# *NUMB3RS* Activity: Location! Location! Location!
# Episode: "Bones of Contention"

**Topic:** Voronoi Diagrams **Grade Level:** 8 - 12
**Objectives:** Understand the basic idea of a Voronoi diagram. Draw Voronoi diagrams in simple cases.
**Time:** About 30 minutes
**Materials:** ruler (Optional: colored pencils, compass, geometry software such as Cabri® Jr. on the TI-84 Plus)

## Introduction

When you go into the takeout section of your favorite pizza store, you might see a map on the wall that highlights the part of the city where this store delivers, as well as the delivery zones for other nearby stores in this chain. A simplified version of such a map could be modeled by a **Voronoi diagram** where a set of points (sites) in the plane are associated with "service regions" or "areas of influence." Every point in a given (convex) region is closer to that site than to any other site. The boundaries between the regions are called **Voronoi edges**. In "Bones of Contention," Charlie talks about Voronoi regions associated with fast food restaurants. The goal of this activity is to investigate and construct Voronoi diagrams with 3 or 4 sites.

## Discuss with Students

Explore this idea with students using a simple drawing. Suppose you had two pizza stores which make deliveries to a certain area. How can you decide which part of the area each store will serve? For example, students may say that the homes closer to one store should get pizzas from that store. However, how do you determine the boundaries of each part of the area so that each home is served by the closer store?

The key to determining the boundaries, or edges, in a Voronoi diagram is finding the perpendicular bisector of a line segment. To find this bisector, you need to know the midpoint and the slope of the segment.

Review with students how to find the perpendicular bisector of a line segment using a, geoboard, a coordinate plane, or a geometry drawing program. For the Cabri Jr. figures to accompany this activity, visit **education.ti.com/exchange** and search for "6275."

The suggestions below can be used in class as review before completing the activities.



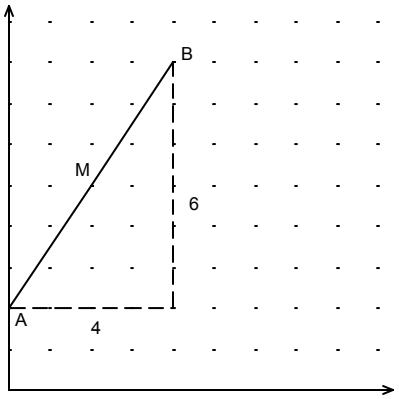
Tom Butts, UT Dallas

## "Geoboard" Approach

Midpoint: The midpoint of $\overline{AB}$ is located at half of the horizontal distance from A to B and half of the vertical distance from A to B. From A, the midpoint has a vertical distance 3 and horizontal distance 2.

Slope: The slope of $\overline{AB}$ is the vertical distance from A to B divided by the horizontal distance from A to B: $\frac{6}{4} = \frac{3}{2}$.

Slope of Perpendicular Bisector: This slope is the opposite reciprocal of the slope of $\overline{AB}$: $-\frac{2}{3}$.



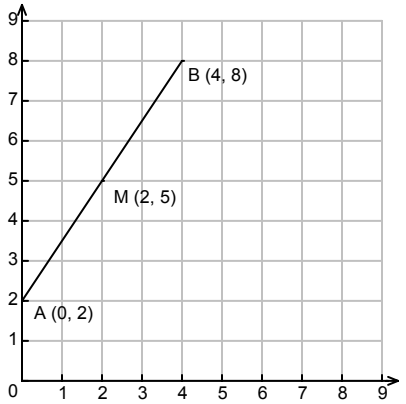

## Coordinate Approach

Coordinates: A(0,2) and B(4,8)

Midpoint: $\left(\frac{0+4}{2}, \frac{2+8}{2}\right) = (2, 5)$

Slope: $\frac{8-2}{4-0} = \frac{6}{4} = \frac{3}{2}$

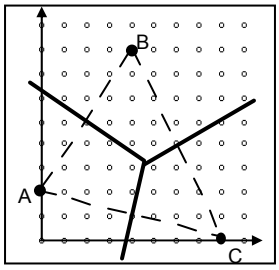
Slope of Perpendicular Bisector: $-\frac{2}{3}$



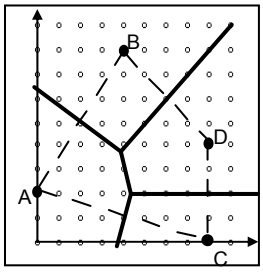

**Student page answers: 1a.** $\overline{AB}$*: slope =* $\frac{3}{2}$*, midpoint = (2, 5), slope of perpendicular bisector =* $-\frac{2}{3}$*;* $\overline{BC}$*: slope =* $-2$*, midpoint = (6, 4), slope of perpendicular bisector =* $\frac{1}{2}$*;* $\overline{AC}$*: slope =* $-\frac{1}{4}$*, midpoint = (4, 1), slope of perpendicular bisector = 4* **1b and 1c.** *See diagram below* **2.** *Store B; the house at (5, 5) is in the same region (zone) as Store B, so Store B is closest to this house.*

