Points of Concurrency in Triangles

by – Marco A. Gonzalez

Activity overview

In this activity, students will use their Nspire handhelds to discover the different points of concurrencies in triangles. The students will take advantage of the dynamic capabilities of this very unique handheld to discover the circumcenter, incenter, and centroid of triangles.

Concepts

Triangles, points of concurrency, circumcenter, incenter, centroid, perpendicular bisectors, angle bisectors, medians

Teacher preparation

This activity is designed for use in a Geometry classroom. Prior to beginning this activity, students should have prior knowledge of perpendicular bisectors, angle bisectors, medians, vertices of triangles, and the different types of triangles.

Classroom management tips

This activity is intended to be a teacher-led activity in which students will make their own discoveries pertaining to the points of concurrency of triangles. Students should be encouraged to make predictions of how the points of concurrency will behave if the vertices of the triangle are translated.

TI-Nspire Applications

Graphs and Geometry Page

Step-by-step directions

The following step-by-step directions are the same as the student directions.

Circumcenter

First, open a Graphs and Geometry page by pressing (home) and scrolling to Graphs and Geometry or simply by pressing (2).

Now, we are going to hide the x & y axes by pressing the (menu) , (2) : View, (4) :Hide Axes .

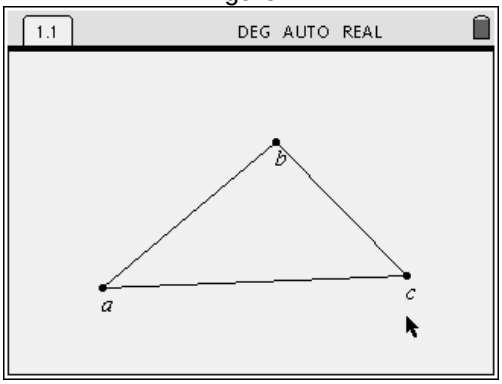
Let's hide the entry line as well by pressing (menu) , (2) : View, (6) : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press (ctrl) (G). This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen.

Now press (menu), (8): Shapes, (2): Triangle.

Figure 1



Using the NavPad, move the cursor to a lower left part of the screen and press ⊙. Before moving the cursor, label the point by pressing the letter Ⓐ. Now move the cursor to another part of the screen, and press ⊙. Again, before moving the cursor, label the point by pressing Ⓑ. Move the cursor to another part of the screen and press ⊙. Again, before moving the cursor, label the point by pressing Ⓒ. Press esc . (see figure 1)

Now let's calculate the perpendicular bisectors of each of the sides of the triangle. Press menu, 9: Construction, 3: Perpendicular Bisector. Using the NavPad, move the cursor to side AB and press enter. Using the NavPad, move the cursor to side BC and press enter . Now, move the cursor to side AC and press enter. Press esc. (see figure 2)

To determine the point of concurrency, we need to find the point of intersection between the perpendicular bisectors. To do this, press menu, 6 : Points & Lines, 3 : Intersection Points. Move the NavPad to any of the perpendicular bisectors and press ⊙ . Move the cursor to another perpendicular bisector and press ⊙. Before you move the cursor press Ⓠ.

Now let's hide the perpendicular bisectors. Move the cursor to one of the perpendicular bisectors and press ctrl menu. Scroll down or simply press 3: Hide/Show. Repeat the process for the other two perpendicular bisectors. (see figure 3)

Point "Q" is called the circumcenter of the triangle.

Now press menu, 7: Measurement, 1: Length. Move the cursor to point A and press ⊙. Now move the cursor to point Q and press ⊙. There should be an "ghost" image of a number. Move the image to a free part of the screen and lock the number in place by pressing ⊙. Move the cursor to point B and press ⊙. Now move the cursor to point Q and press ⊙. There should be a "ghost" image of a number. Move the image below the first measurement number and lock it in place by pressing ⊙. Move the cursor to point C and press ⊙. Now move the cursor to point Q and press ⊙. There should be a "ghost" image of a number. Move the image below the other two measurements and lock it in place by pressing the ⊙. (see figure 4)

