Finding Extraneous Solutions
ID: 9091

Activity Overview

Students will solve different types of equations step by step graphically. They will discover that some of the equations have an extraneous solution and they will investigate at which step in solving the equation that these "extra" solutions appear.

Topic: Rational & Radical Functions

- *Evaluate a rational function for any real value of the variable*
- *Evaluate a radical function for any real value of its variable.*
- *Solve radical equations and inequalities algebraically and check for extraneous roots.*

Teacher Preparation and Notes

- *This activity could be used in Algebra 2 as an extension after students have solved different types of equations. This activity could also be used in Precalculus as review of the different types of equations that students solved in Algebra 2.*
- *Begin by reviewing with students how to solve equations graphically by treating each side of an equation as a separate function—the solution to the equation is the functions' point of intersection.*
- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9091" in the keyword search box.***

Associated Materials

- *ExtraneousSolns_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Roots of Radical Equations (TI-84 Plus family) — 12214*
- *Solving Equations by Graphing (TI-84 Plus family) — 6342*
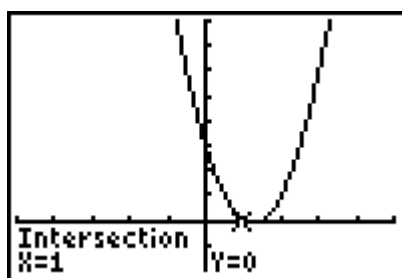- *Intersecting the Solution (TI-84 Plus family) — 11516*

Problem 1 – Solving a quadratic equation

Students will work through the solving of the equation $2x^2 + 3 = 5x$ graphically and step-by-step. They simply need to change the equations in **Y1** and **Y2** to represent the left- and right-hand sides, respectively, of the equation in each step. The functions can be altered by pressing $\boxed{y=}$. They will use the **intersect** command as instructed on the worksheet to find the intersection point of the two graphs.

Students will find that the *x*-coordinates of the intersection points remain the same each time the equation is manipulated graphically.

You may wish to work though this first problem with students if they are unfamiliar with solving equations graphically. Encourage them to write the solution(s)—i.e., the *x*-coordinates of the point(s) of intersection of functions **Y1** and **Y2**—on their worksheet.

The graphs of the functions at each step in the solution are shown at right and below.

> **Solve:** $2x^2 + 3 = 5x$
>
> Step 1: $2x^2 + 3 = 5x$
>
> Step 2: $2x^2 - 5x + 3 = 0$
>
> Step 3: $(2x - 3)(x - 1) = 0$
>
> Step 4: $x = \dfrac{3}{2}$ and $x = 1$

Step 1

Step 2

Step 3

Students can now check their solutions algebraically and by using a table of values. In the table of values, they should notice that at $x = 1$, the *y*-values are equal, and the same happens for $x = 1.5$.

On the Home screen, when students test if **Y1 = Y2**, a value of "1" means the equation is true, and a value of "0" means the equation is false.

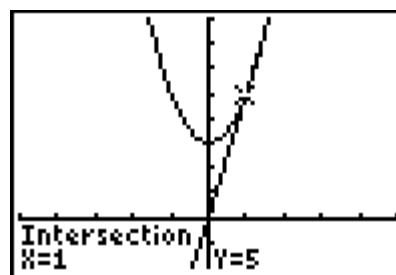Students should conclude that $x = 1$ and $x = 1.5$ are both solutions to the equation $2x^2 + 3 = 5x$.

| X	Y1	Y2
1	5	5
1.5	7.5	7.5
2	11	10
2.5	15.5	12.5
3	21	15
3.5	27.5	17.5
4	35	20

X=1

Problem 2 – Solving a radical equation

In Problem 2, students will approach the equation $\sqrt{x+11}+1=x$ in the same manner, but this time they will observe the appearance of an extraneous solution during the graphing of the functions in Step 3.

Again, the graphs of the functions at each step in the solution are shown below.

