EXPLORATIONS

Name ______________________________

Date ______________________________

Activity 5

Heads Up!

In this activity, you will study some important concepts in a branch of mathematics known as *probability*. You are using probability when you say things like: "It is impossible for me to meet you after school," "I am certain that our book reports are due next Monday," or "It is not likely that my parents will let me go to the game." Probability is used on the television or in the newspaper with statements like "The chance of rain tomorrow is 60%" or "The chance of selecting a winning ticket for the lottery jackpot is 1 in 10,700,000." In all of these situations, a statement is being made about how sure we are that a particular event will or will not occur.

The Problem

You and a partner are going to toss a coin ten times, keep a record of the results of each toss, and write down the number of times that heads came up during the ten tosses. Decide which of you will toss the coin and which will keep record of the results.

- Before you begin tossing the coin, decide how many heads you think you will see over the ten coin tosses and how certain you are of your prediction. Answer #1 through #4 in the **Questions** section.

You may want to spend some time discussing the statements listed above and others that the students generate. To help develop a "probability sense," you may want to focus on benchmark probabilities of 0 (impossible), 50% or ½ (likely or possible), and 100% (certain).

The common terms students will place with given probabilities are interesting; for example, "It is not likely my parents will let me go to the game." What probability would your students assign to the phrase "not likely." A 20% chance, a 50% chance? What is their probability sense?

Do your students believe that 1 out of 2, 50%,½, and 0.5 are equivalent expressions? How are they determining that?

Many people believe that when you toss a coin, it should come up heads as often as it comes up tails. As your class' coin tossing experiments most likely demonstrated, this does not always occur. So why do we still say that the chance of a tossed coin coming up heads is 1 out of 2 (50%, ½, or 0.5)? You will study this question in the next part of this investigation.

Using the Calculator

If at all possible, you should have a graphing calculator with a ViewScreen to work with the students. We have found great success when a student gets to enter values into the calculator with the ViewScreen and the teacher walks around observing, helping, and questioning.

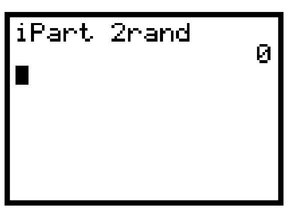
Many calculators can generate numbers that behave the same way as the sequence of heads and tails from repeated tossings of a coin. You and your partner are going to use such a calculator, rather than a coin, to investigate what happens to the portion of heads when you toss a coin many times. This process of using numbers rather than actual coins is an example of a *simulation*.

Calculating the Results

*If you are using a TI-83, the key sequence is [CLEAR] [MATH] [▸] 3 2 [MATH] [◂] 1 [ENTER]. The display will include a left parenthesis in the expression, which will look like: **iPart(2rand**. Students may add the right parenthesis [)] if they wish but it is not necessary.*

We will use the calculator command **iPart 2rand** to simulate tossing a coin. This command will randomly generate either a 0 or a 1 each time [ENTER] is pressed. For this experiment, if your calculator shows a 1, then that is the same as getting a head on a coin toss. If it shows a 0, that is the same as obtaining a tail.

1. Press [CLEAR] [MATH] [▸] (selects **NUM**) **2:iPart** [2] then press [MATH] [◂] (selects **PRB**) **1:rand** [ENTER] to enter the command. Your calculator screen should look like one of the following:



or

*Depending upon your students, you may want to explain what the commands **iPart** and **2rand** mean. The **2rand** command generates random numbers greater than or equal to 0 and less than 2.*

*You and your class may want to investigate that by entering just **2rand**. The **ipart** command takes only the integer part of the decimal fraction, hence you will only see 0's and 1's displayed.*

2. Press [ENTER] repeatedly to simulate the tossing of a coin several times. Notice that each of the numbers is either **1** or **0**.

3. Press [CLEAR] to clear your calculator display.

You and your partner are now going to simulate tossing a coin ten times. To do so, one of you must repeatedly press [ENTER] and the other must record the results in Table 5.2, **Results of Simulated Coin Toss**, in the **Questions** section. After you complete Table 5.2, answer #5 through #8 in the **Questions** section.

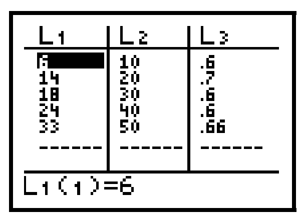
Displaying a Graph

You can examine a graph of the results from your coin tossing simulation on your calculator. To do so, you must first place some of your table entries into your calculator's **LIST** storage.