Now, use the NavPad to move the cursor to point C

Figure 2

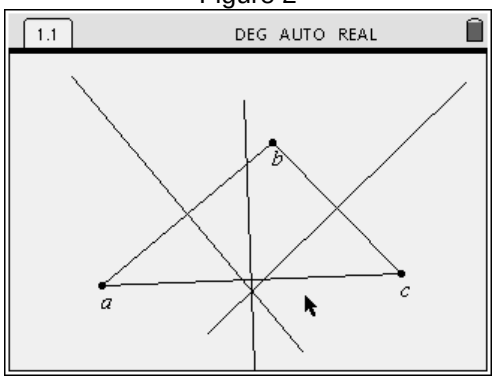


Figure 3

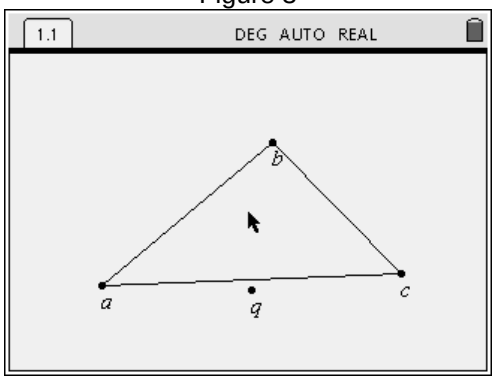


Figure 4

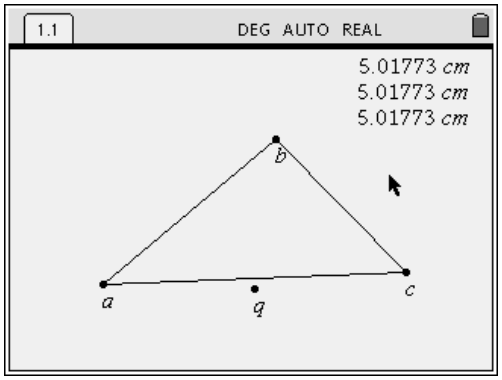


Figure 5

of the triangle. Click ⓔ and hold for 1.7 seconds until the point is "grabbed". Now translate the point and notice the measurements on the screen. (see figure 5)

What conjecture can you make about the circumcenter of a triangle?

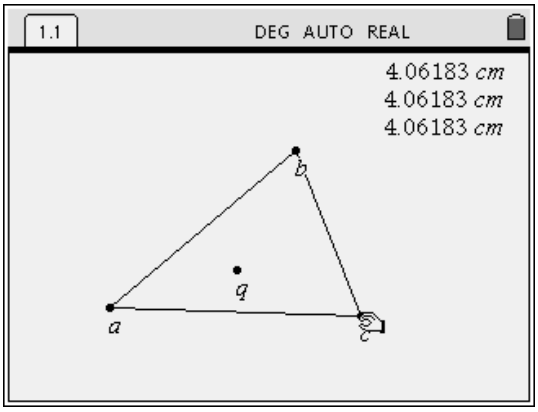


Figure 6

Incenter

Open a new Graphs and Geometry page by pressing ⓗ and scrolling to Graphs and Geometry or simply by pressing ⓶.

Now, we are going to hide the x & y axes by pressing the ⓜ , ⓶ : View, ④ :Hide Axes .

Let's hide the entry line as well by pressing ⓜ , ⓶ : View, ⑥ : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press ⓒ Ⓖ. This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen. (see figure 6)

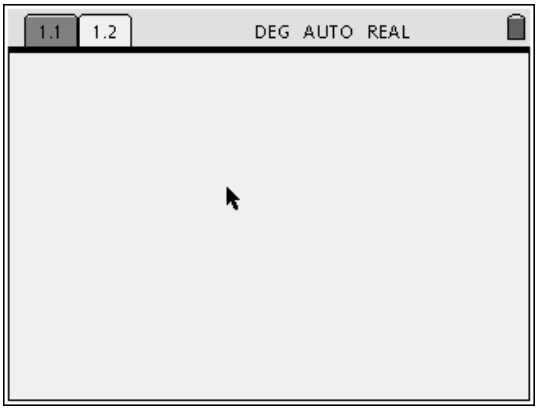


Figure 7

Now press ⓜ, ⑧: Shapes, ⓶: Triangle.

