What’s So Special about 11?

Overview

Students will compute multiples of numbers in search of patterns. As a class, they’ll discover patterns in multiples of 9; then they’ll do the same with patterns in multiples of 11. They will then practice writing the rule for 11, both verbally and algebraically, to summarize the discovered pattern.

Math Concepts

- patterns
- problem solving

Materials

- TI-30XS MultiView™
- calculator
- pencil
- paper

Activity

Begin with a discussion about patterns in numbers.

Much of what happens in math is based on patterns. Often, we are told the patterns. It is more interesting and more beneficial, though, to discover those patterns ourselves.

Begin by talking about multiples of 9.

Here’s a pattern that might be familiar to you. Do you know any procedures or patterns to help you remember multiples of 9?

There are certain answers to expect. One is “to multiply 9 by 3, hold up your hands and put the third digit from the left down. There are two fingers to the left and seven digits to the right; hence 9(3) = 27.” Another is “the answer will add to 9, so 9 • 7 = 63 because you start with the digit one less than 7, then make the sum 9.”

But what about two-digit numbers?

9 • 11	*= 99*	
9 • 12	*= 108*	
9 • 13	*= 117*	
9 • 14	*= 126*	
9 • 15	*= 135*	
9 • 57	*= 513*	

Have students give you answers using mental math or through use of patterns. The third column of the table is for recording their suggested patterns.

Do you see any pattern? Explain.

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Now, begin to explore using the calculator.

Since we’re going to check many multiples of 9, it makes sense to use the constant feature of the TI-30XS MultiView, rather than to enter each line individually.

In addition to the patterns students suggested, share this: Each answer is 10 times the factor, minus that factor.

Notice that 12(9) is the same as 12(10) – 12.

And 13(9) is the same as 13(10) – 13.

And 57(9) is the same as 57(10) – 57.

Using that pattern, can you calculate 77(9) without a calculator? How about 91(9)?

Students may still need direction, but the pattern should begin to emerge for them.

Follow these steps:

1. Press % λ to access the constant feature.
2. Delete any operation currently stored, and press ς 9 <.
3. Press % Θ.
4. Notice the small *K* at the top of the screen:

5. Now press 11 < 12 <, etc.

6. Press % λ □to quit constant mode.

I believe that 77(9) is the same as 77(10) – 77, which is 770 – 77, which is 693. Let’s check.

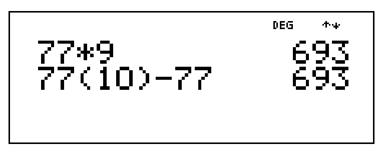
In addition to the constant feature, we can also use the cursor to copy an entry from history, paste it, and simply edit to fit our needs. This is a huge time-saver, and it allows us to use fewer keystrokes.

Point out any other patterns, such as the fact that the sum of all digits of any multiple of 9 will be a factor of 9. For instance, 77(9) = 693, and if you add 6 + 9 + 3, you get 18, which is a multiple of 9, also. This is always true.

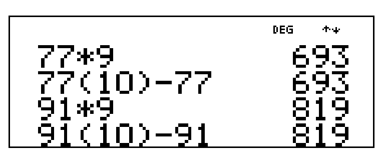
In summary, since any two-digit number multiplied by 9 is equal to 10 times that number, minus the number, we can express that algebraically as 9x = 10x – x for any two-digit x.

Follow these steps:

1. With λ set up as ς 9, press 77 <.
2. Turn off λ □by pressing % λ.
3. Press 77 Δ 10 E Y 77 <.
4. The calculator should display this:



5. Pressing # # # # < allows you to copy 77 Δ 9 E and pull it down for editing.
6. Press ∀ ∀ ∀ ∀, type 91 over the 77, then press <.
7. Press # # # # <□ again to copy 77 Δ 10 E Y 77 and pull it down.
8. Type 91 over the 77 both times, and press <.
9. The calculator should display this:



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Name ______________________

Date ______________________

1. Compute by hand:

11(1) = _____	11(5) = _____	11(9) =______
11(2) = _____	11(6) = _____	11(10) =_____
11(3) = _____	11(7) =______	11(11) =_____
11(4) = _____	11(8) =______	11(12) =_____

2. Summarize any pattern you see when multiplying 11 by a single-digit number.
____
____
____

3. Express that pattern as an algebraic equation. ___________________

4. Use your TI-30XS MultiView™ to multiply 11 by several two-digit numbers; then record a possible pattern. Record your results in the chart below. The first one has been done for you as an example.

11 • 11	= 121	10 • 11 + 11
11 • 12	=	
11 • 13	=	
11 • 14	=	
11 • 15	=	

5. Can you find 11 • 42 from your pattern? _______ What about 11•72? ______

6. Summarize any pattern you see when multiplying 11 by a two-digit number.
____
____
____

7. Express that pattern as an algebraic equation. For any two-digit number *x*, 11 • *x* = ______________.

8. Based on your answers above, what do you think happens when you multiply a three-digit number by 111? Explain. ____
____

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Answer Key

1. Compute by hand:

11(1) = 11	11(5) = 55	11(9) = 99
11(2) = 22	11(6) = 66	11(10) = 110
11(3) = 33	11(7) = 77	11(11) = 121
11(4) = 44	11(8) = 88	11(12) = 132

2. Summarize any pattern you see when multiplying 11 by a single-digit number.

   Each number you multiply by 11 is 10 times that number, plus the number again. For instance, 11(8) = 10(8) + 8 = 80 + 8 = 88.

3. Express that pattern as an algebraic equation. $11x = 10x + x$

4. Use your TI-30XS MultiView™ to multiply 11 by several two-digit numbers; then record a possible pattern. Record your results in the chart below. The first one has been done for you as an example.

11 • 11	= 121	10 • 11 + 11
11 • 12	= 132	10 • 12 + 12
11 • 13	= 143	10 • 13 + 13
11 • 14	= 154	10 • 14 + 14
11 • 15	= 165	10 • 15 + 15

5. Can you find 11 • 42 from your pattern? 462 What about 11•72? 792

6. Summarize any pattern you see when multiplying 11 by a two-digit number.

   The pattern is the same as for a one-digit number. So when multiplying a two-digit number by 11, it's the same as multiplying that number by 10, then adding the number. For instance, 11(42) = 10(42) + 42 = 420 + 42 = 462.

7. Express that pattern as an algebraic equation. For any two-digit number x, $11 \cdot x = 10x + x$.

8. Based on your answers above, what do you think happens when you multiply a three-digit number by 111? Explain.

   Answers will vary, but the pattern continues similar to before—111 times a two-digit number is 100 times that number + 10 times that number + the number.