

NUMB3RS Activity: Logistics Growth Episode: "Vector"

Topic: Logistics Growth

Grade Level: 9 - 12

Objective: In this activity, students will find a logistics growth model from a data set that represents the spread of a virus over time, using the TI-Navigator™ system.

Time: 25 - 30 minutes

Materials: TI-83/84 Plus graphing calculators, TI-Navigator system, and the following activity settings files: *Virus1.act*, *Virus2.act*.

To download these files, go to <http://education.ti.com/exchange> and search for "7709."

Introduction

In "Vector," an outbreak of a virus prompts Charlie to study a complex epidemic graph representation or *Susceptible-Infected-Removed* (SIR) model to help trace the origin of the virus. The three components of a SIR model are defined as:

Susceptible – A susceptible person (someone who can become infected)

Infected – An infected person (someone who is currently sick and can infect a susceptible person)

Removed – A person who can no longer infect someone or become infected, by developing immunity to the disease, physical separation from other people, or, in some cases, death.

There are several variables and complex parameters that affect the spread of a disease. This activity will look at a much more simplistic version of a spreading virus. For example, it will be assumed that "contact" between a susceptible person and an infected person will always result in the susceptible person becoming infected. This is typically not true in real life.

In Part I of this activity, students use Activity Center to conduct a simulation involving only *susceptible* and *infected* people. The data will be collected and subsequent analysis will reveal a logistics growth model. Students will model the data with a logistics equation using the regression capabilities of the TI-83/TI-84 Plus graphing calculator. Particular emphasis will be placed on studying the rate of change of the data over time, identifying a logistics growth pattern from a graph, and using regression to analyze a logistics growth equation.

Part I: Tracking the Spread of a Virus

1.
 - a. Students should work individually during this part of the activity.
 - b. Launch TI-Navigator on the computer and press **Begin Class** to start the session.
 - c. Have students log into NavNet on their calculators.

2. a. Load the ***Virus1.act*** activity settings file into Activity Center and click the 'List – Graph' tab. This will display a 10×10 grid, which is a good size for a class of 20 to 30 students. If the class is smaller than 20, then consider changing the minimum and maximum x and y values to -4 and 4 , respectively. Likewise, consider changing the minimum and maximum x and y values to -6 and 6 for very large classes.
- b. Go to the 'View' menu and select 'Individualize Student Cursors.' In the same menu, also select 'Show Student Names' to reveal the identity of each student.
- c. Assign one student to be the first "infected" person. Explain that, when the activity starts, students will move their cursors randomly around the grid, with the teacher pausing the activity at periodic intervals. When the activity is paused, each student will look to see if an adjoining square (or the same square) contains an "infected" student. If so, then this student will become infected as well.
- d. Press **▶ Start Activity** and slide the Activity Center window towards the bottom of the computer screen so that students cannot see the movement of other student cursors. This is important because you do not want students to try to "run away" from infected students! (Note: You can also click the 'Equation' tab to hide the movement of cursors.)
- e. After a few moments, Press **⏸ Pause Activity** and slide the Activity Center window back into view. Highlight the student that was originally infected (by clicking on his or her name) and check to see if any students occupy one of the eight adjacent squares or the same square. If so, then these students are now infected.
- f. Repeat steps **d** and **e** until all students have been infected. Each time you pause the activity, highlight those students that were previously infected by holding down the 'Ctrl' key and clicking on the names of the infected students. This will reveal their cursors in blue, which will make it easier for the remaining students to see if they have become infected.
- g. During this activity, the teacher must use his or her own calculator to record the round number (in L_1) and the total number of students infected (in L_2). Start with the ordered pair $(1, 1)$ to represent the one person that was initially infected.

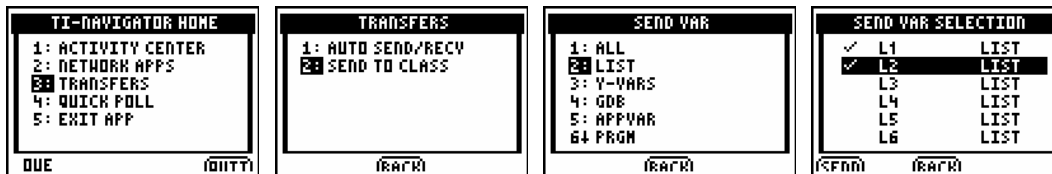
L1	L2	3
1	1	-----
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	1	
9	1	
10	1	
L2 =		

The screen shown above shows an example in which a total of 26 people were infected after the seventh round.