Using the NavPad, move the cursor to any part of the screen and press ⓔ. Before moving the cursor, label the point by pressing the letter Ⓐ. Now move the cursor to another part of the screen, and press ⓔ. Again, before moving the cursor, label the point by pressing Ⓑ. Move the cursor to another part of the screen and press ⓔ. Again, before moving the cursor, label the point by pressing Ⓒ. Press ⓔ . (see figure 7)

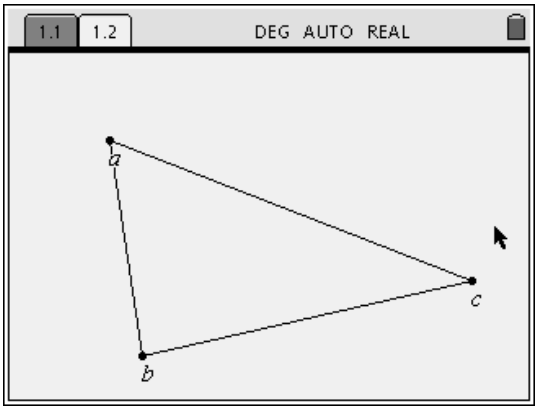


Figure 8

Now press ⑨: Construction, ④: Angle Bisector. Move the cursor to point B and press ⓔ, move the cursor to point A and press ⓔ, move the cursor to point C and press ⓔ. A segment bisecting angle A should be drawn. (see figure 8) Repeat the process for the other two angles. After all three angle bisectors are drawn, press ⓔ. (see figure 9)

To determine the point of concurrency, we need to find the point of intersection between the angle bisectors. To do this, press (menu), (6) : Points & Lines, (3) : Intersection Points. Move the NavPad to any of the angle bisectors and press (•) . Move the cursor to another angle bisector and press (•). Before you move the cursor press (esc).

Now let's hide the angle bisectors. Move the cursor to one of the angle bisectors and press (ctrl)(menu). Scroll down or simply press (3): Hide/Show. Repeat the process for the other two angle bisectors. (see figure 10)

Point "Q" is called the Incenter of the triangle.

Now, use the NavPad to move the cursor to point C of the triangle. Click (•) and hold for 1.7 seconds until the point is "grabbed". Now translate the point. What do you notice?

Extension

Draw a perpendicular segment from the Incenter to each of the sides of the triangle. Find a point of intersection between the line segment and each of the sides. Now measure the distance between the Incenter and each of the points of intersection on the sides of the triangle. What do you notice?

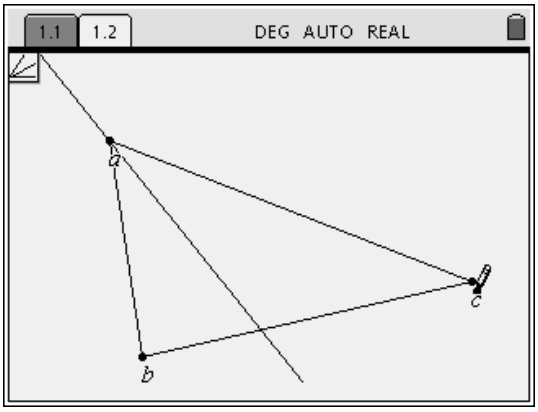


Figure 9

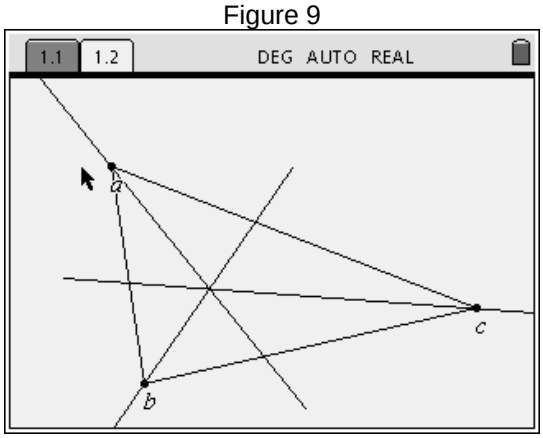


Figure 10

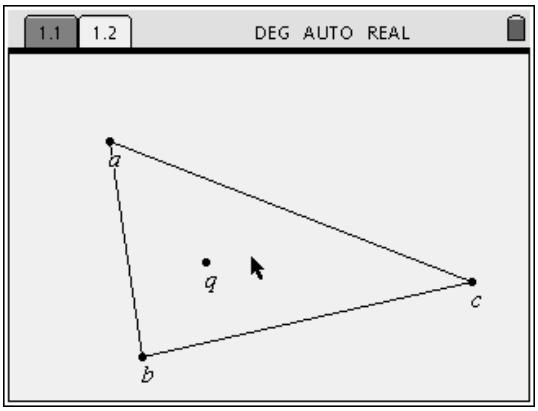


Figure 11

Centroid

Open a new Graphs and Geometry page by pressing (home) and scrolling to Graphs and Geometry or simply by pressing (2).

Now, we are going to hide the x & y axes by pressing the (menu) , (2) : View, (4) :Hide Axes .

Let's hide the entry line as well by pressing menu , 2 : View, 6 : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press ctrl G. This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen.