**3.** *Either Store A or Store B; because this line is the perpendicular bisector of* $\overline{AB}$*, a point on the line will be the same distance from Store A and Store B* **4.** *See diagram below* **5a.** *The 'best' location is the Voronoi vertex – located near (4.6,3.3)* **5b.** *See diagram below* **6.** *This will happen only when the four points lie on a circle.* **7.** *Sample answer: Driving times are assumed to be uniform and independent of traffic.*

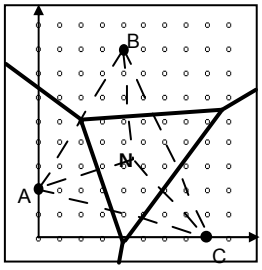
**1c.**




**4.**




**5.**

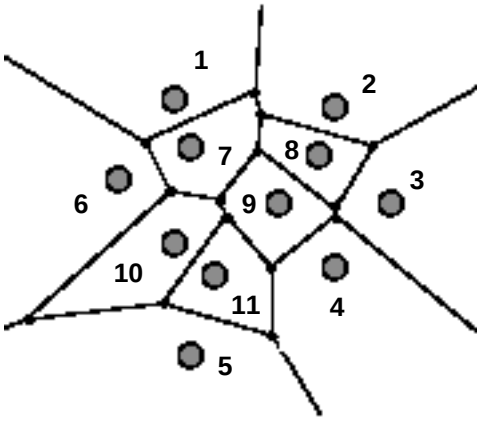



*Note: the dashed lines are not part of the Voronoi diagram.*

Name: _____ Date: _____

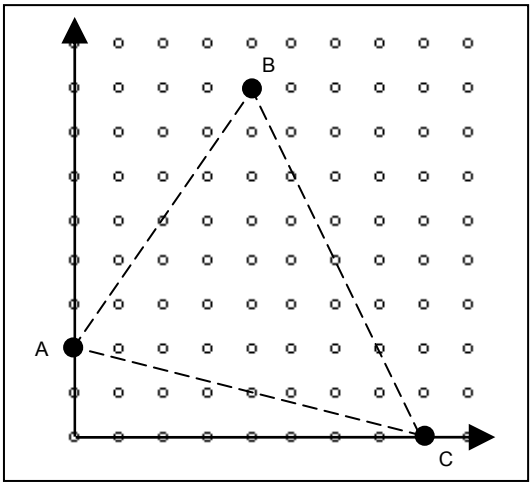
# *NUMB3RS* Activity: Location! Location! Location!

When you go into the takeout section of your favorite pizza store, you might see a map on the wall that highlights the part of the city where this store delivers, as well as the delivery zones for other nearby stores in this chain. A simplified version of such a map could be modeled by a **Voronoi diagram**. In this activity, you will learn how to draw a Voronoi diagram for 3 and 4 stores.




First, we will investigate constructing a delivery zone map for 3 stores [sites].

**1.** Pizza World owns 3 pizza stores, A, B, and C. Construct a delivery zone map for stores A, B, and C.




**a)** Complete the table below.

| | $\overline{AB}$ | $\overline{BC}$ | $\overline{AC}$ |
|---|---|---|---|
| Slope of line segment: | | | |
| Midpoint: | | | |
| Slope of Perpendicular Bisector: | | | |

**b)** Draw the perpendicular of bisector of AB. Then do the same for AC and BC.

**c)** In the final diagram, the three perpendicular bisectors will intersect in a common point in the interior of $\Delta ABC$ called a *Voronoi vertex*. Each Voronoi edge should stop at this vertex.

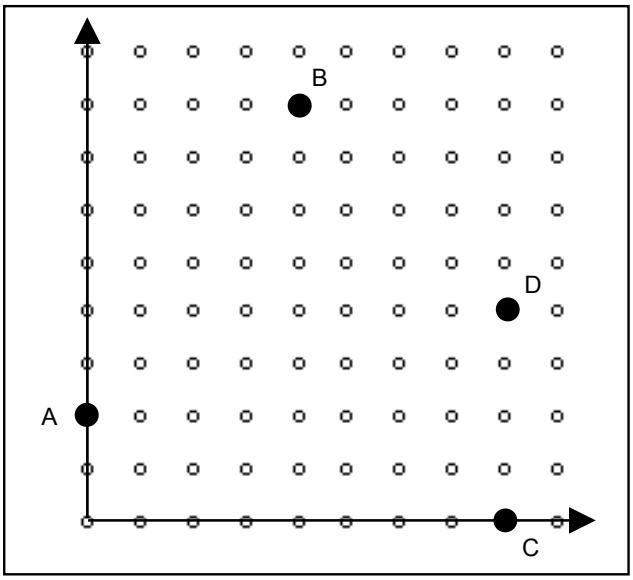
Use your Voronoi diagram from Question #1 to answer the following questions.