In graphing the equations for Step 4, students should observe that the graph of the function $y = (x-1)^2$ did not change position as it was altered to the function $y = x^2 - 2x + 1$. Ask students to explain why this happened. (The expressions $(x-1)^2$ and $x^2 - 2x + 1$ are equivalent—the former is merely the factored form of the latter. Thus, their graphs are identical.)
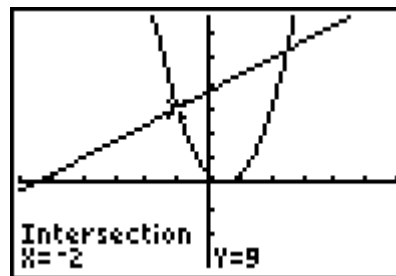
> **Solve:** $\sqrt{x+11}+1=x$
>
> Step 1: $\sqrt{x+11}+1=x$
>
> Step 2: $\sqrt{x+11}=x-1$
>
> Step 3: $x+11=(x-1)^2$
>
> Step 4: $x + 11 = x^2 - 2x + 1$
>
> Step 5: $0 = x^2 - 3x - 10$
>
> Step 6: $0 = (x-5)(x+2)$
>
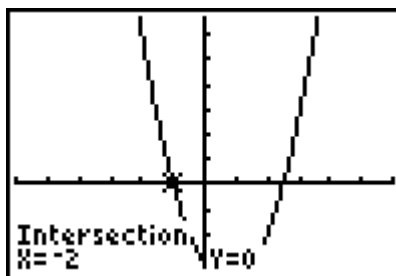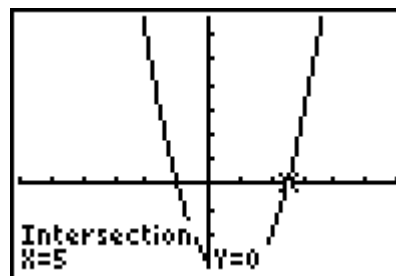> Step 7: $x = 5$ and $x = -2$
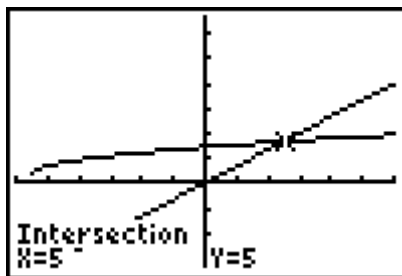
Step 1
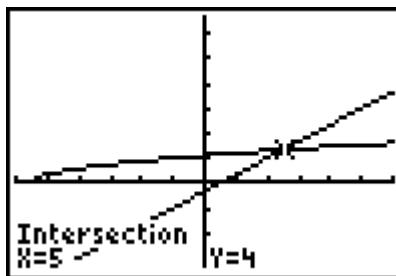
Step 2

Step 3
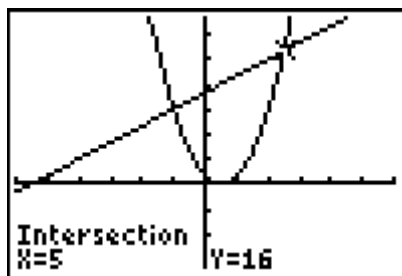
Step 4

Step 5

Step 6

Students should once again check their solutions—remind them to reset the functions **Y1** and **Y2** to be as they were in Step 1. This time, students should find that only *one* of their solutions, $x = 5$, makes the equation true. Tell them that $x = -2$ is called an "extra," or *extraneous*, solution.

If using Mathprint OS:

Students can enter Y1 and Y2 using the shortcut menus. To do this, press alpha f4 and select **Y1** or **Y2**. Then enter the value of the input.
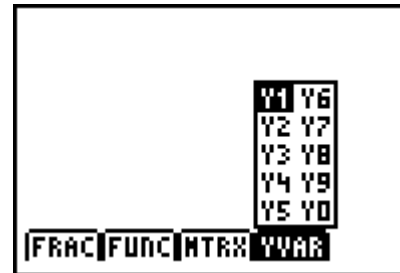
Extension

As an optional extension, Problem 3 provides students with a step-by-step solution to a rational equation. They are to use the graphing screen as done in the previous problems. Have them identify the step in which the extraneous solution appeared. They should also view the function table (the Home screen will not work) to verify whether the solutions are "true" solutions or extraneous ones. ($x = -3$ is a true solution; $x = 3$ is an extraneous solution—both expressions are undefined for $x = 3$.)

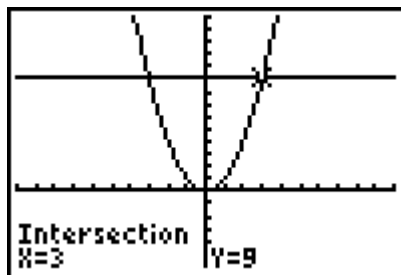**Solve:** $\dfrac{3x}{x-3} = \dfrac{2x-3}{x-3}$

Step 1: $\dfrac{3x}{x-3} = \dfrac{2x-3}{x-3}$

Step 2: $(3x)(x-3) = (2x-3)(x-3)$

Step 3: $3x^2 - 9x = 2x^2 - 9x + 9$

Step 4: $x^2 = 9$

Step 5: $x = -3$ and $x = 3$