Now press menu, 8: Shapes, 2: Triangle.

Using the NavPad, move the cursor to any part of the screen and press ·. Before moving the cursor, label the point by pressing the letter A. Now move the cursor to another part of the screen, and press ·. Again, before moving the cursor, label the point by pressing B. Move the cursor to another part of the screen and press ·. Again, before moving the cursor, label the point by pressing C. Press esc . (see figure 11)

Now, let's determine the midpoints of each of the sides of the triangle. Press menu, 9: Construction, 5: Midpoint. Move the cursor to point A of the triangle and press ·. Move the cursor to point B and press ·. Since the cursor is already at point B, press · and move the cursor to point C. Press ·. Since the cursor is already at point C, press · and move the cursor back to point A. Press ·. Press esc. You triangle should have a midpoint in each of its sides. (see figure 12)

Now, we need to draw the medians of the triangle. Press menu, 6: Points & Lines, 5: Segment. Move the cursor to point A on the triangle and press ·. Now move the cursor to the midpoint of the side opposite vertex A of the triangle and press ·. Move the cursor to point B on the triangle and press ·. Now move the cursor to the midpoint of the side opposite vertex B of the triangle and press ·. Move the cursor to point C on the triangle and press ·. Now move the cursor to the midpoint of the side opposite vertex C of the triangle and press ·. (see figure 13)

To determine the point of concurrency, we need to find the point of intersection between the medians of the triangle. To do this, press menu, 6 : Points & Lines, 3 : Intersection Points. Move the NavPad to any of the three medians and press · . Move the cursor to another median and press ·. Before you move the cursor press Q.

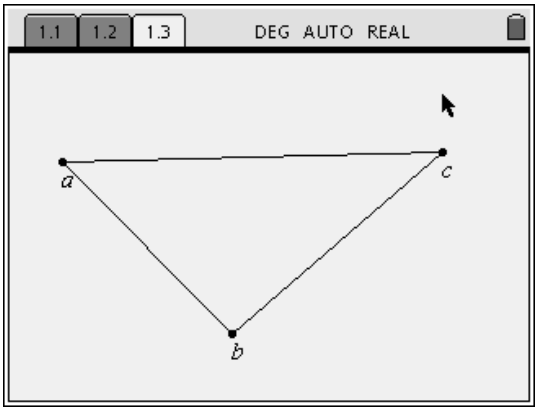


Figure 12

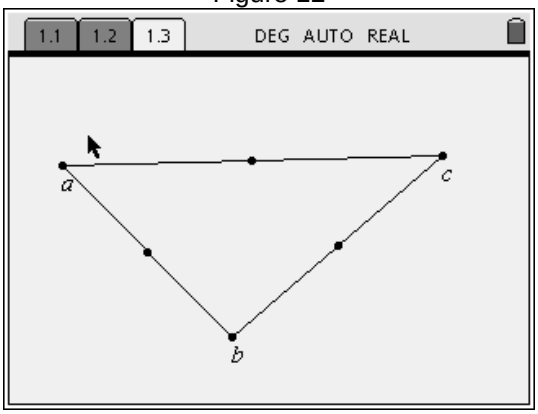


Figure 13

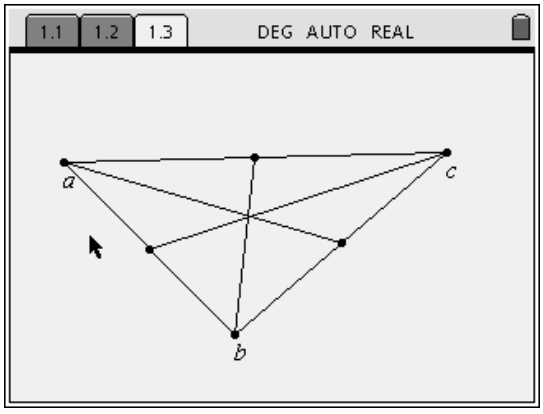


Figure 14

Now let's hide the medians. Move the cursor to one of the medians and press ctrl menu. Scroll down or simply press 3: Hide/Show. Repeat the process for the other two medians. (see figure 14)

Point "Q" is called the Centroid of the triangle.

Note: The centroid of a triangle is known as the "center of gravity" because this is where the triangle will balance.

Now, use the NavPad to move the cursor to point C of the triangle. Click and hold for 1.7 seconds until the point is "grabbed". Now translate the point. What do you notice?

