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Transformational Geometry is a way to study geometry by focusing on geometric "movements" or "transformations" and observing/studying properties about these figures.

There are four geometric transformations:
<Reflections <Translations < Rotations < Dilations

## Play - Investigate - Explore - Discover PIED



In the figure to the right, $\triangle A B C$ is rotated about point $P$.
$\triangle A B C$ is called the pre-image while $\Delta A^{\prime} B^{\prime} C^{\prime}$ is called the image (of rotation).
$\Delta A^{\prime} B^{\prime} C^{\prime}$ is read "triangle A prime, B prime, C prime."
Point $P$ is called the point of rotation.


Download and install the red TI-Nspire student software and the Rotations


TNS file from the website where you obtained this document.
Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the videos.

A conjecture is an opinion or conclusion based on what is observed.

1. What conjecture(s) can you make based upon what you observed about a triangle and its image after being rotated?

2 a . When a figure is rotated about a point through an angle of positive measure, the figure rotates in a
$\qquad$ direction.
b. When a figure is rotated about a point through an angle of negative measure, the figure rotates in a
$\qquad$ direction.
3. a) After observing the angle measures being shown, what conjecture can you make?
b) After observing the side measures being shown, what conjecture can you make?
c) Note: do not say all the angles are equal, or do not say all the sides are equal. They aren't. The sides and angles that correspond to one another have equal lengths and measures, respectively.
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d) What is true about the triangles? State that using symbols.
4. a) After observing the perimeters being shown, what conjecture can you make? Why should this be true?
b) After observing the areas being shown, what conjecture can you make? Why should this be true?
5. After observing how triangles have been rotated about a point, you should be able to rotate a triangle about a point through an angle of $90^{\circ}$, or a multiple of $90^{\circ}$. Ideally you will need a compass and straightedge.
a. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 90^{\circ}$.

Label this image $\Delta A^{\prime} B^{\prime} C^{\prime}$.
b. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 180^{\circ}$.

Label this image $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
c. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 270^{\circ}$. Label this image $\Delta A^{\prime \prime \prime} B^{\prime \prime} C^{\prime \prime}$.

$\stackrel{\bullet}{P}$
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6. If you do not have access to a compass, do problem 5 above using the figure below.

You will need a straightedge (or ruler).
Concentric circles are circles in the same plane, with the same center, but different length radii.
a. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 90^{\circ}$. Label this image $\Delta A^{\prime} B^{\prime} C^{\prime}$.
b. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 180^{\circ}$. Label this image $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
c. Rotate $\triangle \mathrm{ABC}$ about point $\mathrm{P} 270^{\circ}$. Label this image $\Delta A^{\prime \prime \prime} B^{\prime \prime} C^{\prime \prime \prime}$.

7. Based on your explorations and observations:
a. What appears to be true about segments

PA and PA'? $\qquad$ PB and PB '? $\qquad$ PC and PC' $\qquad$

Why should these 3 statements be true?
(hint: look at the results to either \#5 or \#6 above)
8. Grids and Coordinates Rotate $90^{\circ}$ Complete the following.
a) If a triangle is rotated $90^{\circ}$ about the origin, the $x$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
b) If a triangle is rotated $90^{\circ}$ about the origin, the $y$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
c) Or in symbols, If $(x, y)$ is a point on the pre-image, then $\qquad$ is a point on the image triangle.
$(x, y) \rightarrow \quad$ where ' $\rightarrow$ ' means 'maps to'
9. Grids and Coordinates Rotate $180^{\circ}$ Complete the following.
a) If a triangle is rotated $180^{\circ}$ about the origin, the $x$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
b) If a triangle is rotated $180^{\circ}$ about the origin, the $y$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
c) Or in symbols, If $(x, y)$ is a point on the pre-image, then $\qquad$ is a point on the image triangle.
$(x, y) \rightarrow \ldots \quad$ where ${ }^{\prime} \rightarrow$ ' means 'maps to'
10. Grids and Coordinates Rotate $270^{\circ}$ Complete the following.
a) If a triangle is rotated $270^{\circ}$ about the origin, the $x$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
b) If a triangle is rotated $270^{\circ}$ about the origin, the $y$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
c) Or in symbols, If $(x, y)$ is a point on the pre-image, then $\qquad$ is a point on the image triangle.

