Segments in Circles

ID: 8107

<div style="text-align:right">

Time required
45 minutes

</div>

Topic: Circles

- *Prove and apply the Tangent-Chord Theorem and its converse.*
- *Prove and apply the Chord-Chord, Secant-Secant and the Secant-Tangent Product Theorems.*

Activity Overview

In this activity, students will explore the relationships among special segments in circles. The special segments include tangent segments, segments created by two intersecting chords, and secant segments. Students will store variables and use both the automated and manual data capture features in a spreadsheet and write formulas in spreadsheets to confirm relationships among segments.

Teacher Preparation

- *This activity is designed to be used in a high-school geometry classroom. This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*

- *Students should already be familiar with parts of a circle, including tangents and secants. If needed, introduce the following terms: tangent segment, secant segment, internal secant segment, and external secant segment.*

- *Remind students that secants and tangents are lines and because lines continue indefinitely, they **cannot** be measured. Segments are parts of lines and the distances between their endpoints **can** be measured.*

- *This activity assumes a basic working knowledge of the TI-Nspire device, such as drawing shapes and finding lengths of segments.*

- *Due to the amount of memory that is taken up by using automated data capture, it is recommended that students either not save this file when they are finished or that they clear the data from the spreadsheet before saving (by pressing ENTER twice in the formula cells of the columns containing the data).*

- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8107" in the quick search box.***

Associated Materials

- *SegmentsInCircles.tns*
- *SegmentsInCircles_Student.doc*

This activity allows students to interactively discover, confirm, and explore the theorems that describe the relationships involving different segments in circles. You may need to help students use the handheld's different tools; instructions are provided in this document at point of first use.

Problem 1 – Chords and tangents

Step 1: Have students use the **Circle** tool (⊙) from the Shapes menu to draw a circle on page 1.2. Then instruct them to draw two tangents (**MENU > Points & Lines > Tangent**) to the circle and extend the lines until they intersect. Now tell them to use the **Intersection Point(s)** tool (⤬) from the Points & Lines menu to plot the intersection point.
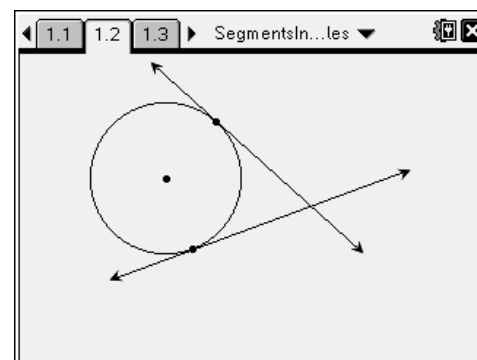
Step 2: Direct students to use the **Length** tool (✐) to measure the distance from this intersection point to the two points of tangency. (After selecting the tool, you must click on each endpoint of a segment to find its length.) Then have them grab the circle and move the cursor to change the size. Have them record their findings on their worksheet. (Students should find that the lengths of the tangent segments remain equal.)
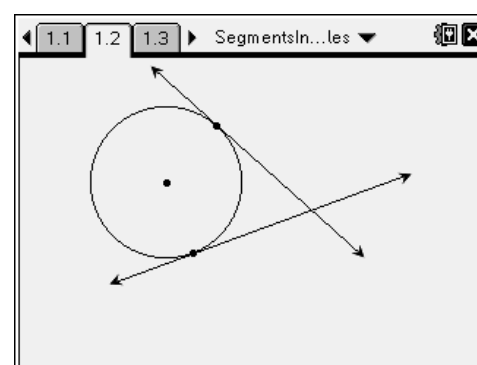
Step 3: On page 1.3, two chords are drawn in a circle. Their lengths have already been measured and stored as variables as shown.
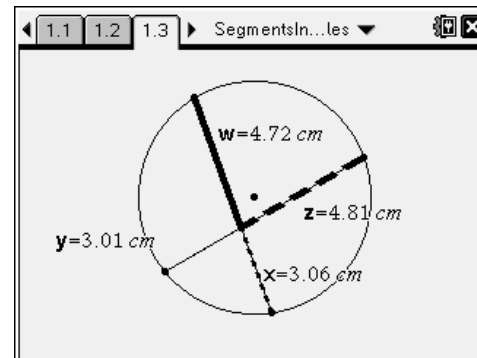
Step 4: Have students move to page 1.4 and set up the spreadsheet to automatically capture each length. In the gray formula cell in Column A, they should select **MENU > Data > Data Capture > Automated Data Capture** and enter *w* as the variable to be captured. (The "1" that follows each variable indicates that the data is being captured *automatically*, rather than manually.)

