Exploring Complex Roots

Student Worksheet

Name ______________________

Class ______________________

Problem Statement

While we have developed algebraic tools for finding complex solutions of quadratic equations, how do these solutions relate to the graphical representation of the parabola? In this exploration, we will investigate how simple transformations of the graph of a parabola can yield the same complex solutions we usually get by algebraic manipulation.

Finding Real Solutions

1. Recall that the real solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are the x-intercepts of the graph of the function $f(x) = ax^2 + bx + c$ and can be represented by one of two situations. Advance to page 1.3 by pressing ctrl and the right side of the NavPad.

a. Examine the graph of the function $f(x) = x^2 + 2x - 8$ and locate the exact zeros by tracing until **zero** displays. What are the roots of the equation $x^2 + 2x - 8 = 0$?

b. Next, locate the vertex of the parabola by tracing until **minimum** appears on the screen. Give the axis of symmetry and the coordinates of the vertex of the graph of $f(x) = x^2 + 2x - 8$.

c. Describe the relationship you see between the location of the real zeros and the axis of symmetry.

2. Advance to page 2.1 of the file *CollegeAlg_ComplexRoots.tns* by pressing ctrl and the right side of the NavPad. Examine the graph of the function $f(x) = x^2 - 4x + 4$ and locate the exact zeros and vertex as you did in question 1.

a. How many distinct real zeros are there? Give the distinct real zeros.

b. Give the axis of symmetry and the coordinates of the vertex of the graph of $f(x) = x^2 - 4x + 4$.

c. Describe the relationship you see between the location of the real zeros and the axis of symmetry.

Finding Complex Solutions with Imaginary Parts

3. Advance to page 3.1 of the file *CollegeAlg_ComplexRoots.tns* by pressing ctrl and the right side of the NavPad, and view the graph.

a. Locate the vertex of the parabola by tracing until **minimum** appears on the screen. Give the axis of symmetry and the coordinates of the vertex of the graph of $f(x) = x^2 + 4x + 5$.

b. Is it possible to find the solutions to $x^2 + 4x + 5 = 0$ as you did earlier in questions 1 and 2? Explain.

c. Describe how you can decide whether or not a quadratic equation of the form $ax^2 + bx + c = 0$ has real solutions by looking at the graph of $f(x) = ax^2 + bx + c$.

d. Advance to page 3.2 of the file *CollegeAlg_ComplexRoots.tns*, find the complex solutions of $x^2 + 4x + 5 = 0$ using the **cSolve(** command, and express them in $a + bi$ form. Give the complex solutions of $x^2 + 4x + 5 = 0$.

Visualizing Complex Roots

4. Advance to page 3.3 of the file *CollegeAlg_ComplexRoots.tns*, use the k value in the vertex form of the graph of the function $f(x) = x^2 + 4x + 5 = (x + 2)^2 + 1$, and reflect the parabola over the line $y = k$. Recall that to reflect the graph of $f(x) = a(x - h)^2 + k$ over $y = k$, we can first reflect it about the x-axis, getting $-(a(x - h)^2 + k) = -a(x - h)^2 - k$. However, the vertex of the graph of this expression would be $2k$ units below (or above, depending on the sign of k) the old vertex, so we need to adjust by adding $2k$ to the function's expression. This gives $-a(x - h)^2 - k + 2k = -a(x - h)^2 + k$. So here we simply need to change the sign in front of the $a(x - h)^2$ part of the expression.

a. Enter the reflected function for **f2**(x) and graph it. Sketch and label your graph below.

Complex numbers of the form $a + bi$ are graphed by using the x-axis as the real axis for a and the y-axis as the imaginary axis for bi. Advance to page 3.4 and plot the complex roots by selecting **MENU** > **Points & Lines** > **Point**. Move the pencil to each complex root (***intersection point*** will display) and press [enter] or [click].

Now draw the segment joining the plotted complex roots by selecting **MENU** > **Points & Lines** > **Segment**. Cursor over to each plotted complex root and press [enter] or [click] and [esc] to exit this menu. Locate the midpoint of the segment joining the plotted complex roots by selecting **MENU** > **Construction** > **Midpoint**. Cursor over to the segment and press [enter] or [click] and [esc] to exit this menu.

Rotate clockwise the segment joining the plotted complex roots about its midpoint by selecting **MENU** > **Transformation** > **Rotation**. Select the segment, then select the center point of the rotation (segment midpoint), and then select three points that determine a clockwise rotation by 90° (top endpoint and midpoint of the segment and any other point on the x-axis for the rotation angle.

b. Describe where the endpoints of the rotated segment joining the plotted complex roots are located.

c. Locate the zeros of the reflected function **f2**(x) as you did earlier. What are the coordinates of the zeros of the reflected function?

d. What can you conclude about the location of the roots of the function $f(x) = x^2 + 4x + 5$ and the endpoints of the rotated segment?

e. Explain how the complex roots of a quadratic equation can be found using the graph of its reflected function.