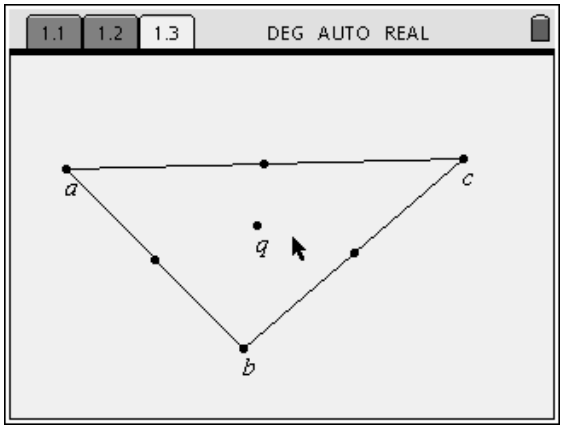


Figure 1

Activity extensions

- *The student should be encouraged to make predictions during the process of this activity.*
- *Students should be encouraged to explore other possible extensions of points of concurrencies (e.g. orthocenter, etc.)*
- *If time permits, the student should be encouraged to experiment and attempt to apply these points of concurrency to other polygons.*

Student TI-Nspire Document

Circumcenter

First, open a Graphs and Geometry page by pressing and scrolling to Graphs and Geometry or simply by pressing 2.

Now, we are going to hide the x & y axes by pressing the menu , 2 : View, 4 :Hide Axes .

Let's hide the entry line as well by pressing menu , 2 : View, 6 : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press ctrl G. This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen.

Now press menu, 8: Shapes, 2: Triangle.

Using the NavPad, move the cursor to a lower left part of the screen and press ·. Before moving the cursor, label the point by pressing the letter A. Now move the cursor to another part of the screen, and press ·. Again, before moving the cursor, label the point by pressing B. Move the cursor to another part of the screen and press ·. Again, before moving the cursor, label the point by pressing C. Press esc . (see figure 1)

Now let's calculate the perpendicular bisectors of each of the sides of the triangle. Press menu, 9: Construction, 3: Perpendicular Bisector. Using the NavPad, move the cursor to side AB and press enter. Using the NavPad, move the cursor to side BC and press enter . Now, move the cursor to side AC and press enter. Press esc. (see figure 2)

To determine the point of concurrency, we need to find the point of intersection between the perpendicular bisectors. To do this, press menu, 6 : Points & Lines, 3 : Intersection Points. Move the NavPad to any of the perpendicular bisectors and press · . Move the cursor to another perpendicular bisector and press ·. Before you move the cursor press Q.

Now let's hide the perpendicular bisectors. Move the cursor to one of the perpendicular bisectors and press ctrl menu. Scroll down or simply press 3: Hide/Show. Repeat the process for the other two perpendicular bisectors. (see figure 3)

Point "Q" is called the circumcenter of the triangle.

Now press menu, 7: Measurement, 1: Length. Move the cursor to point A and press ·. Now move the cursor to point Q and press ·. There should be an "ghost" image of a number. Move the image to a free part of the screen and lock the number in place by pressing ·. Move the cursor to point B and press ·. Now move the cursor to point Q and press ·. There should be a "ghost" image of a number. Move the image below the first measurement number and lock it in place by

