Activity 12
What is the Number "e"

Answers to Instructions: Part A

2. $\int f(x)dx = 1.38629$

3. $b = 2.7$

4. $F(0) = 0$

$$F(x) = \int_0^x \sqrt[3]{t^2 + \cos(t)} \cdot dt$$

$$F'(x) = \sqrt[3]{x^2 + cos(x)}$$

$$F'(\pi/4) = \sqrt[3]{\pi^2 + \frac{\sqrt{2}}{2}}$$

7. $(x, y) = (1, 0)$ The value of any integral from 1 to 1 is zero.

10. 2.718

Activity 12
What is the Number "e"
(Continued)

Answers to Instructions: Part A

11. $\frac{d}{dx}\ln(x) = \frac{1}{x}$

$\frac{d}{dx}\text{myst}(x) = \frac{1}{x}$

$\text{myst}(x) = \ln(x)$

Answers to Part A—Extra Practice

1. $f(0) = 0, f(1) = 1.11145, g(0) = -1.11145, g(1) = 0$
2. $f\,'(x) = \sqrt{x^3+1}$ and $g\,'(x) = \sqrt{x^3+1}$
3. They differ by a constant, $\int_0^1 \sqrt{x^3+1}\cdot dx = 1.11145$
4. $f\,''(x) = \frac{3\cdot x^2}{2\cdot\sqrt{x^3+1}}$
5. $u = 2\cdot x \Rightarrow f(u) = \int_0^{2\cdot x}\sqrt{t^3+1}\cdot dt$ and

   $\frac{d}{dx}f(u) = \int_0^{2\cdot x}\sqrt{t^3+1}\cdot dt = \sqrt{8\cdot x^3+1}$

$u(x)$	$f(u)$	$\frac{df(u)}{dx}$
$u = x^2$	$\int_0^{x^2}\sqrt{t^3+1}\cdot dt$	$2\cdot x\cdot\sqrt{x^6+1}$
$u = x^3$	$\int_0^{x^3}\sqrt{t^3+1}\cdot dt$	$3\cdot x^2\cdot\sqrt{x^9+1}$
$u = e^x$	$\int_0^{e^x}\sqrt{t^3+1}\cdot dt$	$e^x\cdot\sqrt{e^{3\cdot x}+1}$
$u = \sin(x)$	$\int_0^{\sin(x)}\sqrt{t^3+1}\cdot dt$	$\cos(x)\cdot\sqrt{\sin^3(x)+1}$

$$\frac{d}{dx}\left(\int_0^{u(x)}\sqrt{t^3+1}\cdot dt\right) = u\,'(x)\cdot\sqrt{(u(x))^3+1}$$

6. $\frac{d}{dx}\left(\int_a^{u(x)} g(t)\cdot dt\right) = g(u(x))\cdot u\,'(x)$

Activity 12
What is the Number "e"
(Continued)

Answers to Part B—Extra Practice

1. Let y = the number of bacteria after t hours.
   Then $\frac{dy}{dt} = 0.082 * y$.

   Separate the variables: $\frac{dy}{y} = 0.082 * dt$

   Integrate both sides of the equation:
   $\ln(|y|) = 0.082 * t + k$

   Solve the resulting equation for y given $y > 0$:
   $y = e^{082t + k}$

   Solve for k, given that $y = 100$ when $t = 0$:
   $k = 2 * \ln(10)$.

   Define $k = 2\ln(10)$. The growth equation is
   $y = 100 * (1.08546)^t$

   Solve for t when $y = 10000$, $t = 56.1606$. The bacteria will increase from 100 to 10000 in 56.16 hours.

2. Let y = the percentage of Carbon 14 t years from now, then $\frac{dy}{dt} = r * y$

   Separate the variables $\frac{dy}{y} = r * dt$

   Integrate both sides of the equation:
   $\ln(|y|) = r * t + k$

   Solve the resulting equation for y given $y > 0$:
   $y = e^{r * 1 + k}$

   Solve for k given $y = 4.7$ when $t = 0$: $k = 1.54756$.
   Define $k = 1.54756$.

   Solve for r given $y = 9.4$ when $t = -5700$.
   $r = -.000122$.

   Define $r = -.000122$.

   The exponential decay equation is:
   $y = 4.7 * (.999878)^t$

   Take the natural logarithm of both sides of the equation: $\ln(y) = -.000122 * t + 1.54756$

   Solve this logarithmic equation for t given $y = 100$:
   $t = -25143.8$

   The campsite is approximately 25,000 years old.