Potential and Kinetic Energy – ID: 9601

By Peter Fox

Time required
30 minutes

Topic: Work and Energy

- *Use the equation $K_e = (1/2)mv^2$ to solve problems involving mass, velocity, and kinetic energy.*
- *Determine the gravitational potential energy of an object.*
- *Calculate the amount of mechanical energy contained in a system.*
- *Solve problems using the conservation of mechanical energy.*
- *Describe, predict, and calculate the path of a projectile.*

Activity Overview

In this activity, students explore the relationship between the mass, energy, and angle of a projected object fired from an idealized cannon. The projectile does not lose energy in the firing process or while it is in flight. This provides for some relatively simple conceptual development in relation to the balance in Ep *(potential energy) and* Ek *(kinetic energy).*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Data & Statistics, Lists & Spreadsheet

Teacher Preparation

Students should be familiar with the equations relating gravitational potential energy to mass and height and relating kinetic energy to mass and velocity. More advanced students may benefit from a discussion of how the energy of the projectile would change if there were drag in the system (i.e., if the projectile were not ideal).

- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7 and 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9601" in the search box.***

Classroom Management

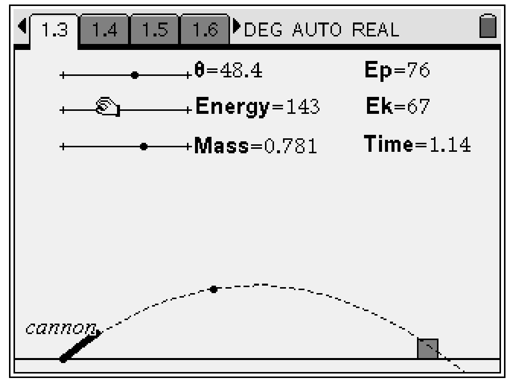
- *This activity is designed to be* ***teacher-led*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

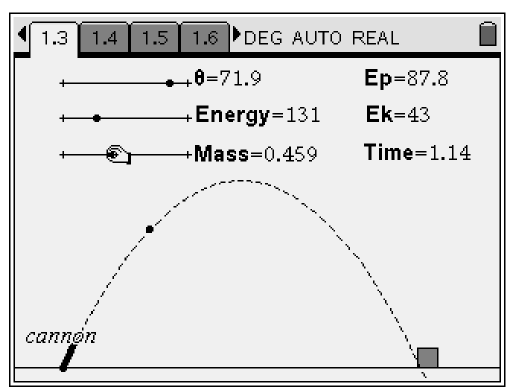
- What is the relationship between velocity and energy?
- How are kinetic and potential energy related?

Students will vary the energy, mass, and angle of projection of an ideal projectile. Students will then construct plots of kinetic and potential energy to observe the relationship between the two.

Part 1 – Qualitative study of projectile energy



Step 1: Students should open the file **PhyAct_9601_potential_kinetic.tns** and read the first two pages. Page 1.3 shows a simulation of an ideal cannon firing a projectile. Students are able to change the initial energy of the projectile, the mass of the projectile, and the angle of firing. Students can also move the cannonball along the trajectory to find how the values of *Ek* (kinetic energy) and *Ep* (potential energy) change during flight. Students should adjust the values of **θ, Energy,** and **Mass** using the sliders and attempt to get the projectile to "land" in the target box. Encourage them to find several combinations of **θ, Energy,** and **Mass** that produce this result. The following symbols are used in the simulation:
θ = initial angle of the projectile in degrees
Energy = initial blast energy of the cannon in joules
Mass = mass of the projectile in kilograms
Ep = potential energy of the projectile in joules
Ek = kinetic energy of the projectile in joules
Time = time in seconds since the projectile left the cannon
Note: Make sure students do not modify the *Calculator* or *Lists & Spreadsheet* applications at the end of the .tns file. These applications control the simulation, and if they are modified, the simulation may not work properly.