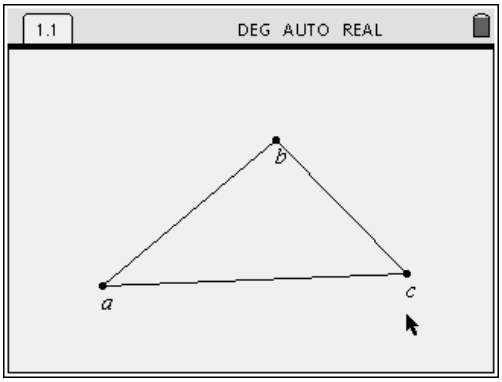


Figure 2

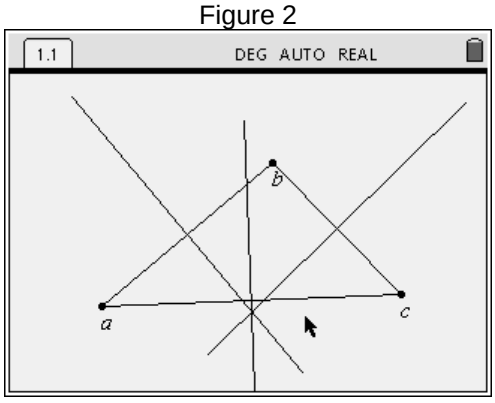


Figure 3

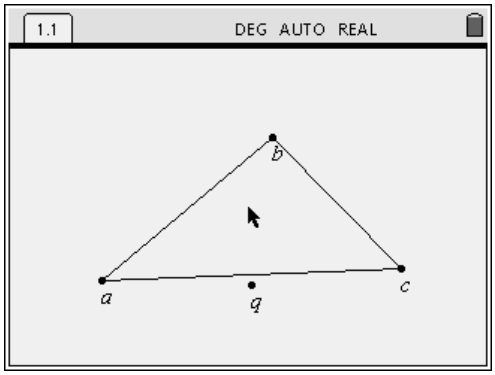


Figure 4

pressing ⊛. Move the cursor to point C and press ⊛. Now move the cursor to point Q and press ⊛. There should be a "ghost" image of a number. Move the image below the other two measurements and lock it in place by pressing the ⊛. (see figure 4)

Now, use the NavPad to move the cursor to point C of the triangle. Click ⊛ and hold for 1.7 seconds until the point is "grabbed". Now translate the point and notice the measurements on the screen. (see figure 5)

What conjecture can you make about the circumcenter of a triangle?

Incenter

Open a new Graphs and Geometry page by pressing ⌂ and scrolling to Graphs and Geometry or simply by pressing 2.

Now, we are going to hide the x & y axes by pressing the menu , 2 : View, 4 :Hide Axes .

Let's hide the entry line as well by pressing menu , 2 : View, 6 : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press ctrl G. This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen.(see figure 6)

Now press menu, 8: Shapes, 2: Triangle.

Using the NavPad, move the cursor to any part of the screen and press ⊛. Before moving the cursor, label the point by pressing the letter A. Now move the cursor to another part of the screen, and press ⊛. Again, before moving the cursor, label the point by pressing B. Move the cursor to another part of the screen and press ⊛. Again, before moving the cursor, label the point by pressing C. Press esc . (see figure 7)

Now press 9: Construction, 4: Angle Bisector. Move the cursor to point B and press ⊛, move the cursor to point A and press ⊛, move the cursor to point C and press ⊛. A segment bisecting angle A should be drawn. (see figure 8) Repeat the process for the other two angles. After all three

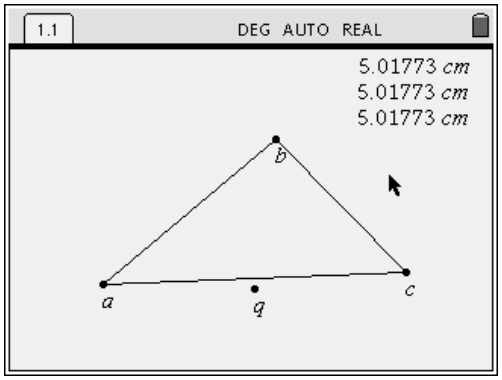


Figure 5

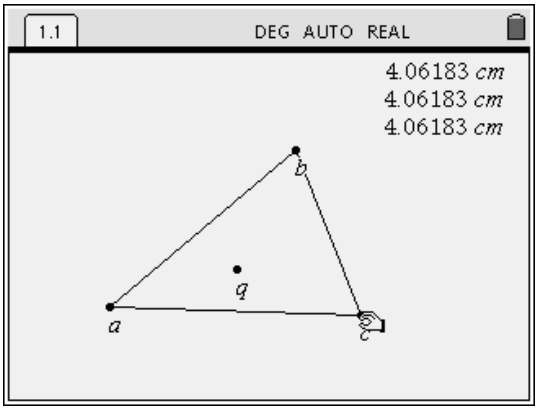


Figure 6

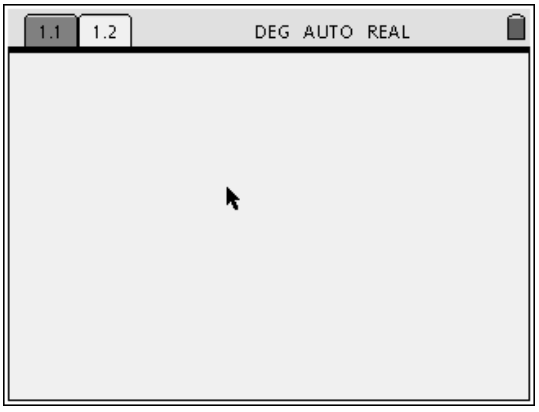


Figure 7

angle bisectors are drawn, press (esc). (see figure 9)

To determine the point of concurrency, we need to find the point of intersection between the angle bisectors. To do this, press (menu), (6) : Points & Lines, (3) : Intersection Points. Move the NavPad to any of the angle bisectors and press (·) . Move the cursor to another angle bisector and press (·). Before you move the cursor press (a).

