Wrapping Functions – ID: 8257

By Mark Howell

Time required
90 minutes

Activity Overview

This activity introduces students to various functions of a circular angle. They are shown a unit circle and a point P that can be dragged around the circle. As the point is dragged, different measures are captured, including angle measures, linear distance, and the area of a sector. The activity concludes with them exploring the x- and y-coordinates of point P, giving rise to the graphs of the cosine and sine functions.

Concepts

- *Circular functions*
- *Radian angle measure*
- *Area of a sector of a circle*
- *Sine and cosine*

Teacher Preparation

This activity is appropriate in an advanced Algebra 2 or Precalculus setting. It is best used as an introduction to graphing the trigonometric functions sine and cosine.

- *Students should have an understanding of the graphs of ordered pairs, radian angle measure, and finding the area of a sector of a circle. They should also be able to name the four quadrants of the Cartesian plane.*
- *The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on page 5 for a preview of the student .tns file.*
- ***To download the .tns file and student worksheet, go to http://education.ti.com/exchange and enter "8257" in the search box.***

Classroom Management

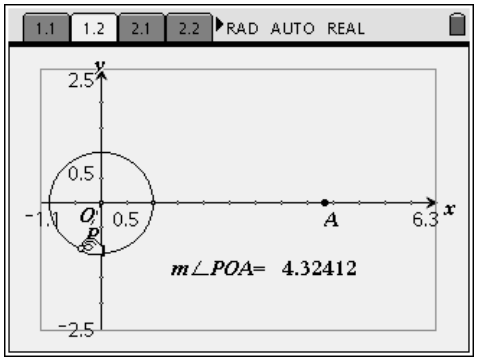
- *This activity is designed to have students explore **individually or in pairs**. It is important, though, that every student is given the opportunity to move point P around the circle and watch as the various graphs appear.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *After each capture and scatter plot is completed, students should return to the L&S page, position the cursor in the formula cell (gray) for Column A, and press ≈ enter twice to clear out the data. They should then repeat this process for Column B. If these steps are not followed, the device might run very slowly.*
- *It is also recommended that students move point P counterclockwise slowly around the circle only once. This will also limit the amount of data that is accumulated in the L&S application.*

TI-Nspire™ Applications

Graphs & Geometry (G&G), Lists & Spreadsheet (L&S), Notes

Problem 1

This problem introduces students to the idea of transferring the measure of an angle to the *x*-coordinate of a point on the *x*-axis. This activity should not be skipped—many students fail to understand the graphs of trigonometric functions simply because they miss this connection.



Page 1.2

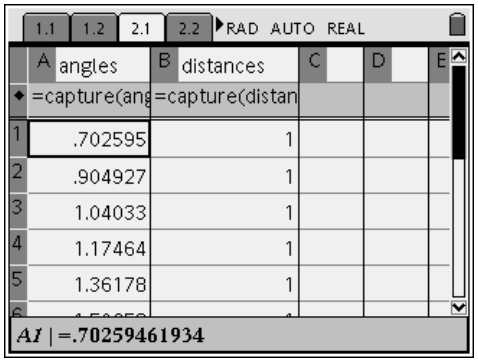
Solutions

1. to the right; increase
2. crossing over the positive *y*-axis, at (0, 1); crossing over the negative *x*-axis, at (–1, 0); crossing over the negative *y*-axis, at (0, –1)
3. The angle measure starts over again, from 0, and point *A* moves back to just near the origin; about 6.28 radians (2π)
4. It moves to the left, since the angle is now decreasing.

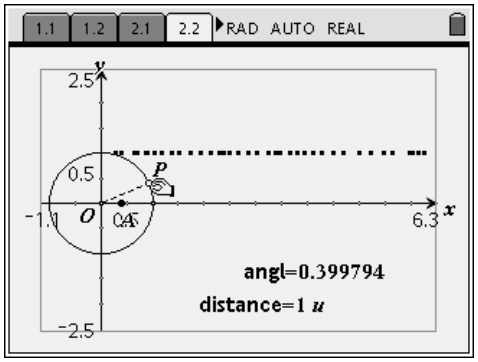
Problem 2

Problem 2 presents a simple function of the angle that students investigated in the previous problem—the distance of the point on the circle from the origin. It should come as no surprise that this is a constant function (as the radius of the circle is always 1). The purpose of this exploration is to encourage students to begin thinking about what such a graph would look like, to prepare them for later exercises.

