# Limits-Vertical and Horizontal Asymptotes of Rational Functions

by

# **Mary Ann Connors**

Department of Mathematics Westfield State College Westfield, MA 01086

# **Textbook Correlation: Key Topic**

- Pre-Requisites: Functions and Equations
- Limits

# **NCTM Principles and Standards:**

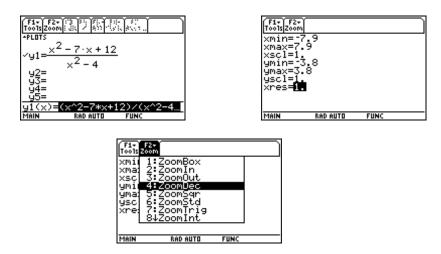
- Process Standard
  - Representation
  - Connections

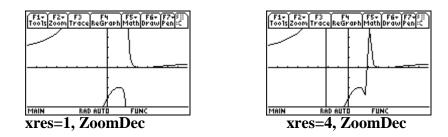
# **Exercises**:

1. Investigate the **vertical asymptotes** of  $f(x) = \frac{x^2 - 7x + 12}{x^2 - 4}$  graphically, numerically and symbolically.

#### Solution:

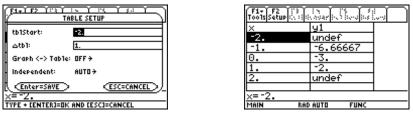
A. Graphical Analysis:





Why does a "vertical" lines appear when you use an **xres** value of 4? Using the Trace feature will help you figure it out.

#### **B.** Numerical Analysis:



Apparent limit as x approaches -2 and 2

# C. Symbolic Analysis (Limit of a function f(x) as x approaches an arbitrary constant, c $[\lim_{x \to c} f(x)]$ ):

Evaluate limits that show the vertical asymptotes (x = 2 and x = -2). Recall that after the limiting value you need to type a comma and a positive number for a right hand limit or a negative number for a left-hand limit.

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2. Investigate the horizontal asymptotes of  $f(x) = \frac{x^2 - 7x + 12}{x^2 - 4}$  graphically, numerically and symbolically.

#### Solution:

A. Graphical Analysis:

Same as in Exercise 1.

# **B.** Numerical Analysis:

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Apparent limit as x increases without bound

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Apparent limit as x decreases without bound

# C. Symbolic Analysis (Limit of a function f(x) as x approaches infinity, $[\lim_{x \to \infty} f(x)]$ ):

Solve for the horizontal asymptote (y = 1) on the Home screen. Press  $\blacklozenge$ , **CATALOG** for the infinity symbol ( $\infty$ ).



Additional Exercise: Investigate all asymptotes and the end behavior of  $f(x) = \frac{x^2 - 2x - 1}{x - 1}$  graphically, numerically, and symbolically using limits.