Mr. Taylor, I Presume?

MrTaylorIPresume.tns

Name ______________________

Class ______________________

Introduction

Polynomials are easy to integrate, to differentiate, and even to tell jokes to (they always laugh!). Wouldn't it be nice if it were possible to transform a very difficult function into a nice, "easygoing" polynomial? Of course it would! But how?

Believe it or not, it is possible to determine any polynomial of the form $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$ by just knowing the value of its derivatives at a point.

Taylor Polynomials Centered at Zero

For example, find a polynomial $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ if $P(0) = 1$, $P'(0) = 3$, $P''(0) = 6$ and $P'''(0) = 9$.

The first step it to find the derivatives:

$$P'(x) = a_1 + 2a_2x + 3a_3x^2 \qquad P''(x) = 2a_2 + 6a_3x \qquad P'''(x) = 6a_3$$

Since we know the value of each derivative when $x = 0$, we can determine the a_n terms:

$$P'(0) = 1 = a_1 + 2a_2(0) + 3a_3(0)^2 \qquad P''(0) = 6 = 2a_2 + 6a_3(0) \qquad P'''(0) = 9 = 6a_3$$

$$1 = a_1 \qquad 6 = 2a_2 \qquad \frac{9}{6} = a_3$$

$$\frac{6}{2} = a_2$$

So the polynomial would be $P(x) = 1 + 1x + \frac{6}{2}x^2 + \frac{9}{6}x^3$.

Notice that the numerator of the a_n term is $f^{(n)}(0)$ and the denominator is $n!$. This will come in handy in the next exercise.

While this is amazing, it is important to note that this polynomial is centered at the value $x = 0$ because that is where we calculated the derivative values.

Enough chatter, let's get to it!

Find a polynomial of degree four that approximates $f(x) = \ln(x + 5)$ when $x = 0$.

Find the values of $f(x)$, $f'(x)$, $f''(x)$, $f'''(x)$ and $f^{(4)}(x)$ when $x = 0$:

$f(0) =$

$f'(0) =$

$f''(0) =$

$f'''(0) =$

$f^{(4)}(0) =$

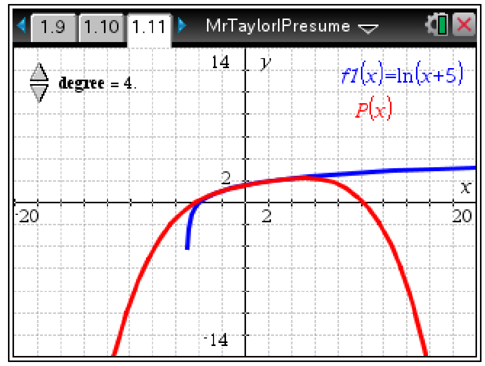
Substitute the derivative values into the numerator and $n!$ into the denominator of each term in $P(x)$. On page 1.7, you can check your answer by manipulating the slider to degree 4. Simplify the polynomial.

$P_4(x) =$

1. Using the spreadsheet on page 1.9, decide where the polynomial and function agree or nearly agree in value.

Notice that the values of the Taylor polynomial and the values of the function do not agree everywhere on the graph. In fact, they are closest where the derivative was evaluated. This is called the *center* of the polynomial.

2. On what interval does the polynomial best approximate the original function?



3. What do you notice about the interval as the degree of the polynomial changes?

Taylor Polynomial Not Centered at Zero

Now it is time to leave the origin and find Taylor polynomials whose centers are not zero.

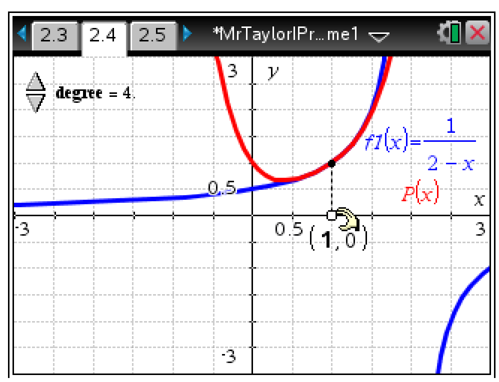
The only adjustment is to change x^n to $(x - a)^n$ where a is the center of the approximation.

Find a 4th degree Taylor polynomial for $f(x) = \dfrac{1}{2-x}$ centered at $x = 1$.

4. Find the values of $f(1)$, $f'(1)$, $f''(1)$, $f'''(1)$ and $f^{(4)}(1)$ when $x = 1$. Substitute the derivative values into the numerator and the n! into the denominator of each term in $P(x)$. Remember to write $(x - 1)^n$ instead of x^n. Simplify your polynomial.

$P(x) =$

Examine the graphs of the function and the Taylor polynomial. Pay close attention to where $x = 1$.



Adjust the center of the Taylor polynomial by dragging the point on the x-axis on page 2.4.

5. What do you notice about the interval as the center is changed?