Step 2: Next, students should select one combination of **θ, Energy,** and **Mass** that allows the projectile to land in the box. If you wish, you may assign different students different combinations. Once the values are established, students should answer questions 1–9.

Q1. Drag the projectile along its path. Where is **Ep** maximum? Explain this observation.

A. ***Ep*** *is maximum when the projectile reaches its maximum height. The equation for potential energy is Ep = mgh. For a projectile of fixed mass in a specified gravitational field,* h *is the only variable; therefore, the higher the object gets, the greater its potential energy is.*

Q2. Where is **Ep** minimum? Explain this observation.

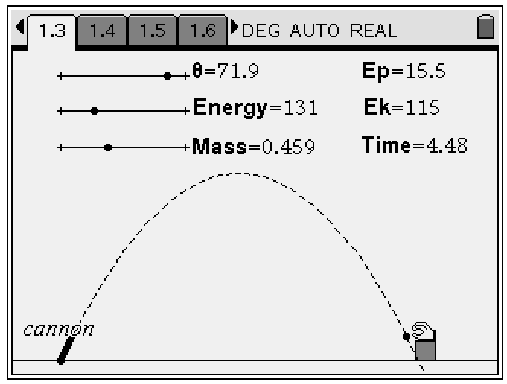
A. ***Ep*** *is minimum when the projectile is closest to the ground. This occurs when the projectile lands in the bottom of the box.*

Q3. Where is **Ek** minimum? Explain this observation.

A. *The total energy of the projectile is constant. It is equal to* Ep + Ek*.* ***Ep*** *is a maximum at the top of the projectile's flight path. Therefore, because the sum of* Ep *and* Ek *is fixed,* ***Ek*** *is minimum at the top of the projectile's flight path.*

Q4. Where is **Ek** maximum? Explain this observation.

A. ***Ek*** *is maximum where* ***Ep*** *is minimum—that is, when the projectile lands (or, more accurately, just before it lands—once it lands, it has no velocity and therefore no kinetic energy). If the bottom of the box and the muzzle of the cannon were at the same height, then* ***Ek*** *would also be maximum just after the projectile left the cannon.*



Q5. Where is the velocity of the projectile minimum? Explain your answer.

A. *Given Ek =* $\frac{1}{2}mv^2$*, the minimum velocity will occur when* Ek *is a minimum. This occurs at the top of the projectile's path. If you wish, you may have students calculate this velocity for the parameters they have set in their simulation.*

Q6. Where is the velocity maximum? Explain your answer.

A. *The maximum velocity will occur when* Ek *is maximum. This happens when* Ep *is minimum. The minimum potential energy occurs when the height of the projectile is minimum. This occurs just before the projectile lands in the bottom of the box. As in question 4, if the muzzle of the cannon were at the same height as the bottom of the box, then the velocity of the projectile would be maximum just after it left the cannon.*

Q7. Explain why the mass of the projectile affects the distance the projectile travels.

A. *The distance the projectile travels depends on its initial horizontal velocity. This, in turn, depends on its initial kinetic energy, which is fixed and is equal to* $\frac{1}{2}mv^2$, *where* v *is the initial velocity of the projectile. Because the total energy in the system is fixed, the greater the mass is, the lower the velocity is.*

Q8. At what point along the path of a projectile is the vertical velocity zero?

A. *at the highest point in its path*

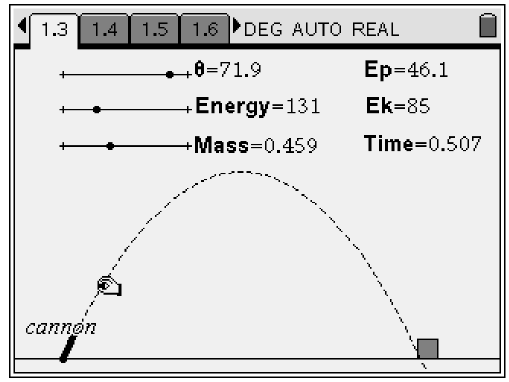
Q9. What is the horizontal velocity for a projectile with mass 1 kg fired with an initial energy of 324 J at an angle of 74.2°?

