Optimization

ID: 8996

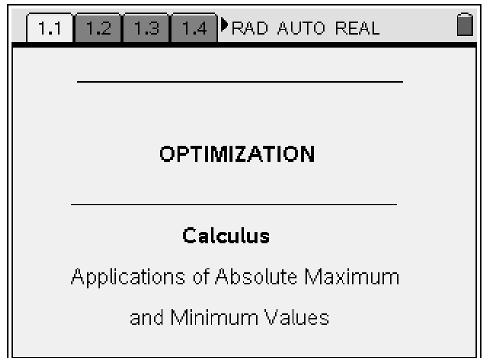
Name ______________________

Class ______________________

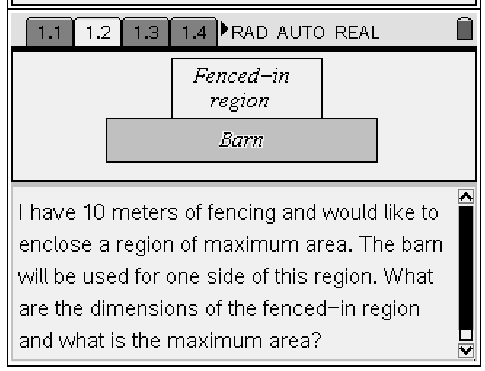
In this activity, you will explore:

- *optimization problems*

Open the file *CalcAct06_Optimization_EN.tns* on your handheld and follow along with your teacher for the first two screens. Use this document as a guide to the activity and as a place to record your answers. Move to page 1.2 and wait for further instructions from your teacher.

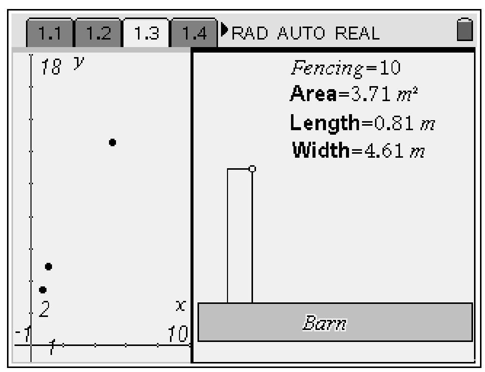


Page 1.2 (shown at right) displays a rectangular area enclosed on three sides by a fence and one side by a barn. Consider that there a 10 meters of fence material to work with. What are the dimensions that will produce the maximum area?



Problem 1 – Maximizing the area of a fenced-in region

Advance to page 1.3. You will see a screen similar to the one at right, with the right side showing the fenced in area and the left side showing a scatter plot of length verses area. The fence length has been locked at 10 meters. Notice that the length, width, and area of the rectangular region are also displayed on the screen. Also note that three points (*length, area*) have already been plotted on the grid.



Since the rectangle has a small length, it makes sense to drag the vertex with the open circle slowly to the right to increase this dimension.

- What do you think will happen to the width and area of the rectangle as you do this?

- Can you tell where additional data points will fall on the scatter plot?

Drag the rectangle and periodically press ctrl + . to manually capture the length and area of the rectangle. Notice that this action inserts a scatter plot point on the grid. Continue alternately dragging the rectangle and pressing ctrl + . until the length is as large as possible.

You should notice a pattern emerge in the scatter plot of length verses area.

- What does the domain of this scatter plot appear to be? Explain.

- What dimensions (length and width) appear to produce the maximum area?

- What shape does the scatter plot appear to be? Explain.

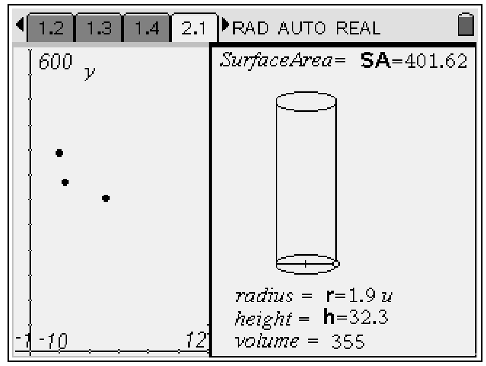
The next step is to find a symbolic model for the length verses area model and use calculus to find the dimensions that produce the maximum area.

The area of the rectangle is given by length times width, or $A = length \cdot width$. We want to find an expression for the area in terms of one variable, preferably *length*, since this is the independent variable that was used in the scatter plot. Use the fact that the fence length is locked at 10 meters to write a second equation in terms of *length* and *width*. Solve this equation for *width* and use substitution to write your area formula in terms of *length* only. Write your equation here.

Verify your equation is correct by graphing it as **f1**(x) on page 1.3.(Note: Use x instead of *length* for the purpose of graphing the equation.) Does your equation match the scatter plot?

Finally, use methods from calculus to find the value of l that produces the absolute maximum on the interval given by the domain of your area function. Show your work here.

Problem 2 – Minimizing the surface area of a cylinder



Advance to page 2.1. You will see a screen similar to the one at right, showing a cylinder and a grid where a scatter plot of radius verses surface area will be graphed. The volume has been locked at 355 cm^3. Notice that the radius, height, and surface area of the cylinder are also displayed on the screen. Also note that three points (*radius, surface area*) have already been plotted on the grid. Why do you think it will be advantageous to minimize the surface area of the cylinder for this given volume?

Since the cylinder is tall and thin, it makes sense to drag the point with the open circle to make the cylinder shorter and wider.

- What do you think will happen to the surface area of the cylinder as you do this?

- Can you tell where additional data points will fall on the scatter plot?

Drag the cylinder and periodically press ctrl + . to manually capture the radius and surface area of the cylinder. Notice that this action inserts a scatter plot point on the grid. Continue alternately dragging the cylinder and pressing ctrl + . until the radius is as large as possible.

You should notice a pattern emerge in the scatter plot of radius verses surface area.

- What does the domain of this scatter plot appear to be? Explain.

- What dimensions (radius and height) appear to produce the minimum surface area?

- What does the minimum surface area appear to be?

As in Problem 1, you will use calculus to find the minimum surface area of this figure. Find a formula for the surface area in terms of radius and height. Then use the fact that the volume is constant to eliminate *h* from this equation and write it strictly in terms of *r*. Graph this equation in **f1**(*x*) on page 2.1, using *x* in place of the variable *r*.

- Does **f1**(*x*) pass through the scatter plot as you would expect?

Use calculus to find the value of *r* that corresponds to the absolute minimum value of the surface area function.

- What is the minimum surface area?

Find the height of the cylinder corresponding to the value of *r* that gives the minimum surface area.

- How are these two quantities related?

- Explain how you can generalize this relationship.

