Limits-Vertical and Horizontal Asymptotes of Rational Functions

by

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Textbook Correlation: Key Topic

- Pre-Requisites: Functions and Equations
- Limits

NCTM Principles and Standards:

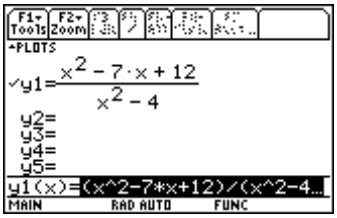
- Process Standard
  - Representation
  - Connections

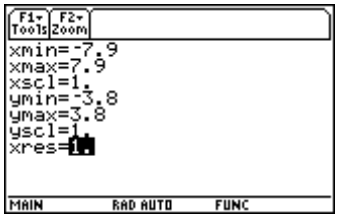
Exercises:

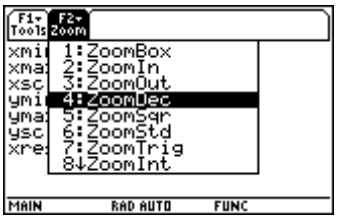
1. Investigate the **vertical asymptotes** of $f(x) = \dfrac{x^2 - 7x + 12}{x^2 - 4}$ graphically, numerically and symbolically.

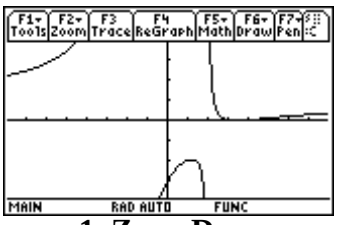
Solution:

A. Graphical Analysis:

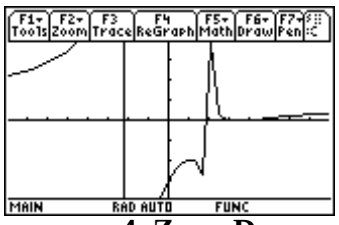








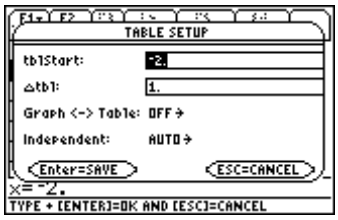
xres=1, ZoomDec

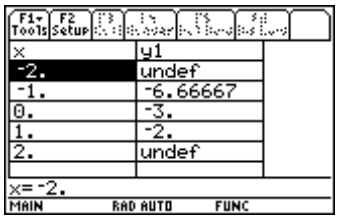


xres=4, ZoomDec

Why does a "vertical" lines appear when you use an **xres** value of 4? Using the Trace feature will help you figure it out.

B. Numerical Analysis:

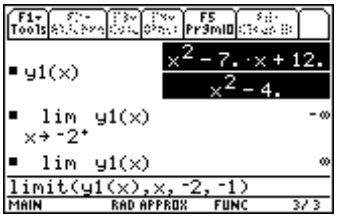


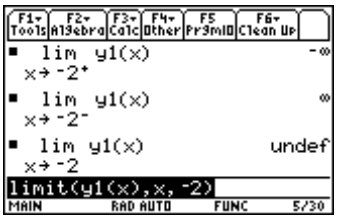


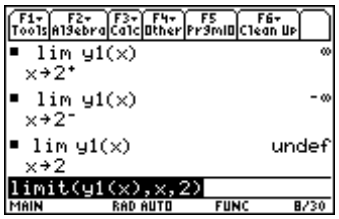
Apparent limit as *x* approaches -2 and 2

C. Symbolic Analysis (Limit of a function f(*x*) as *x* approaches an arbitrary constant, *c* [$\lim_{x \to c} f(x)$]):

Evaluate limits that show the vertical asymptotes ($x = 2$ and $x = -2$). Recall that after the limiting value you need to type a comma and a positive number for a right hand limit or a negative number for a left-hand limit.







2. Investigate the horizontal asymptotes of $f(x) = \dfrac{x^2 - 7x + 12}{x^2 - 4}$ graphically, numerically and symbolically.

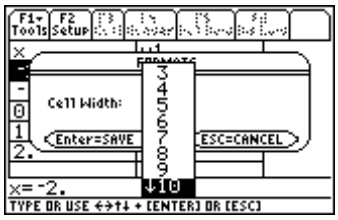
Solution:

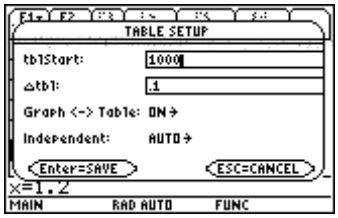
A. Graphical Analysis:

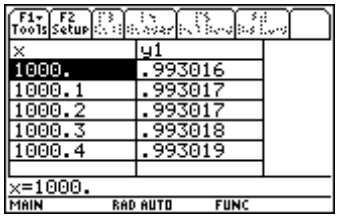
Same as in Exercise 1.

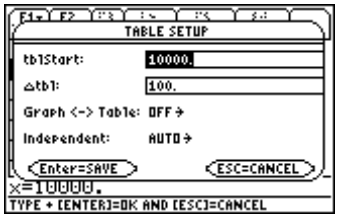
B. Numerical Analysis:

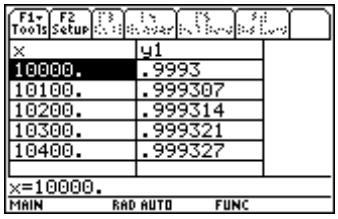
You can adjust the cell width for the table by using **F1(Tools), 9:Format**.



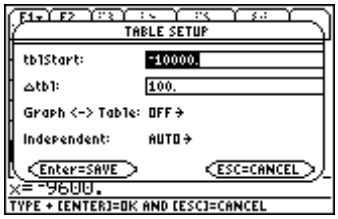


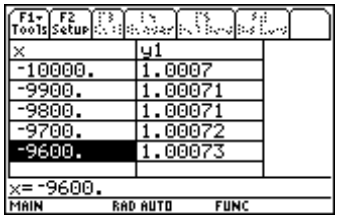






Apparent limit as *x* increases without bound





Apparent limit as *x* decreases without bound

C. Symbolic Analysis (Limit of a function f(x) as x approaches infinity, [$\lim_{x \to \infty} f(x)$]):

Solve for the horizontal asymptote (y = 1) on the Home screen. Press ♦,**CATALOG** for the infinity symbol (∞).

Additional Exercise: Investigate all asymptotes and the end behavior of $f(x) = \dfrac{x^2 - 2x - 1}{x - 1}$ graphically, numerically, and symbolically using limits.