NUMB3RS Activity: Exponential Growth
Episode: "Identity Crisis"

Topic: Exponential Growth   **Grade Level:** 9 - 10

Objective: In this activity, students will learn that exponential growth is characterized by successive values changing by a constant multiplier. A comparison will be made to a linear function in which the rate of growth is constant; that is, successive values are changing by a constant addend.

Time: 25 - 30 minutes

Materials: TI-83/84 Plus graphing calculators, TI-Navigator™ system, and the following activity settings files: *Allowance.act* and *Exponential.act*
To download these files, go to **http://education.ti.com/exchange** and search for "7727."

Introduction

In "Identity Crisis," Charlie talks about how exponential growth is used by thieves in fraud cases. He discusses a pyramid scheme, where each person is recruiting 2 or more people to enroll into the scheme, getting them to pay for a product which is usually never being delivered. He goes on to use the analogy of paper folding to explain the geometric progression.

In this activity, students will explore exponential growth by investigating two different options for receiving a weekly allowance.

Suppose your parents are going to pay you a weekly allowance for doing your household chores this coming year. Also, imagine that they are going to give you a choice of two payment schemes. With Choice A, your parents will pay you $10 every week. Therefore, you will have earned $10 after the first week, $20 after the second week, $30 after the third week, and so on. With Choice B, just to be nice, they are going to pay you $0.01 before you even start working. After you complete chores for week one, they will double your money, thus giving you a total of $0.02. After you complete chores for week two, they will once again double your money, thus giving you a total of $0.04. They will continue to double your money at the end of every week. Which payment scheme would you choose?

Part I: What Would You Choose?

1. a. Launch TI-Navigator on the computer and press **Begin Class** to start the session.

   b. Have students log into NavNet on their calculators.

   c. Ask students which payment scheme they prefer, using **Quick Poll** with *Multiple Choice A Thru B* as the question type where *A* represents a preference for Choice A and *B* represents a preference for Choice B.

   d. If the results are roughly equal, then tell the students that they are either on *Team A* or *Team B*, based on how they responded to the question. If there is a significant imbalance, then try to persuade some students to switch to the other choice.

   e. For each team, assign each student a number from 0 to 15. If there are fewer than 15 students per team, start back with student 0 and begin assigning the remaining numbers to successive students until you reach 15.

Part II: The Competition – Weeks 0 through 8

1. a. Load the ***Allowance.act*** activity settings file into Activity Center. This activity
   setting will allow students to submit a form. The form will ask students to input their
   Week Number and the **Total Earned**.

  b. Click on the "List – Graph" tab.

2. a. Press $\boxed{\blacktriangleright\textbf{Start Activity}}$, click the "Configure Plots" button, and set "X-List" to L_1
   and "Y-List" to L_2.

  b. Instruct the Week 0 students from each team to enter their week number and total
   earnings. The screens below show the inputs for each team. After students press
   $\boxed{\text{ZOOM}}$ **Send**, their points will be plotted in the Activity Center window. (Note: If a
   student sends in an incorrect point, you can press $\boxed{\blacksquare\ \textbf{Stop Activity}}$, press
   $\boxed{\blacktriangleright\textbf{Start Activity}}$,and "Hide" the incorrect point. The activity data will not be lost
   and the student will have an opportunity to re-send the point.)

      Team A – Week 0   Team B – Week 0

  c. Next, instruct the Week 1 students from each team to enter their week number and
   total earnings.

      Team A – Week 1   Team B – Week 1

  d. Repeat **Step b** until the Week 8 students have inputted their data. Notice that the
   student on Team B multiplied the previous value by 2, which may be helpful if they
   are having trouble with the arithmetic. Students can see the previous value
   because it is visible in the "List – Graph" view.