A. *v = 6.9 m/s; students should solve this using the simulation on page 1.3. Remind students that the horizontal velocity of a projectile does not change as it travels along its path and that at the top of the projectile's path, the projectile's vertical velocity is zero. Therefore, any kinetic energy the projectile has at the top of its path is due to its horizontal velocity. Students should use the simulation to determine the kinetic energy of the projectile at the top of its path and then use the equation* $Ek = \frac{1}{2}mv^2$ *to calculate the horizontal velocity. At the top of its path, the projectile's kinetic energy is 24 J. Therefore,*

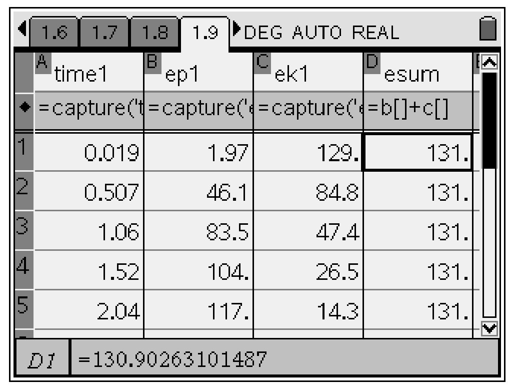
$$v = \sqrt{\frac{2Ek}{m}} = \sqrt{\frac{(2)(24)}{(1)}} = \sqrt{48} = 6.9\ m/s.$$

Part 2 – Graphs of potential and kinetic energy

Step 1: Next, students should move back to the simulation on page 1.3. They should move the projectile back to the beginning of its path. Again, you may wish to assign different students different values of **θ, Energy,** and **Mass** for this part of the activity. Students should drag the projectile along its path, stopping approximately every 0.5 sec (i.e., at 0 sec, 0.5 sec, 1 sec, etc.). At each half-second time point, students should use manual data capture (ctrl .) to capture the values of **Time, Ep,** and **Ek** for the projectile. Once students have moved the projectile through its entire path, they should move to page 1.9, which contains a *Lists & Spreadsheet* application. The values for **Time, Ep,** and **Ek** that the students captured should be stored in Columns A, B, and C, respectively, of this spreadsheet.



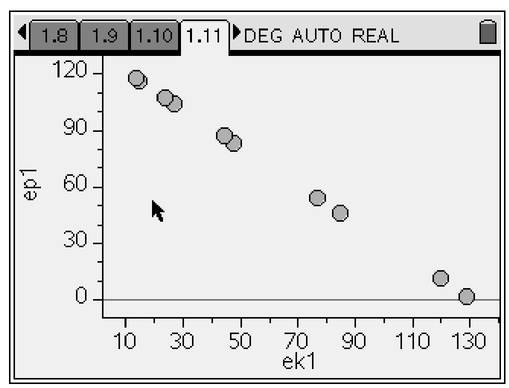
Step 2: Next, students should use a formula to calculate the projectile's total energy at each time point in Column D. To do this, students should enter the formula **=b[]+c[]** into the formula bar of Column D. Then, they should answer question 10.



Q10. What do you notice about the relationship between **Ep, Ek,** and **Energy**?

A. *The combined energy,* ***Ep*** *+* ***Ek,*** *is equal to the original blast energy (****Energy****). Therefore, no energy is lost in the simulation.*

Step 3: Next, students should move to page 1.11, which contains an empty *Data & Statistics* application. Students should plot **ep1** vs. **ek1** on this graph. Then, they should answer questions 11 and 12.



