then up 3. It is important for them to understand that both ways are correct, but they must be careful to ensure that they do not reverse the 2 and the 3. Suggestion: since slope is defined as change in y divided by change in x, encourage students to move in the vertical direction first, i.e., up 3 then right 2.

4. Use the same average rate to plot the next three points. Describe the pattern you followed to plot these points. Record the ordered pairs below

Answer: Points should be plotted at (14, 7), (16, 10), and (18, 13).

Teacher Tip: Give students the opportunity to explain their strategies. Students might say they moved up 3 for the number of songs and right 2 for the time to find those three songs, or they might say they moved right for the time to find the 2 songs first then up 3 for the number of songs they downloaded in those two minutes. Be sure that students use the words that relate to the context in describing the rate of change. Rate of Change MATH NSPIRED

5. Is it possible to have the point (21, 16) on the graph if you continue with this pattern? Why or why not?

Sample Answers: Responses will vary. One example might be: The points have to be in a straight line where, as the first coordinate of any point changes by 2 for the number of songs, the second coordinate has to change by 3, the time to download those songs. So the next point would be (20,16) and then (22,19). In 21 minutes, you would have downloaded 16 songs and be half way to downloading the next 3 or have about 1.5 songs downloaded, so the point would be (21,17.5) to be on the line, not (21,16).



TI-Nspire Navigator Opportunity: *Screen Capture* See Note 1 at the end of this lesson.

- 6. A rate of change is a ratio that specifies how much one quantity changes as another quantity changes.
 - a. Determine your rate of change for downloading songs after the site was found.

Answer: The rate of change is 3 songs per 2 minutes. Some students might say 2 minutes per 3 songs, which is also correct. However, in the context of this problem, time is the independent variable, so expressing the rate as 3 songs per 2 minutes corresponds to the slope of the graph.

Teacher Tip: Be sure that students understand that it takes two numbers—the ratio—to completely describe the rate of change.

b. Look at your graph. How is the slope of the line through the points you plotted related to your answer to part *a*?

Answer: The slope would be 3/2 and the slope and rate of 3 songs per 2 minutes is the same as the slope.

- 7. What is the total number of songs you have downloaded in each case below? Explain how you found your answer.
 - a. 24 minutes after you started searching.

<u>Answer:</u> 22 songs. Some students might continue using the rate of change until the number of minutes is 24 and others might make a table and write out the possibilities. Some students might figure 1 song in the first 10 minutes, then use a proportion $(\frac{m}{14} = \frac{3}{2})$ to find the number of songs in the next 14 minutes. Students might use $\frac{3}{2}$ as a constant of variation ($s = \frac{3}{2}m$) where *x* is 14.

b. An hour after you started searching.

<u>Answer:</u> 76 songs. Again some students might write out all of the possibilities, but others will look for more efficient strategies. Some students might figure 1 song in the first 10 minutes, then solve using a proportion $\left(\frac{m}{50} = \frac{3}{2}\right)$. Students might use $\frac{3}{2}$ as a constant of variation ($s = \frac{3}{2}m$).

Teacher Tip: Allow students to share their strategies with the class.

- 8. Rates are often measured *per single unit*. For example, speed is usually described as 45 miles per hour rather than 90 miles per 2 hours.
 - a. Express the download rate as the number of songs per minute.

Answer: 1.5 songs per minute.

Teacher Tip: Give students the opportunity to share their strategies for determining their rates.

b. How long did it take you to find the site? What point on the graph would represent this time? Explain your reasoning.

Answer: Because the rate per minute is 1.5 songs, and you found one song in the first 10 minutes, it took you 2/3 minutes to download the first song so the search for the site took 10-2/3 or 9 1/3 minutes. The point would be (9 1/3,0), 9 1/3 minutes and 0 songs.

9. Suppose you were able to double the number of songs you could download in two minutes after you found the site.



a. What would be the new rate of change for downloading songs?

Answer: 3 songs per minute.

b. How would the new graph compare to the original?

<u>Answer:</u> The new graph would be steeper than the original graph because you could increase the number of songs (the vertical change in the slope ratio) for the same time (horizontal change in the slope ratio).

TI-Nspire Navigator Opportunity: *Quick Polls* See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should insure that students are able to:

- Use a known rate of change to generate data points.
- Express slope as a rate of change.
- Interpret units of slope in the context of a real-world problem.

Assessment

The mix of closed and open-ended questions in this lesson provides a suitable basis for both formal and informal assessment.

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Note 1

Questions 1 – 5, *Screen Capture*: Take screen captures to see how the students are progressing. Especially take a screen capture once most or all students have completed plotting all the points in question 5.

Note 2

Questions 6 – 9, Quick Poll (Open Response):

<u>For question 6</u>, ask students to answer question 6, to type in the rate of change but using units (words) to describe the rate of change. Look at the Poll Summary to



see how students responded. Discuss different correct answers and various incorrect answers (and why they are correct/incorrect).

<u>For question 7</u>, ask students to answer question 7, to type the number only. Look at the Poll Summary to see how students responded.

<u>For question 8</u>, ask students to answer question 8, to type in the rate of change but using units (words) to describe the rate of change. Look at the Poll Summary to see how students responded. Discuss different correct answers and various incorrect answers (and why they are correct/incorrect).

<u>For question 9</u>, ask students to answer question 9, to type in the rate of change but using units (words) to describe the rate of change. Look at the Poll Summary to see how students responded. Discuss different correct answers and various incorrect answers (and why they are correct/incorrect).



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