When prompted, have them choose *variable reference* (instead of *column reference*) to capture the values of the variables. Otherwise, the device looks for values in Column W.

Repeat the capture for Columns B, C, and D and variables *x*, *y*, and *z*, respectively.

If desired, students can name each list at the top of the column, such as *wlist*, *xlist*, etc.

Step 5: Tell students to enter formulas into the gray cells of Columns E and F to find the products of the measures of the segments of each chord, that is, $w \cdot x$ and $y \cdot z$. To find the product of the contents in Column A with the corresponding value in Column B, enter **=a[]*b[]**. (Note: If students named the lists, they can use the column names. For example, they may write =wlist*xlist.)

Insert the brackets by pressing (ctrl) + (( ) (the left parenthesis). The right bracket automatically appears when the button is pressed.

Students may make a particular column wider by selecting **MENU > Actions > Resize**.

Step 6: Have students return to page 1.3 and slowly change the size of the circle. The new lengths are "captured" and recorded in the spreadsheet as they do this. Instruct them to return to the spreadsheet and look at the data in Columns E and F to make a conjecture. (When two chords intersect, the products of the lengths of the segments of each chord are equal.)

Problem 2 – Intersecting secants and tangents

Step 1: Instruct students to draw a circle on page 2.1. Then have them use the **Line** tool (⌐) and the **Tangent** tool (⌐) to draw a secant and tangent that intersect outside the circle. Once again, students should plot the point of intersection using the **Intersection Point(s)** tool. (All three of these tools are available from the Points & Lines menu.)

Step 2: Have students again use the **Length** tool to measure the length of the tangent segment and the external and internal secant segments. Store each value as a variable (use t for the tangent segment, se for the external secant segment and si for the internal secant segment) by doing the following: click once on a value to select it, press (ctrl) + (var), enter the variable name, and press (enter). Remind students to press (esc) to deselect each value before attempting to store another.

Have them conjecture the relationship between the different segment lengths before continuing.
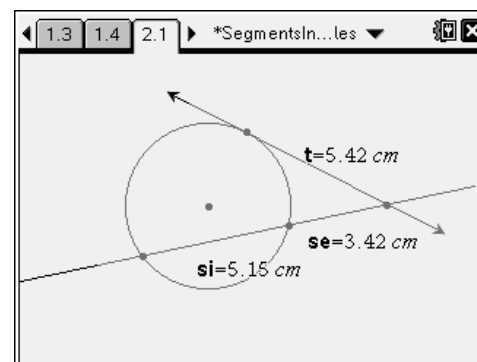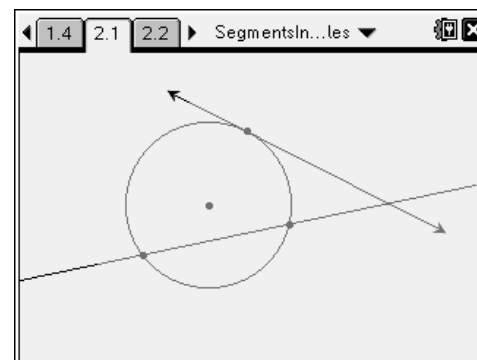
Step 3: Help students set up the spreadsheet on page 2.2 to manually capture each length. For Column A, have them select **MENU > Data > Data Capture > Manual Data Capture** and enter *t* as the variable to be captured. (A "0" now follows each variable, indicating that the data will be captured manually.)

Repeat the capture for Columns B and C and variables *se* and *si*, respectively. Notice that the current values have not been filled in yet. They will need to be collected manually.

Step 4: In Columns D and E, have students enter formulas to find the square of the tangent segment and the product of the *entire* secant segment and the external secant segment. Again, students may wish to **Resize** these two columns.