- h. Press **■ Stop Activity** after everyone in the class has become infected.

Part II: Distribute the Data

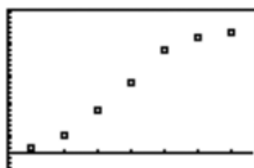
3. a. Log into NavNet as a teacher, using the calculator containing the data from Part I.
- b. From the TI-Navigator home screen, transfer L_1 and L_2 to all the students following the sequence of screen images below.



Note: You can also send the lists to Activity Center by loading the *Virus2.act* activity settings file into Activity Center and pressing **Start Activity**. Have one student enter Activity Center and **SEND** lists L_1 and L_2 . A scatter plot of the data will be displayed in the Activity Center Window. You may need to click the 'Zoom Stat' button to adjust the window settings to reveal all the data. Then press **Stop Activity**.

Part III: Analyze the Results

4. Have students exit NavNet by pressing **2nd** [QUIT] to return to TI-Navigator Home, and selecting 4: EXIT APP.
5. a. Instruct students to make a scatter plot of the data contained in L_1 and L_2 and use **Screen Capture** to view the results. Screens should resemble the one shown below, which follows a logistics growth pattern.



This data can be analyzed in a variety of ways, depending on the level of the class.

- b. In an algebra class, you may choose to run this activity for the purpose of exploring the concept of change informally. For example, you might ask students what happens to the rate at which people become infected over time. Point out that the growth rate is higher during the first few rounds, and then tapers off as more and more students became infected. You may even have students calculate the slopes of several consecutive points to illustrate this point.

- c. In a pre-calculus class, you can use this activity to introduce the concept of logistics growth. Students should already be familiar with exponential growth, which is used to represent situations with *unrestricted* growth. However, in many real-life situations, including the spread of a virus, a logistics growth model is more practical. You could then show them the equation for logistics growth,

$$y = \frac{c}{1 + ae^{-bx}},$$
 and have them use their calculators to perform logistics regression

on the data contained in L₁ and L₂ (press $\boxed{\text{STAT}}$, go to the **CALC** menu, and select **B:Logistic**). The TI-84 Plus uses a logistics growth model of the form

$$y = \frac{c}{1 + ae^{-bx}}$$
 where c is the *carrying capacity*, or the limit as to how many people

can become infected. Students can then use the regression equation to explore its behavior in more detail. For example, as x gets large, students will see that the denominator approaches 1 and the value of y therefore approaches c . Particular attention should also be given to how the values of a and b affect the graph.

- d. In a calculus class, you may choose to demonstrate that an equation of the form

$$y = \frac{c}{1 + ae^{-bx}}$$
 is a solution to the differential equation $\frac{dy}{dx} = cy\left(1 - \frac{y}{c}\right)$. This can be

accomplished by simultaneously constructing a slope field for the differential equation and graphing the logistics growth equation. Students will see that the equation they found from the logistics regression is a particular solution to the differential equation.

- e. Sample questions you may ask students include:

- How does the rate at which people become infected change over time?
- Explain why the rate at which people become infected slows down towards the end of the experiment.
- Describe the significance of the point where the concavity of the graph changes from concave up to concave down.
- The graph appears to approach a horizontal line as time increases. What does this horizontal line represent? How can you use your logistics model to find the equation of this horizontal line?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension

This activity is based on the premise that people infected with a virus will continue to infect people until everyone has caught the virus. This extension will examine a situation where infected people are "removed" if they no longer carry the virus (and have developed immunity to it) or die from the disease.

- Modify Part I of this activity by starting with three infected people and allowing students to remain in Activity Center for only *one more round* after they become infected. These students can be "removed" by clicking **Pause Activity**, highlighting the student name(s), and clicking 'Hide.' After resuming the activity, these students must not press any more keys; otherwise their cursors will reappear in Activity Center. (Note: You can also have students disconnect from the network hub to avoid having them 'sneak' back into the activity.)
- Distribute the data and analyze the results as was done in Parts II and III of this activity. You will most likely find that the data follows a logarithmic pattern, with some students never getting infected by the virus.

Additional Resources

- An additional *NUMB3RS* activity for "Vector" that also examines SIR models can be downloaded for free by going to <http://education.ti.com/exchange> and searching for "7720."
- If you would like to learn more about TI-Navigator, visit <http://education.ti.com/navigator>.