1. Press [STAT] **4:ClrList** [2nd] [L1] [,] [2nd] [L2] [,] [2nd] [L3] [ENTER] to clear the lists you need.
2. Press [STAT] **1:Edit** to gain access to the calculator's **LIST** storage.
3. Type the numbers from the column labeled **L1** in Table 5.3 into the first list on your calculator. Type a number, press [ENTER], and then repeat until all five numbers have been entered.
4. Press [▶] to move to the second list. Type the numbers from the table column labeled **L2** into the second list on your calculator.

We find it easier to enter in one column of values, then another column of values, rather than entering rows. It is usually faster and you are prone to make fewer mistakes in entering values.

Some sample data:



Instead of entering the fractions from the last column of your table, you can tell the calculator to compute these values.

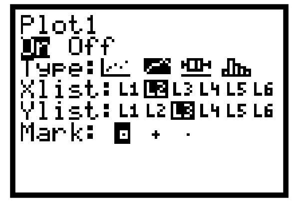
5. Press [▶] to move to the third list. Press [▲] to move to the top so that **L3** is highlighted.
6. Press [2nd] [L1] [÷] [2nd] [L2] [ENTER] to calculate the third list.

Your calculator lists should now contain information from the last three columns of your table. To see a graph that displays some of the information from the lists:

7. Press [2nd] [STAT PLOT] [ENTER].

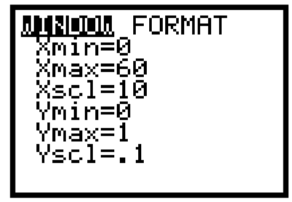
Remind your students to press ENTER *after moving the cursor on top of a selection. Just putting the cursor there does not select the feature. Ask why* ***L2*** *and* ***L3*** *were chosen for the* ***Xlist*** *and* ***Ylist****?*

8. Edit the window so that yours looks like the one at the right. To highlight a selection, use the blue arrow keys to move the blinking cursor on top of the desired location and press ENTER.



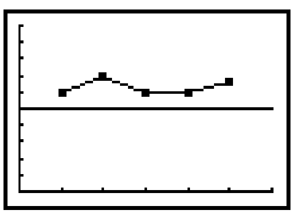
Ask your students why they think these window settings were chosen. Why does the ***Ymax*** *only go to 1? What about the choice of scale? Does* ***Xscl*** *have to be 10? Could it be 5? What makes the difference? Ask them to predict what the graph will look like.*

9. Press WINDOW and edit the numbers to match those shown at the right.



10. Press GRAPH to view a line plot of the fractional part of the coin tosses that came up heads as a function of the total number of coins tossed.

A sample display with the horizontal line at 0.5:



11. Press 2nd [DRAW] 3 ENTER to add a horizontal line that represents the decimal fraction 0.5. By examining the data around this line, you can determine how close the fractions are to ½.

After 50 tosses, the experimental probability will be close to but probably not exactly ½. Yet, if we tossed the coin 1000 times, we would expect the probability to come very close to ½. This stabilizing value is referred to as the theoretical probability. Theoretical probabilities can be determined by comparing the number of favorable outcomes (in this case, heads) to all possible outcomes from the experiment (heads and tails), assuming that all outcomes have an equal chance of occurring. To examine this stabilizing effect you could have the class pool all of their data and compute the summary statistics.

As you probably observed in this activity, the fraction of the tosses coming up heads varied each time a series of coins was tossed. Even with 50 tosses, it is possible that the final fraction is uncomfortably far away from the expected 0.5. If a very large number of trials were to be conducted, this experimental probability would approach and stabilize at 0.5. Some persistent people have attempted to demonstrate this principle. In the early twentieth century, the English statistician Karl Pearson tossed a coin 24,000 times and got 12,012 heads, resulting in an overall experimental value of 0.5005.

Questions

1. If a coin is tossed ten times, we expect to see heads come up _____ times.

Now toss your coin ten times and record the results after each toss. Use **H** for head and **T** for tail:

Table 5.1. Coin Toss Results

Toss	1	2	3	4	5	6	7	8	9	10	Total Heads
Result											

2. Did the results from your tosses agree with your guess?

____

3. Compare your results with those of others in your classroom.

a. What was the smallest number of heads anyone got?

____

b. What was the largest number of heads anyone got?

____

4. Why do you think the results varied as much as they did?

____

____

____

____

Return to page 42.