Step 5: Returning to page 2.1, tell students that they must press ⓒₜᵣₗ + (.) each time they wish to capture specific data values. Have them press ⓒₜᵣₗ + (.) once and look at the spreadsheet to verify that those values have been captured. Then have them change the size of the circle (capturing at least five different data values), refer to the calculations using the captured data (collected in Columns D and E), and record their findings on the worksheet. Ask why this theorem is often called "outside squared = outside × whole."

**Step 6:** Page 2.3 displays a diagram that shows how the segment lengths are related to the sides of rectangles. If time permits, have students study the picture and drag the circle, discussing how the theorem is related to the areas of the rectangles.

Selecting **Hide/Show** from the **Tools** menu will show how the rectangles were constructed, thereby helping students to see that the width of the bottom rectangle equals the length of the external secant segment while the top rectangle is actually a square. You can also move the circle and see that the area of the rectangle is always equal to that of the square.
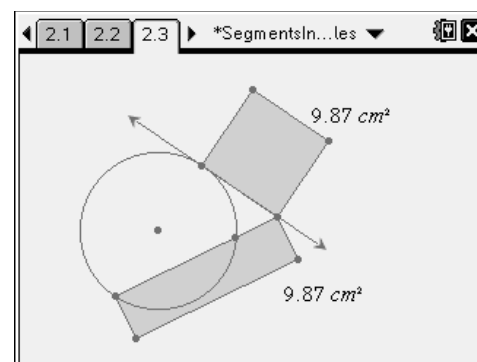
Step 7: On page 2.4, instruct students to draw a circle, two secants that intersect outside the circle, and their point of intersection. Have them measure the **Length** of each of the four secant segments and **Store** them as variables. (Note that they cannot use the variables *se* or *si* because those were already used in Problem 2. Suggest students use *w*, *x*, *y*, and *z*.) Walk around the room to assist students as needed.
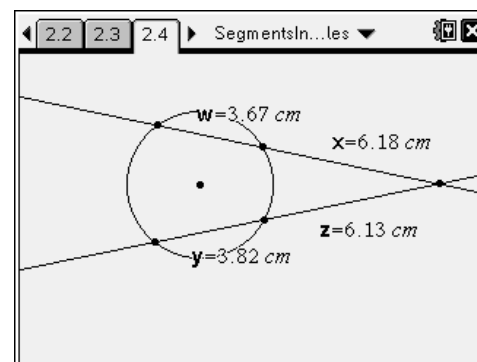
Step 8: Instruct students to use the spreadsheet on page 2.5 to prove that the product of the lengths of one entire secant segment and its external secant segment is equal to the product of the lengths of the other entire secant segment and its external secant segment. Students should capture (either automated or manual) the values of *w–z* in Columns A–D, respectively, and set up the formulas in Columns E and F as shown. Assist them as needed. Share that this theorem is often referred to as "outside × whole = outside × whole."

Step 9: In the above problem, students had choices in which lengths to measure. Instead of the way shown here, they could have found the lengths of the two external secant segments and the lengths of the entire secant segments. As in the previous problem, rectangles with the dimensions of *whole secant segment* by *external secant segment* have the same areas. Students who finish early can be challenged to construct those rectangles.

Problem 3 – Extension

On page 3.2, students are asked to construct a circle, a radius, and a tangent to the circle such that the point of tangency is the intersection of the radius and circle. Have them measure the angle that is formed by selecting **MENU > Measurement > Angle** (they may need to change their settings to *Degree* mode: and choose **Settings and Status > Settings > General**). Have students grab the point of tangency and move it around the circle. They should find that the angle always remains a right angle.
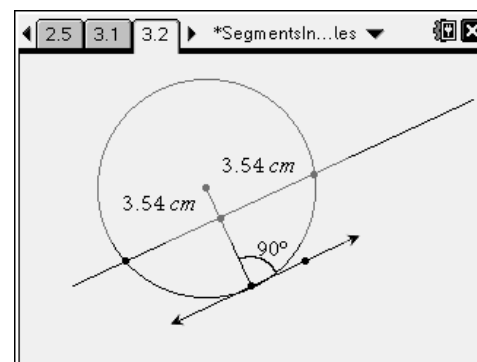
Next, have students construct a perpendicular to the radius, using the **Perpendicular** tool ( ) from the Construction menu. Have them plot the points of intersection of this line with the circle and then measure the lengths of each segment of the chord that is formed. Students should find these lengths to be equal, even as the perpendicular line is moved across the length of the radius!