Now let's hide the angle bisectors. Move the cursor to one of the angle bisectors and press (ctrl)(menu). Scroll down or simply press (3): Hide/Show. Repeat the process for the other two angle bisectors. (see figure 10)

Point "Q" is called the Incenter of the triangle.

Now, use the NavPad to move the cursor to point C of the triangle. Click (·) and hold for 1.7 seconds until the point is "grabbed". Now translate the point. What do you notice?

Extension

Draw a perpendicular segment from the Incenter to each of the sides of the triangle. Find a point of intersection between the line segment and each of the sides. Now measure the distance between the Incenter and each of the points of intersection on the sides of the triangle. What do you notice?

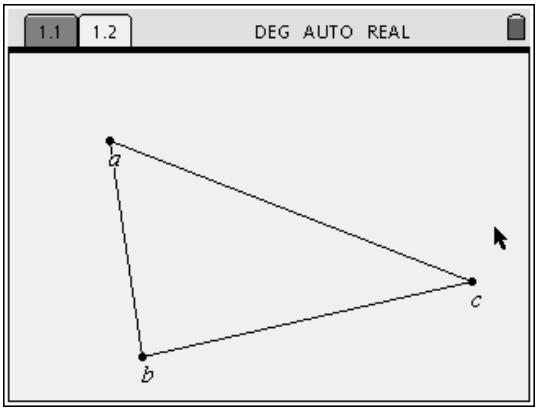


Figure 8

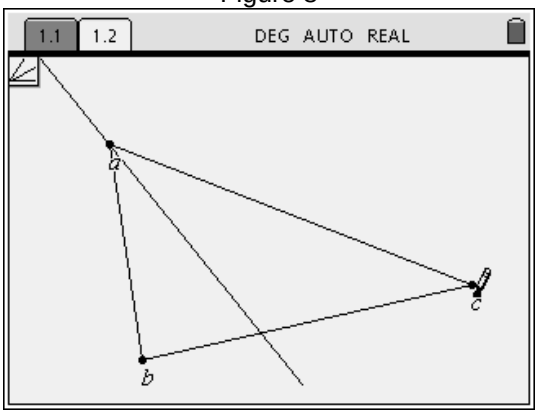


Figure 9

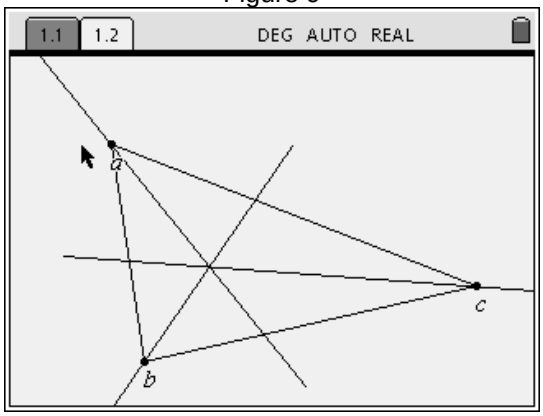


Figure 10

Centroid

Open a new Graphs and Geometry page by pressing (⌂) and scrolling to Graphs and Geometry or simply by pressing (2).

Now, we are going to hide the x & y axes by

pressing the (menu) , (2) : View, (4) :Hide Axes .

Let's hide the entry line as well by pressing (menu) , (2) : View, (6) : Hide Entry Line .

There is also another way to hide the entry line and that is to simply press (ctrl) (G). This will remove the entry line and re-insert it when necessary.

Your screen should be a blank screen.

Now press (menu), (8): Shapes, (2): Triangle.