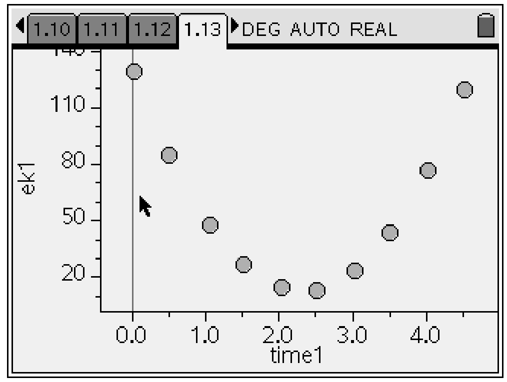
Q11. Does the graph of **ep1** vs. **ek1** support the observation that the total energy of the projectile remains constant throughout its path? Explain your answer.

A. *Yes; as potential energy increases, kinetic energy decreases. The data points lie along a straight line with a slope of –1, which implies that their sum is constant:* ***ep1*** *= –1·****ek1*** *+ 136, so* ***ep1*** *+* ***ek1*** *= 136.*

Q12. Predict what a graph of **ek1** vs. **time1** would look like. Explain your reasoning.

A. *Student answers will vary. Encourage student discussion of their predictions.*

Step 4: Next, students make a graph of **ek1** vs. **time1** on page 1.13. Then, they should answer questions 13 and 14.



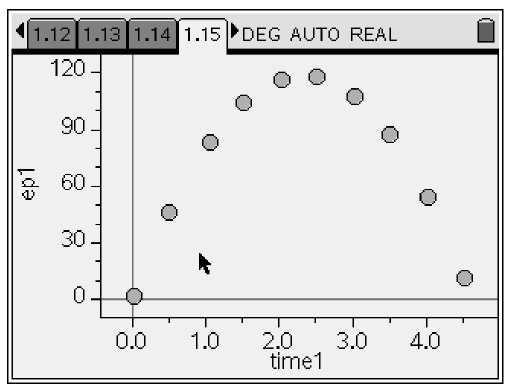
Q13. Was the prediction you made in question 12 correct? If not, explain any errors in your reasoning.

A. *Student answers will vary. Encourage them to think about what led them to make the predictions they made and to identify their errors in reasoning.*

Q14. Predict what a graph of **ep1** vs. **time1** would look like. Explain your reasoning.

A. *Student answers will vary. Encourage student discussion of their predictions.*

Step 5: Next, students make a graph of **ep1** vs. **time1** on page 1.15. Then, they should answer question 15.

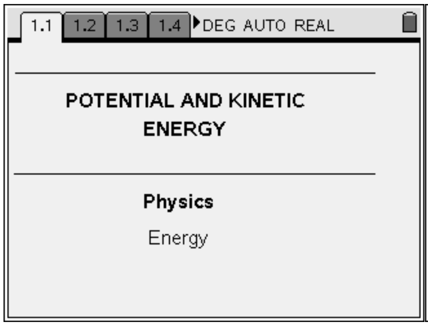


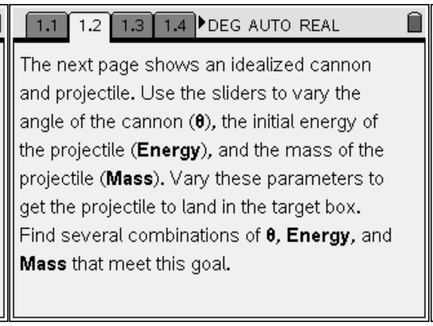
Q15. Was the prediction you made in question 14 correct? If not, explain any errors in your reasoning.