**2.** Which store should deliver to a house located at (5, 5)? Why?

**3.** Which store should deliver to a house on the boundary between A and B? Why?

**4.** The owner of Pizza World decided to open a fourth suburban store at D. Draw the Voronoi diagram or delivery zone map for these four stores.

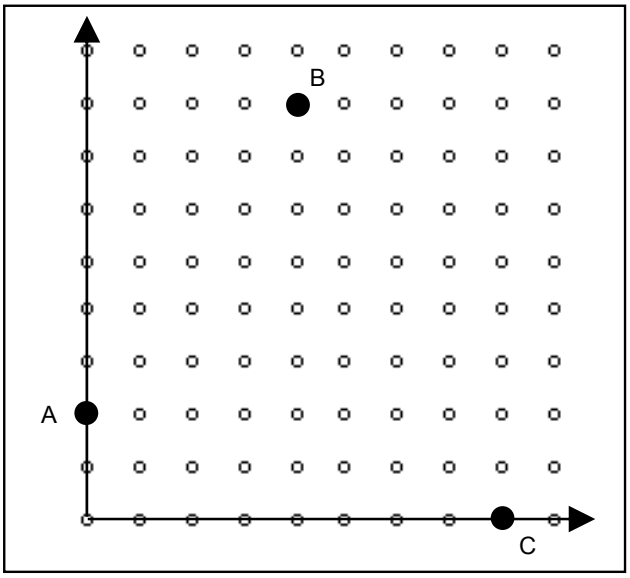
Hint: Think about how you can modify your map for three stores.




**5.** The owner then changed his mind and decided to locate the fourth store downtown near the center of the city

**a.** Where should the downtown store be located so it is in the center of the city but as far away from each of the other three stores as possible? Place this point in the diagram and estimate its coordinates.

**b.** Draw the Voronoi diagram or delivery zone map for these four stores.



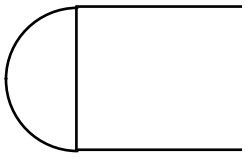

**6.** Suppose the Voronoi diagram or zone map for four stores X, Y, Z, and W has exactly four Voronoi edges. How are the locations of the sites X, Y, Z, and W related to each other? Draw an example of such a Voronoi diagram.

**7.** A zone map or Voronoi diagram is a mathematical model in which several simplifying assumptions are made. One of these assumptions is that the population is uniformly distributed throughout the city – at least the population who wish to buy pizza. What is another simplifying assumption made in using this model?

***The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.***

# Extensions

## For the Teacher



Another way to introduce the idea of a Voronoi diagram is through an activity involving the use of a zone defense in basketball. See the website for more details.
**http://dimacs.rutgers.edu/Institute/96/classroom/voronoi/lessons/pat2**

## For the Student

• Find several elementary schools, post offices, fire stations, etc. in your city. Draw a Voronoi diagram using their locations as sites on a city map. Then check to see how the Voronoi regions compare with the school attendance zones, postal delivery zones, fire station response zones, etc.

• A Voronoi diagram with n sites cannot have more than 3n – 6 Voronoi edges. What is the minimum number of edges in a Voronoi diagram with n sites? How would the sites be positioned so that it would be possible for a minimum number to exist?

## Additional Resources

- There are applications of Voronoi diagrams in many areas including archaeology, astronomy, biology, and marketing. Examine "Applications from Archeology through Zoology" at **http://mathforum.org/mathed/dimacs/lecture.4.html** or **http://www.ics.uci.edu/~eppstein/gina/scot.drysdale.html**.

- Voronoi Diagrams and Proximity Problems, Matthew Dickerson and Scot Drysdale, COMAP, 1996. Check **www.comap.com** for information about how to purchase this module.

- Geometry and Its Applications, Walter Meyer, Harcourt Academic Press, 1999 Chapter 2 has a section on Voronoi diagrams.

- In a Voronoi diagram, each Voronoi vertex is the circumcenter of a circle containing three sites. Students can explore this concept using these applets for constructing Voronoi diagrams and Delaunay Triangulations:
  **http://www.cs.cornell.edu/Info/People/chew/Delaunay.html** and
  **http://www.pi6.fernuni-hagen.de/GeomLab/VoroGlide/index.html.en**

- The Cabri® Jr. App for the TI-83 Plus and TI-84 Plus calculators can be downloaded for free from **http://education.ti.com/cabrijr**.

## Related Topic

The **Delaunay Triangulation/Tessellation** is the geometric dual of the Voronoi Diagram. It can be defined as a triangulation of the sites (constructing triangles with the sites as vertices) with the additional property that for each triangle of the triangulation, the circumcircle of that triangle does not contain any other site.
See: **http://mathworld.wolfram.com/DelaunayTriangulation.html**



Tom Butts, UT Dallas