Using the NavPad, move the cursor to any part of the screen and press (•). Before moving the cursor, label the point by pressing the letter (A). Now move the cursor to another part of the screen, and press (•). Again, before moving the cursor, label the point by pressing (B). Move the cursor to another part of the screen and press (•). Again, before moving the cursor, label the point by pressing (C). Press (esc) . (see figure 11)

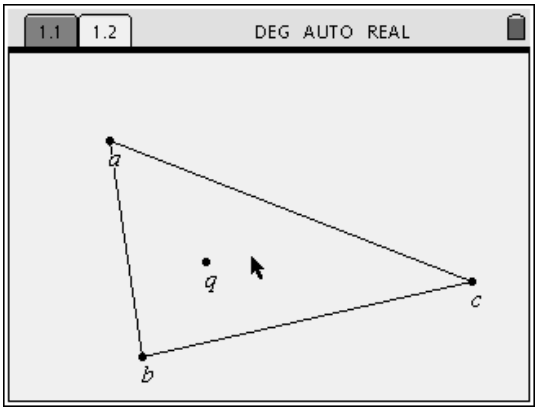


Figure 11

Now, let's determine the midpoints of each of the sides of the triangle. Press (menu), (9): Construction, (5): Midpoint. Move the cursor to point A of the triangle and press (•). Move the cursor to point B and press (•). Since the cursor is already at point B, press (•) and move the cursor to point C. Press (•). Since the cursor is already at point C, press (•) and move the cursor back to point A. Press (•). Press (esc). You triangle should have a midpoint in each of its sides. (see figure 12)

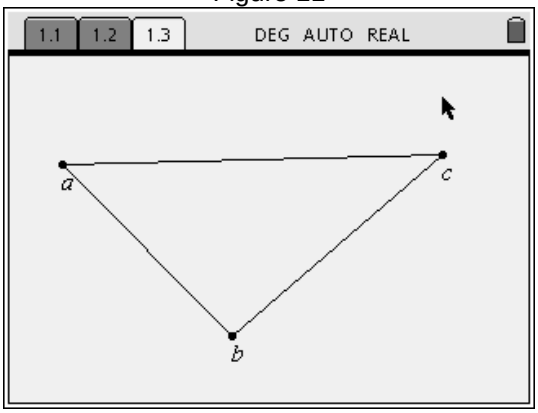


Figure 12

Now, we need to draw the medians of the triangle. Press (menu), (6): Points & Lines, (5): Segment. Move the cursor to point A on the triangle and press (•). Now move the cursor to the midpoint of the side opposite vertex A of the triangle and press (•). Move the cursor to point B on the triangle and press (•). Now move the cursor to the midpoint of the side opposite vertex B of the triangle and press (•). Move the cursor to point C on the triangle and press (•). Now move the cursor to the midpoint of the side opposite vertex C of the triangle and press (•). (see figure 13)

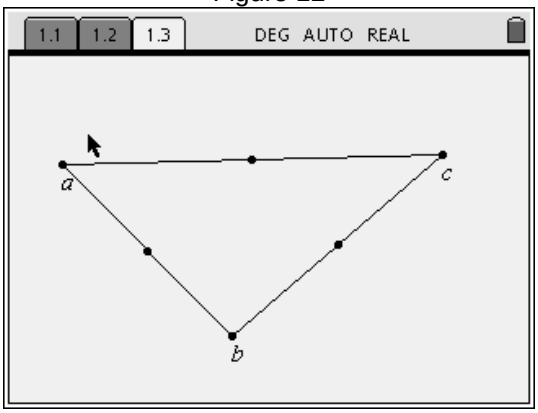


Figure 13

To determine the point of concurrency, we need to find the point of intersection between the medians of the triangle. To do this, press (menu), (6) : Points & Lines, (3) : Intersection Points. Move the NavPad to any of the three medians and press (•) . Move the cursor to another median and press (•). Before you move the cursor press (Q).

Now let's hide the medians. Move the cursor to one of the medians and press ctrl menu. Scroll down or simply press 3: Hide/Show. Repeat the process for the other two medians. (see figure 14)

Point "Q" is called the Centroid of the triangle.

Note: The centroid of a triangle is known as the "center of gravity" because this is where the triangle will balance.

Now, use the NavPad to move the cursor to point C of the triangle. Click and hold for 1.7 seconds until the point is "grabbed". Now translate the point. What do you notice?

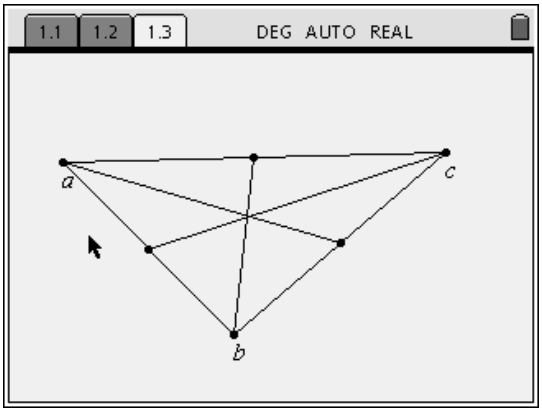


Figure 14