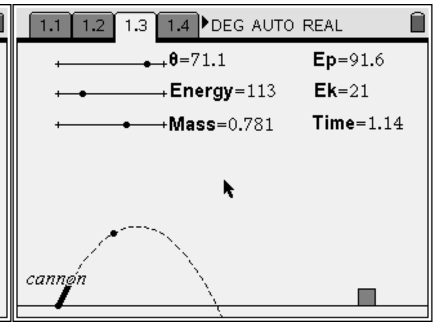
A. *Student answers will vary. Encourage them to think about what led them to make the predictions they made and to identify their errors in reasoning.*

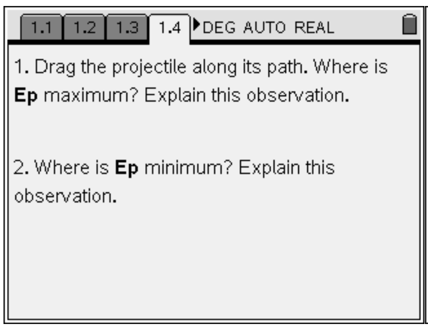
Potential and Kinetic Energy – ID: 9601

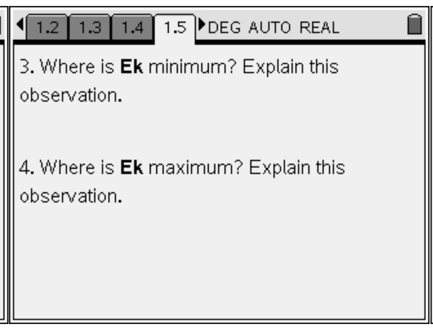
(Student)TI-Nspire File: *PhyAct_9601_potential_kinetic.tns*

