EXPLORATIONS™

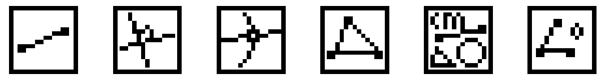
Activity 17

Ratios in Right Triangles

Objective

- To investigate the trigonometric ratios that exist between sides and angles in right triangles

Cabri® Jr. Tools



Introduction

The study of properties of right triangles is a major topic in mathematics. Right triangle trigonometry is one of the many useful topics of mathematics. In this Exploration, you will investigate three common right triangle trigonometric ratios.

Construction

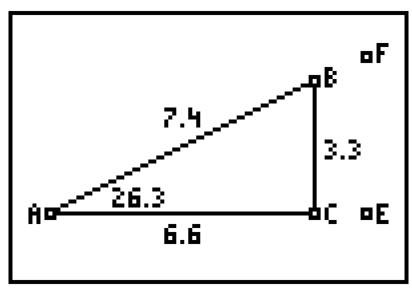
Construct a right triangle.

Draw a horizontal line segment $\overline{AE}$. Construct a second line segment $\overline{AF}$ forming $\angle EAF$.

Draw point C on $\overline{AE}$.

Construct a line perpendicular to $\overline{AE}$ through a point C on $\overline{AE}$.

Construct point B at the intersection of $\overline{AF}$ and the line perpendicular to $\overline{AE}$.



Construct ΔABC.

Hide $\overline{AE}$, $\overline{AF}$, and $\overleftrightarrow{BC}$. Do not hide points E and F or the sides of ΔABC.

Measure the lengths of segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

Measure $\angle BAC$.

Exploration

Observe the changes to the measures as you move point C along $\overline{AE}$. Move point F to a different location and again observe the measures as you move point C.

Calculate the ratios $\frac{BC}{AB}$, $\frac{AC}{AB}$, and $\frac{BC}{AC}$. Observe what happens to the ratios as you move point *C*. Change $\angle BAC$ by moving point *F*. Move point *C* again and observe any relationships that exist.

Questions and Conjectures

1. Make a conjecture about what happens to the lengths and ratios of the sides as point *C* is moved. Explain your reasoning.
2. Complete the following table of values for the ratios given the following measures of $\angle BAC$: 0°, 30°, 45°, 60°, 90°. Compare these values to the values your graphing calculator displays when you use the SIN, COS and TAN functions. (Be sure your graphing calculator is set to Degree mode.)

Angle (θ)	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$	Sin (θ)	Cos (θ)	Tan (θ)
0°						
30°						
45°						
60°						
90°						

Extension

Construct a figure that will allow you to investigate the ratios for angles that are between 90° and 180°. Compare these values to the values between 0° and 90°.

Teacher Notes

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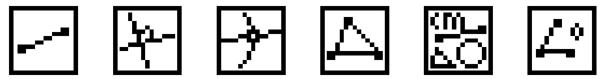


Additional Information

The point *C* can be animated in this construction.

Students will need to leave the Cabri Jr. application and return to the home screen of their graphing calculator to compare trigonometric ratios with the values from this investigation.

Answers to Questions and Conjectures

1. Make a conjecture about what happens to the lengths and ratios of the sides as point *C* is moved. Explain your reasoning.

   Students should see that as point *C* moves, the lengths of the sides change, but the ratios remain constant. By changing the angle, students can see that the constant ratio is true for other angles. Students should be able to explain the constant ratios based on the properties of similar triangles.

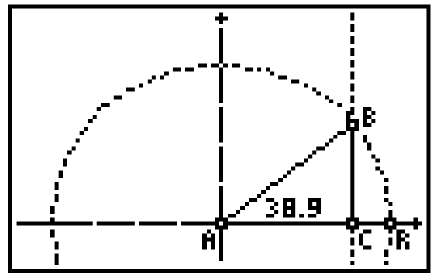
2. Complete the following table of values for the ratios given the following measures of $\angle BAC$: 0°, 30°, 45°, 60°, 90°. Compare these values to the values your graphing calculator displays when you use the SIN, COS and TAN functions. (Be sure your graphing calculator is set to Degree mode.)

Angle (θ)	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$	Sin (θ)	Cos (θ)	Tan (θ)
0°	0	1	0	0	1	0
30°	0.5	0.9	0.6	0.5	0.866...	0.5677...
45°	0.7	0.7	1	0.707...	0.707...	1
60°	0.9	0.5	1.7	0.866...	0.5	1.732...
90°	1	0	Does not exist	1	0	Undefined

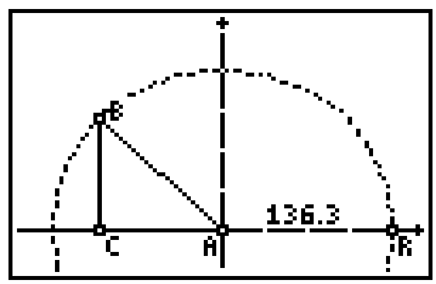
Have students investigate the branch of mathematics called trigonometry. Find the connection between what they have been investigating and the trigonometric ratios.

Answers to Extension

Construct a figure that will allow you to investigate the ratios for angles that are between 90° and 180°. Compare these values to the values between 0° and 90°.



One way to construct a Cabri® Jr. figure to explore angles larger than 90º is to attach the construction to the coordinate axes. Show the axis system on the screen. Construct a circle centered at the origin with radius point *R* on the positive *x*-axis. Construct point *B* on the circle in the first quadrant. Construct a line perpendicular to the *x*-axis through point *B*. Construct point *C* at the intersection of this perpendicular line with the *x*-axis. Construct $\overline{AR}$ and overlay a triangle using points *A*, *B*, and *C* forming right triangle ΔABC. Measure $\angle RAB$ (not $\angle CAB$).



Drag point *B* along the circle into the second quadrant to make ΔRAB larger than 90º. Drag point *R* to change the size of ΔABC and point *B* to change the size of the angle. Measure the lengths of various pairs of sides from ΔABC, like $\overline{BC}$ and $\overline{AB}$, and compute the ratios used in the exploration.

Discussions at this point could go in several directions. How do ratios compare for supplementary angles (reference angles)? Why are some trigonometric ratios negative when computed, but not negative in the figure (quadrants)? What do the coordinates of point *B* reveal when the length of segment $\overline{AR}$ = 1 unit (unit circle trigonometry)? Many other discussions are possible.