

Texas Instruments Activity #12

Title: Taking Discontinuity Away

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Estimated Time: 40-50 Minutes

NCTM Standards:

Connections Standard – Recognize and apply mathematics in contexts outside of mathematics. Use representations to model and interpret physical, social and mathematical phenomena.

Problem Solving Standard – Solve problems that arise in mathematics and other contexts.

Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Topics in Calculus:

Limits, Continuity, Functions

Overview:

In this activity, the students will use the TI-89 graphing calculator to find points of discontinuity of a function, and then create a new function that corrects the discontinuity. This method allows students to complete the assignment with or without the use of the graphing calculator.

Supplies: TI-89 Graphing Calculator

TAKING DISCONTINUITY AWAY

The following steps are used to make discontinuous functions continuous. In this activity, you will make several discontinuous functions continuous. While the process looks to be involved, it is a collection of small steps. Since you will be using a variety of tools on the TI-89, the directions are detailed to walk you through each step.

STEP ONE: Find the domain of f . (Factor the denominator if applicable. To use the TI-89 to factor the denominator, press **[F2]**, select 2:Factor, and press **[ENTER]**. factor(expression **[]** variable **[]** **[ENTER]**).

STEP TWO: Graph the equation and view the area around the point of discontinuity. Press **[♦]**[Y=] and enter the function in Y1=. Press **[♦]** [WINDOW] to set a window to view the point of discontinuity.

STEP THREE: Determine how to define f at the point of discontinuity. Press **[♦]** [TABLE] to view the table of the function values. What value logically “connects” the function (to make it continuous)?

STEP FOUR: Show that the one of the factors of the denominator is also a factor of the numerator. You can use the TI-89’s CAS to find this. Simply type the *numerator* in, on the home screen, followed by **[÷]** *factor of the denominator*, and press **[ENTER]**.

STEP FIVE: Compute the limit of this new function as x approaches the point of discontinuity. Press **[F3]** and select 4:limit(. Press **[◀]** key to highlight the appropriate function and press **[ENTER]** to select the function. Then, press **[,]** **[X]** **[,]** *approaching value* **[]** **[ENTER]** to find the limit. This limit value is the value needed for the function to be continuous.

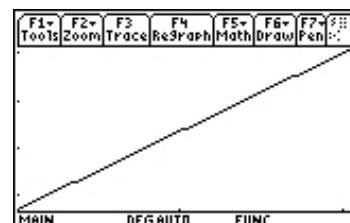
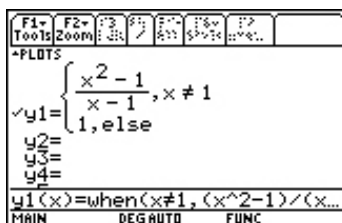
STEP SIX: Write the extended function. In order to write this function, we use the original function everywhere except at the point of discontinuity. The second part of the function is the point from **STEP FIVE** at the value where the original function was discontinuous. This function should be a piecewise function and look similar to:

$$g(x) = \begin{cases} \text{function} & x \neq \text{value} \\ \text{new value} & x = \text{value} \end{cases}$$

STEP SEVEN: Graph the function. You can graph piecewise functions using your TI-89 graphing calculator by pressing **[♦]** [GRAPH] to open the Y= Editor. Select y1= and press **[ENTER]**. Enter the expression as it appears: (To select when(, press **[CATALOG]** and scroll to when(, by pressing **[ENTER]**.)

$$y1(x)=\text{when}(x \neq \text{value}, \text{function}, \text{new value})$$

For example:



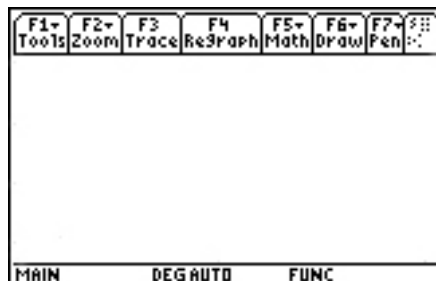
For each of the following functions, determine whether there is a point of discontinuity and create a new function $g(x)$ using $f(x)$ such that $g(x)$ is continuous for all points. Write the new function on the space provided and sketch the graph of the original function $f(x)$ and $g(x)$.

1. $f(x) = \frac{1}{(x+3)^2}$

New function:



Original Function's Graph



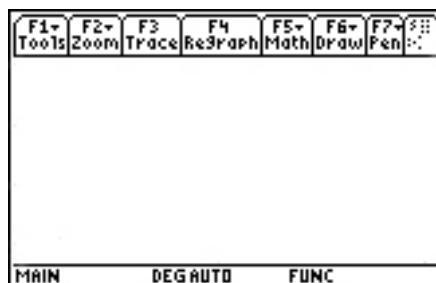
New Function's Graph

2. $f(x) = |x|/x$

New function:



Original Function's Graph



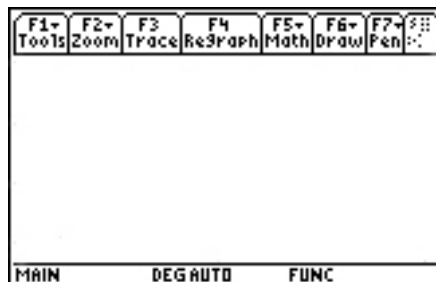
New Function's Graph

3. $f(x) = \frac{x^2 - 9}{x + 3}$

New function:



Original Function's Graph



New Function's Graph

4. Are there any cases in which discontinuity cannot be corrected? Why? _____