Table 5.2. Results of Simulated Coin Toss

Toss	1	2	3	4	5	6	7	8	9	10	Total Heads
Result											

5. What fractional part of the tosses in Table 5.2 came up heads?

   That is, what is $\frac{\text{number of heads}}{\text{total number of tosses}}$?

   ____

6. This fraction is called the *experimental probability* of getting heads. Why do you think it is called an experimental probability?

   ____

   ____

   ____

*Notice that the subheadings in some of the column labels contain **L1**, **L2**, and **L3**. This notation will be useful when entering the data into the calculator since these are the headings for the lists.*

7. Copy the number of heads from Table 5.2 into the first row of Table 5.3, **Cumulative Simulation Results**. Repeat the simulation process for four more sets of ten coin tosses. Enter your results in the table and calculate the remaining table values.

Table 5.3. Cumulative Simulation Results

	Number of Heads	Total Number of Heads (L1)	Total Number of Coins Tossed (L2)	Fraction of Coins Coming up Heads (L3)
1st set of 10 tosses			10	$\frac{}{10}$
2nd set of 10 tosses			20	$\frac{}{20}$
3rd set of 10 tosses			30	$\frac{}{30}$
4th set of 10 tosses			40	$\frac{}{40}$
5th set of 10 tosses			50	$\frac{}{50}$

8. Did the fractional part of the coins coming up heads get closer to ½ as more coins were tossed? Explain how you know. Is that what you expected to happen?

____

____

____

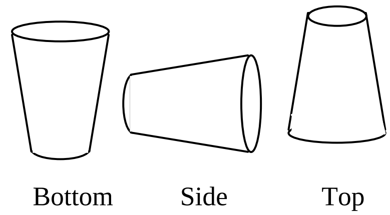
____

*How are the students determining that the fractions in **L3**, 14/20, 23/40, and so on, are close to ½? This is a good time to practice equivalent or benchmark fraction ideas.*

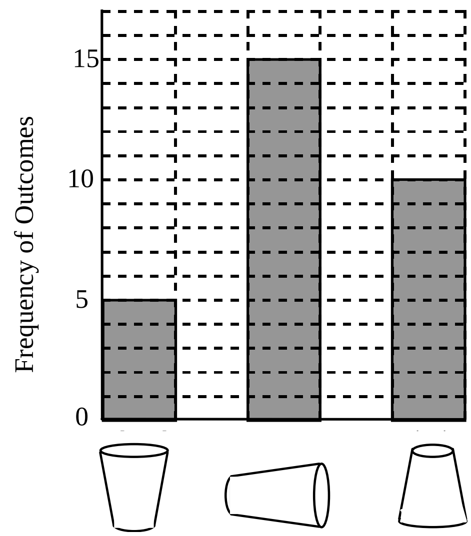
*Return to page 43, **Displaying a Graph.***

Problems for Additional Exploration

1. If a paper cup is dropped onto a flat surface it will land either on its bottom, its top, or its side.



A fifth grade student tossed a paper cup several times and made a bar graph of the results.



30 trials of tossing the cup were carried out.

a. How many trials of the experiment did the student carry out?

____

Based upon these trials:

P(Cup lands on Bottom) ≈ 5/30 = 1/6

P(Cup lands on Side) ≈ 15/30 = 1/2

P(Cup lands on Top) ≈ 10/30 = 1/3

b. Based upon this student's experiment, what are the estimates for each of the following probabilities (where P equals Probability):

P (Cup lands on Bottom):

____

P (Cup lands on Side):

____

P (Cup lands on Top):

____

These probabilities sum to 1.

c. What is the sum of the three probabilities from part b?

____

d. If another student were to toss a similar cup 120 times, about how many times would you expect the cup to land on its bottom?

If 120 tosses were done, you would expect 20 to land on the bottom, 60 to land on the side, and 40 to land on the top.

____

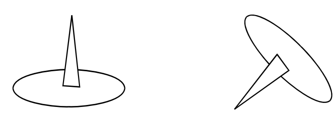
On its side?

____

On its top?

____

2. If a thumbtack is dropped onto a hard flat surface, it will land either "point up" or "point down."



Point up Point down

What probabilities do you think should be assigned to each of theses outcomes? Make a guess. Then design an experiment to approximate the experimental probability that a thumbtack will land point up.

Results will vary depending upon the type of thumbtack used. The outcomes are not equally likely.

For example, using a standard small-head thumbtack, we estimated the probability of the tack landing point up to be 0.7 (to the nearest tenth).