$$
(x, y) \rightarrow \quad \text { where ' } \rightarrow \text { ' means 'maps to' }
$$

11. Grids and Coordinates Rotate $360^{\circ}$ Complete the following.
a) If a triangle is rotated $360^{\circ}$ about the origin, the $x$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
b) If a triangle is rotated $360^{\circ}$ about the origin, the $y$-coordinate of the image is the
$\qquad$ of the pre-image triangle.
c) Or in symbols, If $(x, y)$ is a point on the pre-image, then $\qquad$ is a point on the image triangle.

$$
(x, y) \rightarrow \quad \text { where ' } \rightarrow \text { ' means 'maps to' }
$$

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12. A rotation of $-90^{\circ}$ is equivalent to a rotation of what positive angle measure? $\qquad$
13. A rotation of $-180^{\circ}$ is equivalent to a rotation of what positive angle measure? $\qquad$
14. A rotation of $-270^{\circ}$ is equivalent to a rotation of what positive angle measure? $\qquad$

## Corresponding Sides

We have already shown that corresponding sides of triangle rotated about the origin are equal in length. What else seems to be true about pairs of corresponding sides? Let's explore.
15. Calculate the slopes of corresponding sides either graphically or by slope formula.

Show your work in the space provided below.
Write all answers as fractions in simplest form.

Record your answers in the \#15 row of the table on the bottom of this page. Look for patterns!
$\triangle A B C$ is rotated $90^{\circ}$ about the origin.
15. Find the slopes as fractions of:

a. $\overline{A B}$ and $\overline{A^{\prime} B^{\prime}}$
b. $\overline{B C}$ and $\overline{B^{\prime} C^{\prime}}$
c. $\overline{A C}$ and $\overline{A^{\prime} C^{\prime}}$

| Rotate 90 | $m(\overline{A B})$ | $m\left(\overline{A^{\prime} B^{\prime}}\right)$ | $m(\overline{B C})$ | $m\left(\overline{B^{\prime} C^{\prime}}\right)$ | $m(\overline{A C})$ | $m\left(\overline{A^{\prime} C^{\prime}}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| \#15 |  |  |  |  |  |  |
| \#16 |  |  |  |  |  |  |
| \#17 video |  |  |  |  |  |  |

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16. Calculate the slopes of corresponding sides either graphically or by slope formula.

Show your work in the space provided below.

Write all answers as fractions in simplest form.

Record your answers in the \#16 row of the table on the bottom of the previous page. Look for patterns!
$\triangle A B C$ is rotated $90^{\circ}$ about the origin.
16. Find the slopes as fractions of:
a. $\overline{A B}$ and $\overline{A^{\prime} B^{\prime}}$
b. $\overline{B C}$ and $\overline{B^{\prime} C^{\prime}}$
c. $\overline{A C}$ and $\overline{A^{\prime} C^{\prime}}$
17. There is an example on the video that we want you to record the slopes of sides into the table on the bottom of the previous page. Pause the video and record the slopes as fractions in simplest form.
18. After completing exercises $15-17$ and recording the slopes in the table, then do this exercise.
a. Look at the slopes of corresponding sides in the table on the previous page.

What pattern do you notice?
b. What does that mean is true about lines: $\overleftrightarrow{A B}$ and $\overleftrightarrow{A^{\prime} B^{\prime}} ? ~ \overleftrightarrow{B C}$ and $\overleftrightarrow{B^{\prime} C^{\prime}}$ ? $\overleftrightarrow{A C}$ and $\overleftrightarrow{A^{\prime} C^{\prime}}$ ?

Transformational Geometry Rotations

