REAL LIFE REAL WORLD Activity: Carpentry
Right Triangles and Circles

Topic: Right Triangles & Circles **Grade Level:** 7 - 12
Objective: To solve geometry problems with concepts used by carpenters.
Time: 60-90 minutes

Introduction

There are many applications of geometry that are used by carpenters. The concept of right triangles is used, since carpenters need to be certain that walls are straight and corners are square. A second application is using the properties of circles to create archways and other curved features for windows and doorways.

Discuss with Students

Review the Pythagorean theorem: In a right triangle, $A^2 + B^2 = C^2$, if A and B are the lengths of the legs and C is the length of the hypotenuse.

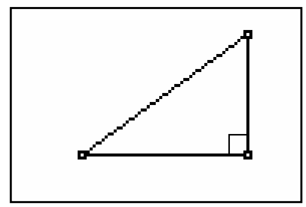

Figure 1

1. In Figure 1, which side would be labeled C?

2. If the legs in Figure 1 are measured as 3.1 and 5.6, use the Pythagorean Theorem to solve for C.

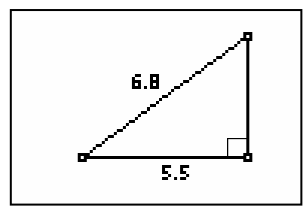

Figure 2

3. In Figure 2, identify which sides are A, B, and C and solve for the missing side.

There are several relationships between chords and diameters. Review these with students. *If desired, have students measure distances to discover these relationships.*

- If a diameter (or radius) is perpendicular to a chord, then it bisects the chord.
- If a diameter (or radius) bisects a chord, then it is perpendicular to the chord.
- The perpendicular bisector of a chord will pass through the center of the circle.

For Activity 2, students may need assistance developing the equation in step 1.

Discuss With Student Answers

1. The side across from the right angle. See Figure 3.

2. $C = 6.4$.

3. Use 5.5 for either A or B and 6.8 for C. The missing side is 4.0

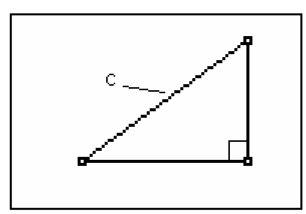

Figure 3

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Student Page Answers

Activity 1

6. The right triangles in the chart should confirm the Pythagorean Theorem so $A^2 + B^2$ will equal C^2.

7. The triangles in which $A^2 + B^2 < C^2$ will be obtuse triangles.

8. The triangles in which $A^2 + B^2 > C^2$ will be acute triangles.

9. A carpenter could lay a piece of wood across a corner of the room to make a triangle. Measure the wood (C) and measure the distances from the corner to each end of the wood (A and B). Calculate $A^2 + B^2$ and C^2 to determine if the corner makes a right angle.

Activity 2

1. The legs of the right triangle are $\frac{W}{2}$ and $R - D$. The hypotenuse is R. The equation is $\left(\frac{W}{2}\right)^2 + \left(R - D\right)^2 = R^2$.

5. When the values are substitute for $\frac{W}{2}$ and D, the equation will look like this (student values will vary):

   $\left(1.8\right)^2 + \left(R - 1.3\right)^2 = R^2$. Expand the binomial $\left(R - 1.3\right)^2$, cancel the R^2 from each side and solve for R. In this example, R will equal 1.9.

7. When the radius is first measured it may look like Figure 4. When it is adjusted it will look like Figure 5.

8. The desired circle is shown in Figure 6. In a circle, all radii are equal.

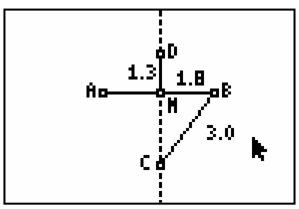

Figure 4

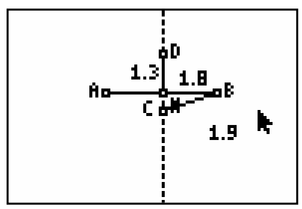

Figure 5

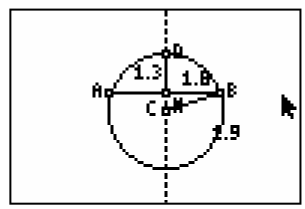

Figure 6

Technology Reference

This activity uses the following Cabri Jr. functions:

F2	Segment	F3	Perpendicular	F5	Hide/Show
	Triangle		Midpoint		Alph-Num
					Display
					Measure: D. & Length
					Measure: Angle
					Calculate

Refer to “Getting Started with Cabri Jr.” for more details.

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Name_____________________________________ Date______________

REAL LIFE REAL WORLD Activity: Carpentry

Carpenters often use the principles of geometry in their work. The concept of right triangles is used because carpenters need to be sure that walls are straight and corners are square. The Pythagorean Theorem relates the lengths of sides of a right triangle. Carpenters also use curves to create archways above windows and doors using the properties of circles. In this activity, you will use Cabri Jr. or Cabri II+ to make measurements and solve problems based on carpentry situations.

Activity 1

In order to determine if corners are square, a carpenter needs to decide if a triangle is a right triangle.

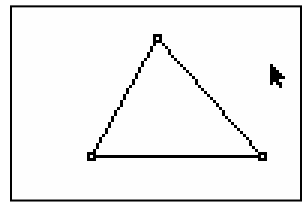

Figure 7

1. Construct a triangle using the triangle tool. See Figure 1.

2. Measure the parts of the triangle. See Figure 2.
   - Measure the three side lengths.
   - Measure the three interior angles.
   - Use this information to classify the triangle as an *acute*, *right* or *obtuse* triangle.
   - Fill in the type of triangle and the three side lengths in the first row of the chart below. Use the *longest* side of the triangle as side *C*.