The data in this problem is collected by students <u>manually</u> by pressing ctrl + . *when they wish to capture values. Students should clear out the data in Columns A and B of the spreadsheet on page 2.1 before proceeding to Problem 3.*



Page 2.1



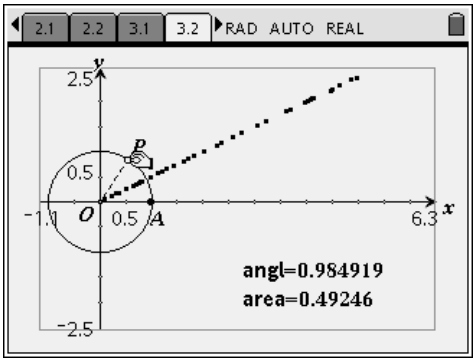
Page 2.2

Solutions

1. The distance of point *P* to the origin is constant—it is equal to the radius of the circle (1 unit) regardless of the measure of the angle; $y = 1$.
2. The points of the scatter plot move up 1 unit, to lie on the horizontal line $y = 2$.

Problem 3

In this problem, students explore a more interesting function of the same angle considered in the previous two problems, namely, the area of a circular sector swept out by $\overline{PO}$. This function is linear since the area of a sector with a central angle θ (in radians) is given by the formula $A = \frac{1}{2}r^2\theta$.



Page 3.2

In our unit circle, the radius is constant (and equal to 1), so the area of the sector is directly proportional to the measure of the angle.

The data in this and subsequent problems is collected automatically; students should still clear out the data from the L&S page before moving to the next problem.

Solutions

1. 0

2. It increases from 0 to about 0.785 (or $\frac{\pi}{4}$).

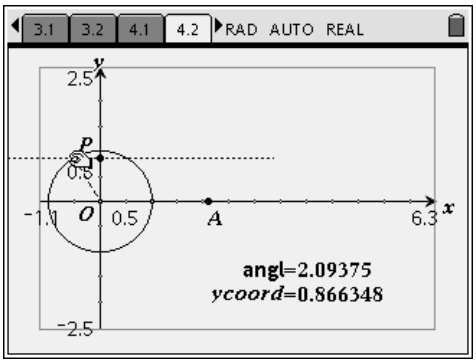
3. twice as high; three times as high

4. The points still lie on a line (the graph is linear) but it is steeper. The slope of the line is 2 instead of $\frac{1}{2}$.

Problem 4

In Problem 4, students begin to look at the behavior of the sine function. The focus here is on the *behavior* of the function; not its graph. When they drag point *P*, it should be clear by the movement of the point on the *y*-axis (and the line through that point) that the behavior of this function obtains all values between –1 and 1.

In the next problem, they will explore the graph of these points.



Page 4.2

Solutions

1. 1; about 1.57 (or $\frac{\pi}{2}$)

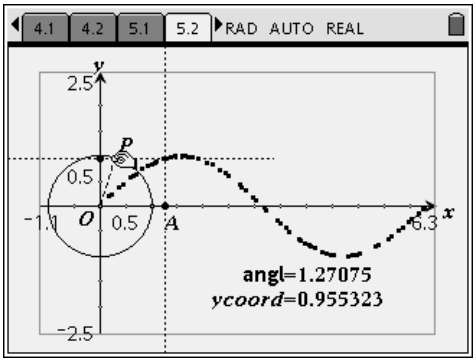
2. –1; about 4.71 (or $\frac{3\pi}{2}$)

3. about 3.14 (or π)

4. decreasing: from 1.57 to 4.71; increasing: from 0 to 1.57 and from 4.71 to 6.28

Problem 5

In this problem, students create a graph of the sine function by grabbing and dragging point *P* around the circle on page 5.2. They are asked to examine the same questions they did in Problem 4, in a slightly different manner.



Page 5.2

Solutions

1. highest: (1.57, 1)

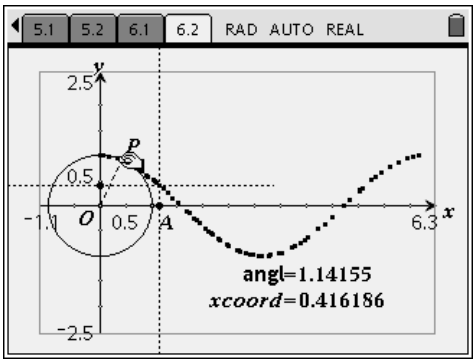
2. lowest: (4.71, –1)

3. (3.14, 0)

4. decreasing: II and III; increasing: I and IV

Problem 6

This problem provides students with the ability to create and explore the cosine graph on page 6.2. They are asked the same questions as in Problem 5, and are further asked to compare the two graphs.



Page 6.2

Solutions

1. highest: (0, 1) and (6.28, 1); lowest (3.14, –1)

2. (1.57, 0) and (4.71, 0)

3. decreasing: I and II; increasing: III and IV

4. Answers may vary. Possible answers: both have a maximum value of 1 and minimum of –1, both end up at the same height at which they started; the cosine graph is a translation of the sine graph about 1.57 (or $\frac{\pi}{2}$) units to the left.

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(Student)TI-Nspire File: *PreCalcAct1_WrappingFxns_EN.tns*