      Team A – Week 8   Team B – Week 8

d. At the conclusion of Week 8, press $\boxed{\textbf{Pause Activity}}$ and ask students if they want to switch their payment scheme. Use **Quick Poll** with *Yes/No.* Based on the screen below, you may find that many of the Team A students do not want to switch payment schemes whereas the Team B students do. This is a good time to have a conversation with students about what is happening with each payment scheme.
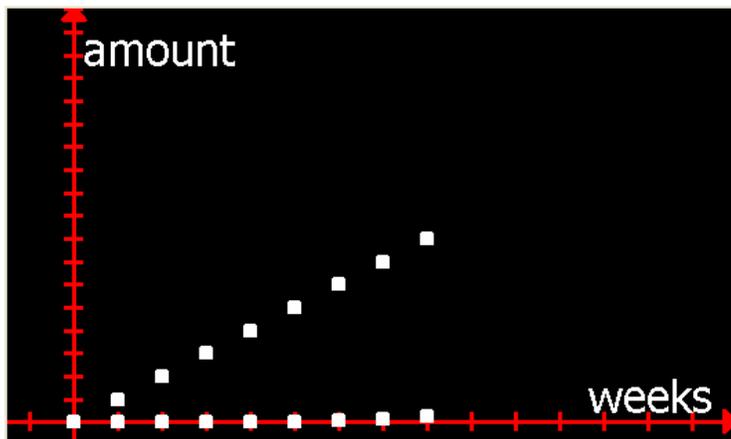
Part III: The Competition – Weeks 9 through 14

3. a. Click $\boxed{\textbf{Resume Activity}}$ and continue having students input the total earnings for each scheme.

b. Press $\boxed{\blacksquare\ \textbf{Stop Activity}}$ after the data from Week 14 has been submitted. The screen below shows that, during Week 14, Team B overtakes Team A in earnings.
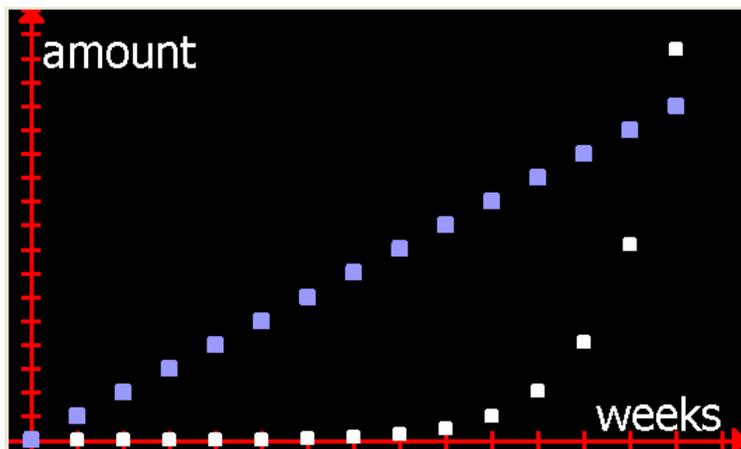
c. Ask students to explain why Team B eventually overtakes Team A. They should see that the Choice A payment scheme increases by the *same amount* each week whereas the Choice B payment scheme increases by twice its value each week. Emphasize with students that this is the basic idea behind exponential growth.

Part IV: Find an Exponential Model

4. a. In Parts I through III, students were introduced to a situation that can be modeled by exponential growth. In this section, you will have students find the equation that models the situation described by Choice B.

   b. Load the ***Exponential.act*** activity settings file into Activity Center. This activity settings file will allow students to submit an equation. It is also configured to allow students to look at the graphs of their equations before they send them to Activity Center.

   c. Press ▶**Start Activity** and instruct students to send in a graph that models the data from Choice B. Depending on the level of your students, you may need to tell them that the equation must be of the form $y = ab^x$. You can also click "Configure," then select the "Equations below" button, and provide part of the exponential equation.

   d. As equations appear in Activity Center, ask students to comment on which graphs seem to model the data well. This can be a very informal conversation about the proximity of the graph in relation to the data points. In fact, some students might have sent the equation $y = 0.01 \cdot 2^x$ into Activity Center. If so, discuss with students the relationship between this equation and the context of the problem. Specifically, they should see that 0.01 is the initial amount and 2 is the constant multiplier by which the total amount earned increases each week.

   e. Show students why 0.01 is the initial amount by replacing x with zero. To illustrate the concept of a constant multiplier, look at a specific week such as week 5. By writing $y = 0.01 \cdot 2^5$ as $y = 0.01 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ students can see that the amount earned each week is twice the previous week.

   f. At this point, you can discuss the general form of the equation for exponential growth, $y = ab^x$.

   g. You may also choose to ask students to find the linear equation that models Choice A, $y = 10x$. Furthermore, you can have students exit NavNet and use their graphing calculators to identify the intersection point of the two graphs.