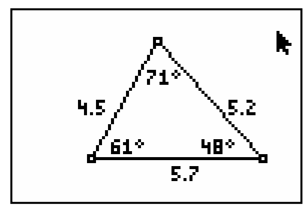

Figure 8

3. Use the Calculate tool to find the value of $A^2 + B^2$ and fill it in the chart.

4. Use the Calculate tool to find the value of C^2 and fill it in the chart.

5. Now that the first row of the chart is filled in, use the Hand Cursor tool to drag one vertex of the triangle to change its shape. Fill in the chart for the new triangle. Make sure to create some triangles of each type (acute, obtuse and right) and complete the chart.

TYPE of TRIANGLE	A	B	C	A^2 + B^2	C^2

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6. Do the right triangles in the chart confirm the Pythagorean Theorem? (Does $A^2 + B^2 = C^2$ for your right triangles?)

7. What do you notice about the triangles where $A^2 + B^2 < C^2$?

8. What do you notice about the triangles where $A^2 + B^2 > C^2$?

9. How could a carpenter use this technique to determine if a corner made by two walls is a right angle?

Activity 2

Archways are sometimes used above doors and windows. One shape used is a segment of a circle, which is the part of a circle cut off by a chord (see Figure 3). A carpenter who knows the available width and height of the archway can calculate the needed radius to mark off the circle.

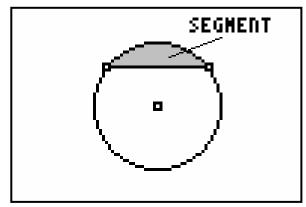

Figure 3

A right triangle is formed by half of the chord, a radius drawn to one endpoint of the chord, and a segment from the center of the circle to the midpoint of the chord (see Figure 4). In this activity, you will make measurements and use the Pythagorean Theorem to find the length of the radius of the circle.

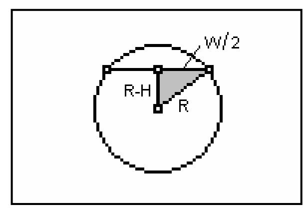

Figure 4

1. Use the labeled parts of the diagram and the Pythagorean Theorem to write an equation using the variables W, H, and R.

2. Construct a horizontal segment to represent the Width (W) of the archway and label it $\overline{AB}$. This is the chord of the desired circle. See Figure 5.

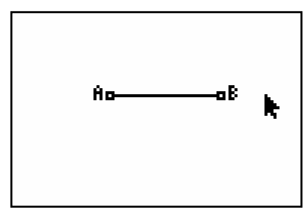

Figure 5

3. Construct a perpendicular segment through the first segment's midpoint to represent the available Height (H). See Figure 6.
   - Find the midpoint of $\overline{AB}$ and label it *M*.
   - Construct a line perpendicular to $\overline{AB}$ through *M*.
   - Create a new segment with one endpoint at *M* and another endpoint on the perpendicular line. Label the second endpoint *D*.
   - If desired, use the Display tool to show the perpendicular as a dashed line.

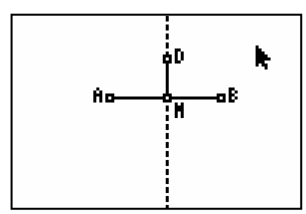

Figure 6

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4. Make measurements.
   - Measure the distance from *B* to *M*. This is half of the chord of the circle (W/2).
   - Measure the distance from *M* to *D*. This is the height of the segment of the circle (H).

5. Use the equation you wrote in step 1 with the values you found for W/2 and H. Solve for R, the needed radius.

6. Construct a segment with one endpoint at *B* and the second endpoint on the perpendicular line. Label the second endpoint *C*. This will become the radius of the needed circle. See Figure 7.

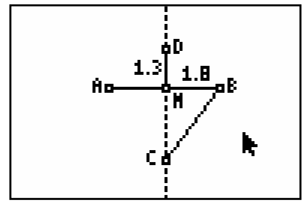

Figure 7

7. Adjust the radius $\overline{BC}$ so it is the desired length.
   - Measure the length of $\overline{BC}$.
   - Use the Hand Cursor to drag point *C* along the perpendicular line until its length is the value you found in step 5.

8. Create a circle. A carpenter can use a length of string like a compass to mark off the needed circle from the correct point *C*.
   - Use the Circle tool with center *C* and radius point at *B*.
   - The circle will also pass through points *D* and *A*.
   - Confirm that the distances from *C* to *A* and *C* to *D* equal the distance from *C* to *B*. What property of circles explains this?

Extensions & Resources

Extension 1
A commonly used carpentry tool is the Carpenter's Square. Explore the uses of this tool.
http://mathdemos.gcsu.edu/mathdemos/carpcircle/carpcircle.html

Extension 2
Another common shape for an archway is half of an ellipse. Investigate how this shape is created on a worksite.
http://mathdemos.gcsu.edu/mathdemos/carpenterellipse/carpellipse_main.html

Extension 3
There are several sets of side lengths that guarantee a right triangle, known as Pythagorean Triples. One of them is 3-4-5. Ancient peoples were known to use a knotted rope with 12 knots to create a right triangle, and carpenters can create a wooden 3-4-5 triangle to use to construct right angles. Investigate this and other Pythagorean triples.