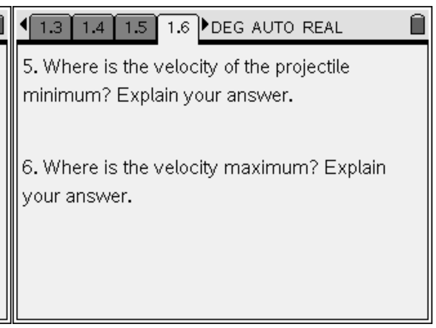


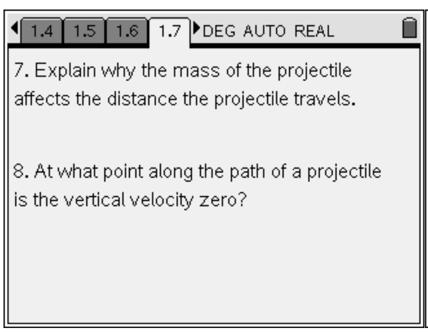


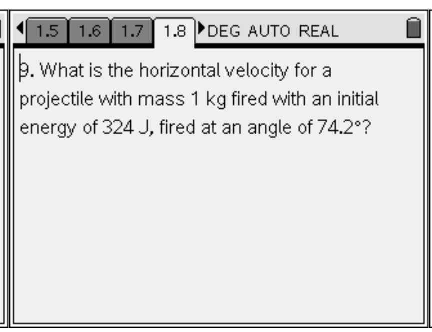


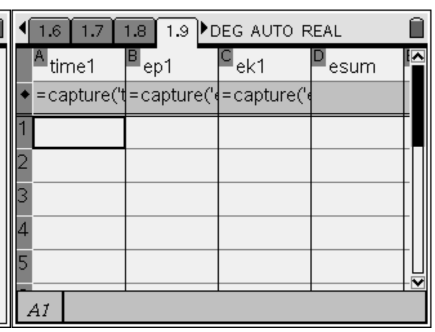


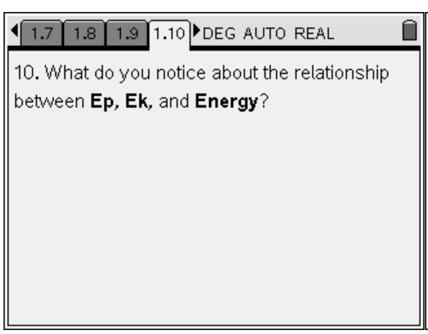


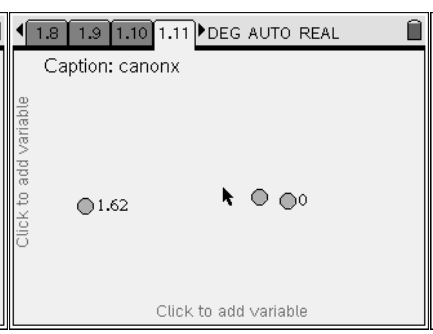


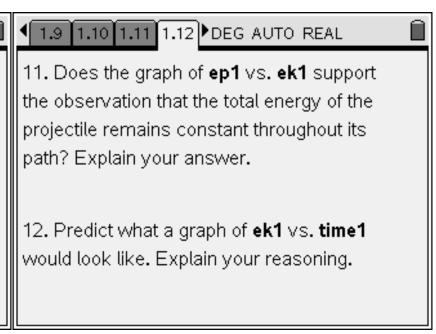


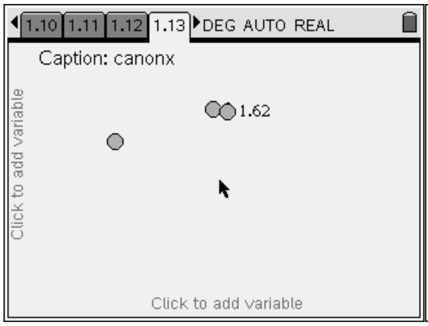








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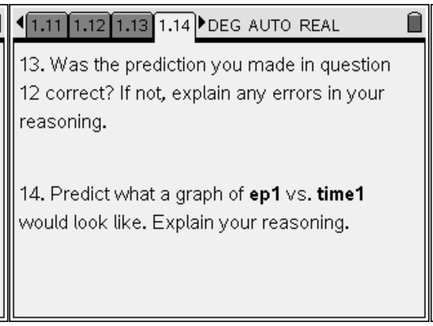
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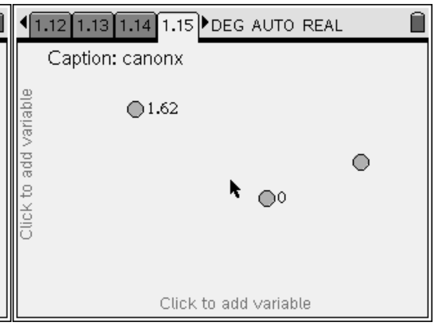
13. Was the prediction you made in question 12 correct? If not, explain any errors in your reasoning.
14. Predict what a graph of **ep1** vs. **time1** would look like. Explain your reasoning.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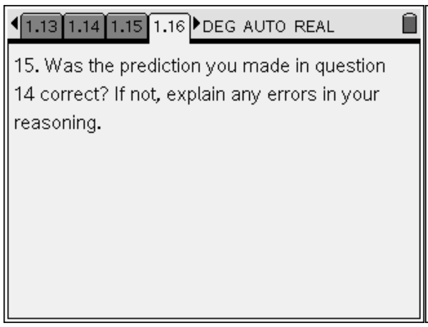
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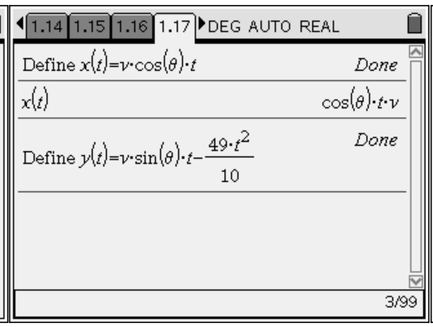
15. Was the prediction you made in question 14 correct? If not, explain any errors in your reasoning.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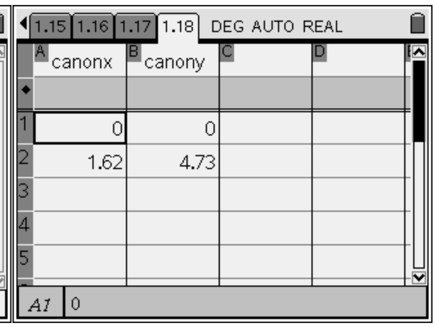
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